Edexcel GCE
Core Mathematics C1
Advanced Subsidiary
Monday 14 January 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Calculators may NOT be used in this examination.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 11 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. Factorise completely \( x - 4x^3 \) (3 marks)

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(Total 3 marks)
2. Express $8^{2x+3}$ in the form $2^y$, stating y in terms of x.

(Total 2 marks)
3. (i) Express 

\[(5 - \sqrt{8})(1 + \sqrt{2})\]

in the form \(a + b\sqrt{2}\), where \(a\) and \(b\) are integers. (3)

(ii) Express 

\[\sqrt{80} + \frac{30}{\sqrt{5}}\]

in the form \(c\sqrt{5}\), where \(c\) is an integer. (3)
Question 3 continued

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(Total 6 marks)
4. A sequence \( u_1, u_2, u_3, \ldots \) satisfies
\[
    u_{n+1} = 2u_n - 1, \quad n \geq 1
\]

Given that \( u_2 = 9 \),

(a) find the value of \( u_3 \) and the value of \( u_4 \),

(b) evaluate \( \sum_{r=1}^{4} u_r \).
Question 4 continued

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5. The line $l_1$ has equation $y = -2x + 3$

The line $l_2$ is perpendicular to $l_1$ and passes through the point (5, 6).

(a) Find an equation for $l_2$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

The line $l_2$ crosses the $x$-axis at the point $A$ and the $y$-axis at the point $B$.

(b) Find the $x$-coordinate of $A$ and the $y$-coordinate of $B$.

Given that $O$ is the origin,

(c) find the area of the triangle $OAB$. 

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Question 5 continued
6.

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}, \ x \neq 0$

The curve $C$ has equation $y = \frac{2}{x} - 5, \ x \neq 0$, and the line $l$ has equation $y = 4x + 2$

(a) Sketch and clearly label the graphs of $C$ and $l$ on a single diagram.

On your diagram, show clearly the coordinates of the points where $C$ and $l$ cross the coordinate axes.

(5)

(b) Write down the equations of the asymptotes of the curve $C$.

(2)

(c) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$

(5)
Question 6 continued
Question 6 continued
Question 6 continued

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(Total 12 marks)
Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship. (2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships. (3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured \(n\) dragons and the total number of points that she scored for capturing all \(n\) dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her \(n\)th dragon,

(c) find the value of \(n\). (3)
Question 7 continued
Question 7 continued

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8. \[
\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \quad x \neq 0
\]

Given that \( y = 7 \) at \( x = 1 \), find \( y \) in terms of \( x \), giving each term in its simplest form. (6)
Question 8 continued

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(Total 6 marks)
9. The equation

\[(k + 3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}\]

has two distinct real solutions for \(x\).

(a) Show that \(k\) satisfies

\[k^2 - 2k - 24 < 0\]

(b) Hence find the set of possible values of \(k\).
Question 9 continued
Question 9 continued
Question 9 continued

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10. \[4x^2 + 8x + 3 = a(x + b)^2 + c\]

(a) Find the values of the constants \(a\), \(b\) and \(c\).

(b) On the axes on page 27, sketch the curve with equation \(y = 4x^2 + 8x + 3\), showing clearly the coordinates of any points where the curve crosses the coordinate axes.
Question 10 continued

\[ x \quad O \quad y \]

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Question 10 continued

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(Total 7 marks)
11. The curve $C$ has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geq 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form. (3)

The point $P$ on $C$ has $x$-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to $C$ at the point $P$, giving your answer in the form $y = ax + b$, where $a$ and $b$ are constants. (4)

The tangent to $C$ at the point $Q$ is parallel to the line with equation $2x - 3y + 18 = 0$

(c) Find the coordinates of $Q$. (5)
Question 11 continued
Edexcel GCE
Core Mathematics C2
Advanced Subsidiary
Monday 14 January 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
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Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
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There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. Find the first 3 terms, in ascending powers of $x$, in the binomial expansion of

$$(2 - 5x)^6$$

Give each term in its simplest form.

(Total 4 marks)
2. \( f(x) = ax^3 + bx^2 - 4x - 3 \), where \( a \) and \( b \) are constants.

Given that \( (x - 1) \) is a factor of \( f(x) \),

(a) show that

\[
    a + b = 7
\]

Given also that, when \( f(x) \) is divided by \( (x + 2) \), the remainder is 9,

(b) find the value of \( a \) and the value of \( b \), showing each step in your working.
Question 2 continued

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(Total 6 marks)
3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

(b) Find the first year in which the yearly predicted profit exceeds £200 000

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.
Question 3 continued
Question 3 continued
4. Solve, for $0 \leq x < 180^\circ$,

$$\cos(3x - 10^\circ) = -0.4$$

giving your answers to 1 decimal place. You should show each step in your working. (7)
Question 4 continued

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(Q4) (Total 7 marks)
5. The circle $C$ has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of $C$ is at the point $M$.

(a) Find

(i) the coordinates of the point $M$,

(ii) the radius of the circle $C$. (5)

$N$ is the point with coordinates $(25, 32)$.

(b) Find the length of the line $MN$. (2)

The tangent to $C$ at a point $P$ on the circle passes through point $N$.

(c) Find the length of the line $NP$. (2)
Question 5 continued
6. Given that \(2 \log_2(x+15) - \log_2 x = 6\)

(a) Show that
\[x^2 - 34x + 225 = 0\] (5)

(b) Hence, or otherwise, solve the equation
\[2 \log_2(x+15) - \log_2 x = 6\] (2)
Question 6 continued
7.

The triangle $XYZ$ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$. The point $W$ lies on the line $XY$.

The circular arc $ZW$, in Figure 1 is a major arc of the circle with centre $X$ and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(b) Find the area, in cm$^2$, of the major sector $XZWX$.

(c) The region enclosed by the major arc $ZW$ of the circle and the lines $WY$ and $YZ$ is shown shaded in Figure 1.

Calculate

(d) the area of this shaded region,

(e) the perimeter $ZWYZ$ of this shaded region.
Question 7 continued
Question 7 continued

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(Total 12 marks)
8. The curve $C$ has equation $y = 6 - 3x - \frac{4}{x^3}, \ x \neq 0$

(a) Use calculus to show that the curve has a turning point $P$ when $x = \sqrt{2}$  

(b) Find the $x$-coordinate of the other turning point $Q$ on the curve. 

(c) Find $\frac{d^2y}{dx^2}$. 

(d) Hence or otherwise, state with justification, the nature of each of these turning points $P$ and $Q$. 

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Question 8 continued

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Question 8 continued
9. The finite region $R$, as shown in Figure 2, is bounded by the $x$-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \quad x > 0$$

The curve crosses the $x$-axis at the points $(1, 0)$ and $(4, 0)$.

(a) Complete the table below, by giving your values of $y$ to 3 decimal places.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>5.866</td>
<td>5.210</td>
<td>1.856</td>
<td>0</td>
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</tbody>
</table>

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of $R$, giving your answer to 2 decimal places.

(c) Use integration to find the exact value for the area of $R$. 

Question 9 continued

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Edexcel GCE
Core Mathematics C3
Advanced
Friday 25 January 2013 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. The curve $C$ has equation

$$y = (2x - 3)^5$$

The point $P$ lies on $C$ and has coordinates $(w, -32)$.

Find

(a) the value of $w$,  

(b) the equation of the tangent to $C$ at the point $P$ in the form $y = mx + c$, where $m$ and $c$ are constants.
Question 1 continued

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(Total 7 marks)
2. \[ g(x) = e^{x-1} + x - 6 \]

(a) Show that the equation \( g(x) = 0 \) can be written as

\[ x = \ln(6 - x) + 1, \quad x < 6 \]  

(2)

The root of \( g(x) = 0 \) is \( a \).

The iterative formula

\[ x_{n+1} = \ln(6 - x_n) + 1, \quad x_0 = 2 \]

is used to find an approximate value for \( a \).

(b) Calculate the values of \( x_1 \), \( x_2 \) and \( x_3 \) to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that \( a = 2.307 \) correct to 3 decimal places.

(3)
Question 2 continued

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3. **Figure 1**

Figure 1 shows part of the curve with equation \( y = f(x), \ x \in \mathbb{R} \).

The curve passes through the points \( Q(0,2) \) and \( P(-3,0) \) as shown.

(a) Find the value of \( f(-3) \).

(2)

On separate diagrams, sketch the curve with equation

(b) \( y = f^{-1}(x) \),

(2)

(c) \( y = f(|x|) - 2 \),

(2)

(d) \( y = 2f\left(\frac{1}{3}x\right) \).

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.
Question 3 continued
Question 3 continued
Question 3 continued
4. (a) Express $6 \cos \theta + 8 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of $\alpha$ to 3 decimal places.

(b) \[ p(\theta) = \frac{4}{12 + 6 \cos \theta + 8 \sin \theta}, \quad 0 \leq \theta \leq 2\pi \]

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of $\theta$ at which the maximum occurs.
Question 4 continued
Question 4 continued

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Question 4 continued
5. (i) Differentiate with respect to $x$

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(ii) Given that $x = \cot y$,

show that $\frac{dy}{dx} = \frac{-1}{1 + x^2}$
Question 5 continued

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Question 5 continued
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(Q5) (Total 11 marks)
6.  (i) Without using a calculator, find the exact value of

\[(\sin 22.5^\circ + \cos 22.5^\circ)^2\]

You must show each stage of your working.

(ii) (a) Show that \(\cos 2\theta + \sin \theta = 1\) may be written in the form

\[k \sin^2 \theta - \sin \theta = 0\]

stating the value of \(k\).

(b) Hence solve, for \(0 \leq \theta < 360^\circ\), the equation

\[\cos 2\theta + \sin \theta = 1\]
Question 6 continued
7. \[ h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0 \]

(a) Show that \( h(x) = \frac{2x}{x^2+5} \) \hspace{1cm} (4)

(b) Hence, or otherwise, find \( h'(x) \) in its simplest form. \hspace{1cm} (3)

\[ y = h(x) \]

Figure 2 shows a graph of the curve with equation \( y = h(x) \).

(c) Calculate the range of \( h(x) \). \hspace{1cm} (5)
Question 7 continued

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Question 7 continued
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(Question 7 continued)

(Total 12 marks)
8. The value of Bob’s car can be calculated from the formula

\[ V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500 \]

where \( V \) is the value of the car in pounds (£) and \( t \) is the age in years.

(a) Find the value of the car when \( t = 0 \)

(b) Calculate the exact value of \( t \) when \( V = 9500 \)

(c) Find the rate at which the value of the car is decreasing at the instant when \( t = 8 \).
   Give your answer in pounds per year to the nearest pound.
Edexcel GCE
Core Mathematics C4
Advanced
Monday 28 January 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. Given

\[ f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3} \]

find the binomial expansion of \( f(x) \), in ascending powers of \( x \), up to and including the term in \( x^3 \).

Give each coefficient as a simplified fraction.

(5)
Question 1 continued

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(Total 5 marks)
2. (a) Use integration to find
\[ \int \frac{1}{x^3} \ln x \, dx \]  

(b) Hence calculate
\[ \int_{1}^{2} \frac{1}{x^3} \ln x \, dx \]
Question 2 continued

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(Total 7 marks)
3. Express \( \frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \) in partial fractions.

(4)
Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of $y$ corresponding to $x = 3$, giving your answer to 4 decimal places.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.5</td>
<td>0.8284</td>
<td>1.3333</td>
<td></td>
</tr>
</tbody>
</table>

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an estimate of the area of the region $R$, giving your answer to 3 decimal places.

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of $R$. 

[Blank lines for additional comments or calculations]
Question 4 continued

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(Total 12 marks)
5.

Figure 2 shows a sketch of part of the curve $C$ with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2t - 1$$

The curve crosses the $y$-axis at the point $A$ and crosses the $x$-axis at the point $B$.

(a) Show that $A$ has coordinates $(0, 3)$. (2)

(b) Find the $x$ coordinate of the point $B$. (2)

(c) Find an equation of the normal to $C$ at the point $A$. (5)

The region $R$, as shown shaded in Figure 2, is bounded by the curve $C$, the line $x = -1$ and the $x$-axis.

(d) Use integration to find the exact area of $R$. (6)
Question 5 continued
Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where $x$ is measured in radians. The curve crosses the $x$-axis at the point $A$ and at the point $B$.

(a) Find, in terms of $\pi$, the $x$ coordinate of the point $A$ and the $x$ coordinate of the point $B$.  

(3)

The finite region $S$ enclosed by the curve and the $x$-axis is shown shaded in Figure 3. The region $S$ is rotated through $2\pi$ radians about the $x$-axis.

(b) Find, by integration, the exact value of the volume of the solid generated.

(6)
Question 6 continued
Question 6 continued
Question 6 continued

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(Total 9 marks)
7. With respect to a fixed origin \( O \), the lines \( l_1 \) and \( l_2 \) are given by the equations

\[
l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})
\]

\[
l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})
\]

where \( \lambda \) and \( \mu \) are scalar parameters.

(a) Given that \( l_1 \) and \( l_2 \) meet, find the position vector of their point of intersection. (5)

(b) Find the acute angle between \( l_1 \) and \( l_2 \), giving your answer in degrees to 1 decimal place. (3)

Given that the point \( A \) has position vector \( 4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k} \) and that the point \( P \) lies on \( l_1 \) such that \( AP \) is perpendicular to \( l_1 \),

(c) find the exact coordinates of \( P \). (6)
Question 7 continued
Question 7 continued
8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and \( t \) minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is \( \theta \) °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

\[
\frac{d\theta}{dt} = \frac{3 - \theta}{125}
\]

(a) By solving the differential equation, show that,

\[
\theta = Ae^{-0.008t} + 3
\]

where \( A \) is a constant. (4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

(b) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute. (5)
Question 8 continued
Question 8 continued
Question 8 continued

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Turn over
Paper Reference(s)

6689/01

Edexcel GCE

Decision Mathematics D1

Advanced/Advanced Subsidiary

Wednesday 23 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination

Nil

Items included with question papers

D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.

Check that you have the correct question paper.

Answer ALL the questions.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 7 questions in this question paper. The total mark for this paper is 75.

There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You should show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.
Write your answers in the D1 answer book for this paper.

1.

![Flow Chart]

Figure 1

Hero’s algorithm for finding a square root is described by the flow chart shown in Figure 1.

Given that \( N = 72 \) and \( E = 8 \),

(a) use the flow chart to complete the table in the answer book, working to at least seven decimal places when necessary. Give the final output correct to seven decimal places.

(4)

The flow chart is used with \( N = 72 \) and \( E = -8 \),

(b) describe how this would affect the output.

(1)

(c) State the value of \( E \) which cannot be used when using this flow chart.

(1)

(Total 6 marks)
2. (a) Starting with a list of all the letters of the alphabet in alphabetical order, demonstrate how a binary search is used to locate the letter P. In each iteration, you must make clear your pivot and the part of the list you are retaining.

(b) Find the maximum number of iterations needed to locate any particular letter of the alphabet. Justify your answer.

(Total 6 marks)

3. Figure 2 shows the possible allocations of six workers, Charlie (C), George (G), Jack (J), Nurry (N), Olivia (O) and Rachel (R), to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 3 shows an initial matching.

(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You should give the alternating path you use and list your improved matching.

(b) Explain why it is not possible to find a complete matching.

After training, Charlie adds task 5 to his possible allocations.

(c) Taking the improved matching found in (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. Give the alternating path you use and list your complete matching.

(Total 8 marks)
4.

(a) Explain what is meant, in a network, by the term **path**.

(2)

Figure 4 represents a network of canals. The number on each arc represents the length, in miles, of the corresponding canal.

(b) Use Dijkstra’s algorithm to find the shortest path from S to T. State your path and its length.

(6)

(c) Write down the length of the shortest path from S to F.

(1)

Next week the canal represented by arc AB will be closed for dredging.

(d) Find a shortest path from S to T avoiding AB and state its length.

(2)

(Total 11 marks)
Figure 5

[The weight of the network is 379]

Figure 5 represents the roads in a highland wildlife conservation park. The vertices represent warden stations. The number on each arc gives the length, in km, of the corresponding road.

During the winter months the park is closed. It is only necessary to ensure road access to the warden stations.

(a) Use Prim’s algorithm, starting at A, to find a minimum connector for the network in Figure 5. You must state the order in which you include the arcs.

(b) Given that it costs £80 per km to keep the selected roads open in winter, calculate the minimum cost of ensuring road access to all the warden stations.

At the end of winter, Ben inspects all the roads before the park re-opens. He needs to travel along each road at least once. He will start and finish at A, and wishes to minimise the length of his route.

(c) Use the route inspection algorithm to find the roads that will be traversed twice. You must make your method and working clear.

(d) Find the length of the shortest inspection route.

If Ben starts and finishes his inspection route at different warden stations, a shorter inspection route is possible.

(e) Determine the two warden stations Ben should choose as his starting and finishing points in order that his route has minimum length. Give a reason for your answer and state the length of the route.

(Total 15 marks)
Lethna is producing floral arrangements for an awards ceremony. She will produce two types of arrangement, Celebration and Party. Let $x$ be the number of Celebration arrangements made. Let $y$ be the number of Party arrangements made.

Figure 6 shows three constraints, other than $x, y \geq 0$

The rejected region has been shaded.

Given that two of the three constraints are $y \leq 30$ and $x \leq 60$,

(a) write down, as an inequality, the third constraint shown in Figure 6.

(2)
Each Celebration arrangement includes 2 white roses and 4 red roses. Each Party arrangement includes 1 white rose and 5 red roses.

Lethna wishes to use at least 70 white roses and at least 200 red roses.

(b) Write down two further inequalities to represent this information. (3)

(c) Add two lines and shading to Diagram 1 in the answer book to represent these two inequalities. (2)

(d) Hence determine the feasible region and label it R. (1)

The times taken to produce each Celebration arrangement and each Party arrangement are 10 minutes and 4 minutes respectively. Lethna wishes to minimise the total time taken to produce the arrangements.

(e) Write down the objective function, T, in terms of \( x \) and \( y \). (1)

(f) Use point testing to find the optimal number of each type of arrangement Lethna should produce, and find the total time she will take. (4)

(Total 13 marks)
7.

Figure 7 is the activity network relating to a building project. The activities are represented by the arcs. The number in brackets on each arc gives the time to complete the activity. Each activity requires one worker.

The project **must** be completed in the shortest possible time.

(a) Explain the reason for the dotted line from event 4 to event 6 as shown in Figure 7. 

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(c) State the critical activities.

(d) Calculate the total float for activity G. You must make the numbers you use in your calculation clear.

(e) Draw a Gantt chart for this project on the grid provided in the answer book.

(f) State the activities that **must** be happening at time 5.5

(g) Use your Gantt chart to determine the minimum number of workers needed to complete the project in the minimum time. You must justify your answer.

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END
**Edexcel GCE**

**Decision Mathematics D1**

**Advanced/Advanced Subsidiary**

Wednesday 23 January 2013 – Morning

**Answer Book**

Do not return the question paper with the answer book
1. You may not need to use all the rows in this table

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<th>E</th>
<th>R</th>
<th>Is $-10^{-6} &lt; R - E &lt; 10^{-6}$?</th>
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Final output ____________________________
Question 1 continued

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(Total 6 marks)
3.

Figure 2

Figure 3
Question 3 continued

C ● ● 1    C ● ● 1
G ● ● 2    G ● ● 2
J ● ● 3    J ● ● 3
N ● ● 4    N ● ● 4
O ● ● 5    O ● ● 5
R ● ● 6    R ● ● 6

(Total 8 marks)
4. (a) ___________________________________________________________________________________________________________
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(b) Key:

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<tr>
<th>Vertex</th>
<th>Order of labelling</th>
<th>Final value</th>
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<td>Working value</td>
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</table>

Shortest path ____________________________
Length of shortest path ____________________________
(c) Shortest distance from S to F

(d) ___________________________________________________________________________________________________________
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Figure 5

[The weight of the network is 379]
Question 5 continued

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(Total 15 marks)
6.

Diagram 1
7. (a) ___________________________________________________________________
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(b) 

Diagram 1

Key:
- Early event time
- Late event time
Question 7 continued

(c) Critical activities

(d) ___________________________________________________________________

(e) ___________________________________________________________________

(f) Activities that must be happening at time 5.5

(g) ___________________________________________________________________

(h) ___________________________________________________________________

(i) ___________________________________________________________________

(j) ___________________________________________________________________

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END
Edexcel GCE
Further Pure Mathematics FP1
Advanced/Advanced Subsidiary
Monday 28 January 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Turn over
1. Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^{2}$, that

$$\sum_{r=1}^{n} 3(2r - 1)^{2} = n(2n+1)(2n-1),$$

for all positive integers $n$. (5)
Question 1 continued

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(Total 5 marks)
2. 

\[ z = \frac{50}{3 + 4i} \]

Find, in the form \( a + bi \) where \( a, b \in \mathbb{R} \),

(a) \( z \),

(b) \( z^2 \).

Find

(c) \( |z| \),

(d) \( \arg z^2 \), giving your answer in degrees to 1 decimal place.
Question 2 continued

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3. \( f(x) = 2x^\frac{3}{2} + x^{-\frac{1}{2}} - 5, \quad x > 0 \)

(a) Find \( f'(x) \).

The equation \( f(x) = 0 \) has a root \( \alpha \) in the interval \([4.5, 5.5]\).

(b) Using \( x_0 = 5 \) as a first approximation to \( \alpha \), apply the Newton-Raphson procedure once to \( f(x) \) to find a second approximation to \( \alpha \), giving your answer to 3 significant figures.
Question 3 continued

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Q3
(Total 6 marks)
4. The transformation $U$, represented by the $2 \times 2$ matrix $P$, is a rotation through $90^\circ$ anticlockwise about the origin.

(a) Write down the matrix $P$. 

The transformation $V$, represented by the $2 \times 2$ matrix $Q$, is a reflection in the line $y = -x$.

(b) Write down the matrix $Q$. 

Given that $U$ followed by $V$ is transformation $T$, which is represented by the matrix $R$,

(c) express $R$ in terms of $P$ and $Q$,

(d) find the matrix $R$,

(e) give a full geometrical description of $T$ as a single transformation.
Question 4 continued
Question 4 continued
5. \[ f(x) = (4x^2 + 9)(x^2 - 6x + 34) \]

(a) Find the four roots of \( f(x) = 0 \)

Give your answers in the form \( x = p + iq \), where \( p \) and \( q \) are real. (5)

(b) Show these four roots on a single Argand diagram. (2)
6. \[ X = \begin{pmatrix} 1 & a \\ 3 & 2 \end{pmatrix}, \text{ where } a \text{ is a constant.} \]

(a) Find the value of \( a \) for which the matrix \( X \) is singular.

\[ Y = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix} \]

(b) Find \( Y^{-1} \).

The transformation represented by \( Y \) maps the point \( A \) onto the point \( B \).

Given that \( B \) has coordinates \((1 - \lambda, 7 \lambda - 2)\), where \( \lambda \) is a constant,

(c) find, in terms of \( \lambda \), the coordinates of point \( A \).
Question 6 continued
Question 6 continued
7. The rectangular hyperbola, $H$, has cartesian equation $xy = 25$

The point $P \left(\frac{5p}{p}, \frac{5}{p}\right)$, and the point $Q \left(\frac{5q}{q}, \frac{5}{q}\right)$, where $p, q \neq 0, p \neq q$, are points on the rectangular hyperbola $H$.

(a) Show that the equation of the tangent at point $P$ is

$$p^2y + x = 10p$$  \hspace{1cm} (4)

(b) Write down the equation of the tangent at point $Q$.  \hspace{1cm} (1)

The tangents at $P$ and $Q$ meet at the point $N$.

Given $p + q \neq 0$,  

(c) show that point $N$ has coordinates $\left(\frac{10pq}{p + q}, \frac{10}{p + q}\right)$.  \hspace{1cm} (4)

The line joining $N$ to the origin is perpendicular to the line $PQ$.

(d) Find the value of $p^2q^2$.  \hspace{1cm} (5)
Question 7 continued
Question 7 continued

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(Total 14 marks)
8. (a) Prove by induction that, for \( n \in \mathbb{Z}^+ \),

\[
\sum_{r=1}^{n} r(r+3) = \frac{1}{2} n(n+1)(n+5)
\]  \hspace{1cm} (6)

(b) A sequence of positive integers is defined by

\[
u_1 = 1, \quad u_{n+1} = u_n + n(3n+1), \quad n \in \mathbb{Z}^+
\]

Prove by induction that

\[
u_n = n^2(n-1)+1, \quad n \in \mathbb{Z}^+
\]  \hspace{1cm} (5)
Question 8 continued

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Question 8 continued
9.

Figure 1 shows a sketch of part of the parabola with equation \( y^2 = 36x \).

The point \( P (4, 12) \) lies on the parabola.

(a) Find an equation for the normal to the parabola at \( P \).  

This normal meets the \( x \)-axis at the point \( N \) and \( S \) is the focus of the parabola, as shown in Figure 1.

(b) Find the area of triangle \( PSN \).
Question 9 continued
Question 9 continued
Edexcel GCE
Mechanics M1
Advanced/Advanced Subsidiary
Wednesday 23 January 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer to each question in the space following the question. Whenever a numerical value of \( g \) is required, take \( g = 9.8 \text{ m s}^{-2} \). When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 7 questions in this question paper. The total mark for this paper is 75. There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.
1. Two particles $P$ and $Q$ have masses $4m$ and $m$ respectively. The particles are moving towards each other on a smooth horizontal plane and collide directly. The speeds of $P$ and $Q$ immediately before the collision are $2u$ and $5u$ respectively. Immediately after the collision, the speed of $P$ is $\frac{1}{2}u$ and its direction of motion is reversed.

(a) Find the speed and direction of motion of $Q$ after the collision.  

(b) Find the magnitude of the impulse exerted on $P$ by $Q$ in the collision.
2. A steel girder $AB$, of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at $C$ and at $D$, where $AC = 2$ m and $DB = 2$ m. A man of mass 80 kg stands on the girder at the point $P$, where $AP = 4$ m, as shown in Figure 1.

The man is modelled as a particle and the girder is modelled as a uniform rod.

(a) Find the magnitude of the reaction on the girder at the support at $C$. (3)

The support at $D$ is now moved to the point $X$ on the girder, where $XB = x$ metres. The man remains on the girder at $P$, as shown in Figure 2.

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

(b) the magnitude of the reaction at the support at $X$, (2)

(c) the value of $x$. (4)
Question 2 continued


3. A particle $P$ of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point $O$. The particle is held in equilibrium, with $OP$ at 30° to the downward vertical, by a force of magnitude $F$ newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.

![Figure 3](image-url)

Find

(i) the value of $F$,

(ii) the tension in the string.

(8)
Question 3 continued
Question 3 continued
Question 3 continued

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(Total 8 marks)
4. A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 m s⁻¹ when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.
Question 4 continued
Question 4 continued
Question 4 continued

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(Total 9 marks)
The velocity-time graph in Figure 4 represents the journey of a train $P$ travelling along a straight horizontal track between two stations which are 1.5 km apart. The train $P$ leaves the first station, accelerating uniformly from rest for 300 m until it reaches a speed of 30 m s$^{-1}$. The train then maintains this speed for $T$ seconds before decelerating uniformly at 1.25 m s$^{-2}$, coming to rest at the next station.

(a) Find the acceleration of $P$ during the first 300 m of its journey.

(b) Find the value of $T$.

A second train $Q$ completes the same journey in the same total time. The train leaves the first station, accelerating uniformly from rest until it reaches a speed of $V$ m s$^{-1}$ and then immediately decelerates uniformly until it comes to rest at the next station.

(c) Sketch on the diagram above, a velocity-time graph which represents the journey of train $Q$.

(d) Find the value of $V$. 
Question 5 continued
Question 5 continued
Question 5 continued

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6. [In this question, \( \mathbf{i} \) and \( \mathbf{j} \) are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]

A ship sets sail at 9 am from a port \( P \) and moves with constant velocity. The position vector of \( P \) is \((4\mathbf{i} - 8\mathbf{j})\) km. At 9.30 am the ship is at the point with position vector \((\mathbf{i} - 4\mathbf{j})\) km.

(a) Find the speed of the ship in km h\(^{-1}\). (4)

(b) Show that the position vector \( \mathbf{r} \) km of the ship, \( t \) hours after 9 am, is given by \( \mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j} \). (2)

At 10 am, a passenger on the ship observes that a lighthouse \( L \) is due west of the ship. At 10.30 am, the passenger observes that \( L \) is now south-west of the ship.

(c) Find the position vector of \( L \). (5)
Question 6 continued
Question 6 continued

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(Total 11 marks)
Figure 5 shows two particles $A$ and $B$, of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially $A$ is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle $\alpha$, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between $A$ and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley $P$ which is fixed at the top of the plane. The part of the string from $A$ to $P$ is parallel to a line of greatest slope of the plane and $B$ hangs vertically below $P$. The system is released from rest with the string taut, with $A$ at the point $X$ and with $B$ at a height $h$ above the ground.

For the motion until $B$ hits the ground,

(a) give a reason why the magnitudes of the accelerations of the two particles are the same, 

$$\quad$$

(b) write down an equation of motion for each particle, 

$$\quad$$

(c) find the acceleration of each particle.

$$\quad$$

Particle $B$ does not rebound when it hits the ground and $A$ continues moving up the plane towards $P$. Given that $A$ comes to rest at the point $Y$, without reaching $P$,

(d) find the distance $XY$ in terms of $h$. 

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Question 7 continued

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Edexcel GCE
Mechanics M2
Advanced/Advanced Subsidiary
Friday 25 January 2013 – Afternoon
Time: 1 hour 30 minutes

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may not gain full credit.
1. Two uniform rods $AB$ and $BC$ are rigidly joined at $B$ so that $\angle ABC = 90^\circ$. Rod $AB$ has length 0.5 m and mass 2 kg. Rod $BC$ has length 2 m and mass 3 kg. The centre of mass of the framework of the two rods is at $G$.

(a) Find the distance of $G$ from $BC$.

The distance of $G$ from $AB$ is 0.6 m.
The framework is suspended from $A$ and hangs freely in equilibrium.

(b) Find the angle between $AB$ and the downward vertical at $A$. 

\[ \text{Leave blank} \]
2. A lorry of mass 1800 kg travels along a straight horizontal road. The lorry’s engine is working at a constant rate of 30 kW. When the lorry’s speed is 20 m s\(^{-1}\), its acceleration is 0.4 m s\(^{-2}\). The magnitude of the resistance to the motion of the lorry is \(R\) newtons.

(a) Find the value of \(R\).  

(b) Find the new rate of working of the lorry’s engine.
Question 2 continued
Question 2 continued

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(Total 9 marks)
A ladder, of length 5 m and mass 18 kg, has one end \( A \) resting on rough horizontal ground and its other end \( B \) resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle \( \alpha \) with the horizontal ground, where \( \tan \alpha = \frac{4}{3} \), as shown in Figure 1. The coefficient of friction between the ladder and the ground is \( \mu \). A woman of mass 60 kg stands on the ladder at the point \( C \), where \( AC = 3 \) m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of \( \mu \).
Question 3 continued

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(Total 9 marks)
4. At time $t$ seconds the velocity of a particle $P$ is $[(4t - 5)i + 3j]$ m s$^{-1}$. When $t = 0$, the position vector of $P$ is $(2i + 5j)$ m, relative to a fixed origin $O$.

(a) Find the value of $t$ when the velocity of $P$ is parallel to the vector $j$. (1)

(b) Find an expression for the position vector of $P$ at time $t$ seconds. (4)

A second particle $Q$ moves with constant velocity $(-2i + cj)$ m s$^{-1}$. When $t = 0$, the position vector of $Q$ is $(1i + 2j)$ m. The particles $P$ and $Q$ collide at the point with position vector $(di + 14j)$ m.

(c) Find

(i) the value of $c$,

(ii) the value of $d$. (5)
Question 4 continued
Question 4 continued

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(Total 10 marks)
5. The point $A$ lies on a rough plane inclined at an angle $\theta$ to the horizontal, where $\sin \theta = \frac{24}{25}$. A particle $P$ is projected from $A$, up a line of greatest slope of the plane, with speed $U$ m s$^{-1}$. The mass of $P$ is 2 kg and the coefficient of friction between $P$ and the plane is $\frac{5}{12}$. The particle comes to instantaneous rest at the point $B$ on the plane, where $AB = 1.5$ m. It then moves back down the plane to $A$.

(a) Find the work done against friction as $P$ moves from $A$ to $B$.  

(b) Use the work-energy principle to find the value of $U$. 

(c) Find the speed of $P$ when it returns to $A$. 

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(Total 11 marks)
6.

![Diagram of a ball thrown from point O, 6 m above horizontal ground, with a thin vertical post 4 m high and 8 m horizontally away from O. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.](image)

A ball is thrown from a point $O$, which is 6 m above horizontal ground. The ball is projected with speed $u \text{ m s}^{-1}$ at an angle $\theta$ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through $O$, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that $\tan \theta = 2.2$  

(b) Find the value of $u$.  

The ball hits the ground $T$ seconds after projection.

(c) Find the value of $T$.  

Immediately before the ball hits the ground the direction of motion of the ball makes an angle $\alpha$ with the horizontal.

(d) Find $\alpha$.  

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Question 6 continued
Question 6 continued
7. A particle $A$ of mass $m$ is moving with speed $u$ on a smooth horizontal floor when it collides directly with another particle $B$, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is $e$. The direction of motion of $A$ is reversed by the collision.

(a) Find, in terms of $e$ and $u$,

(i) the speed of $A$ immediately after the collision,

(ii) the speed of $B$ immediately after the collision.

(b) find the range of possible values of $e$,

(c) determine whether there will be a second collision between $A$ and $B$. 

After being struck by $A$ the particle $B$ collides directly with another particle $C$, of mass $4m$, which is at rest on the floor. The coefficient of restitution between $B$ and $C$ is $2e$. Given that the direction of motion of $B$ is reversed by this collision,
Question 7 continued
Edexcel GCE
Mechanics M3
Advanced/Advanced Subsidiary
Monday 28 January 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Turn over
1. A particle $P$ is moving along the positive $x$-axis. When the displacement of $P$ from the origin is $x$ metres, the velocity of $P$ is $v$ m s$^{-1}$ and the acceleration of $P$ is $9x$ m s$^{-2}$.

When $x = 2$, $v = 6$

Show that $v^2 = 9x^2$. (4)
A uniform solid consists of a right circular cone of radius $r$ and height $kr$, where $k > \sqrt{3}$, fixed to a hemisphere of radius $r$. The centre of the plane face of the hemisphere is $O$ and this plane face coincides with the base of the cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of the solid from $O$ is

$$\frac{(k^2 - 3)r}{4(k + 2)}$$

(b) Find the value of $k$. 

The point $A$ lies on the circumference of the base of the cone. The solid is suspended by a string attached at $A$ and hangs freely in equilibrium. The angle between $AO$ and the vertical is $\theta$, where $\tan \theta = \frac{11}{14}$.
Question 2 continued

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(Q2)
(Total 9 marks)
3. A particle $P$ of mass 0.6 kg is moving along the $x$-axis in the positive direction. At time $t = 0$, $P$ passes through the origin $O$ with speed 15 m s$^{-1}$. At time $t$ seconds the distance $OP$ is $x$ metres, the speed of $P$ is $v$ m s$^{-1}$ and the resultant force acting on $P$ has magnitude $\frac{12}{(t+2)^2}$ newtons. The resultant force is directed towards $O$.

(a) Show that $v = 5\left(\frac{4}{t+2} + 1\right)$.  

(b) Find the value of $x$ when $t = 5$
Question 3 continued
Figure 2

A particle $P$ of mass $m$ is attached to one end of a light elastic string, of natural length $2a$ and modulus of elasticity $6mg$. The other end of the string is attached to a fixed point $A$. The particle moves with constant speed $v$ in a horizontal circle with centre $O$, where $O$ is vertically below $A$ and $OA = 2a$, as shown in Figure 2.

(a) Show that the extension in the string is $\frac{2}{5}a$.  

(b) Find $v^2$ in terms of $a$ and $g$. 

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(Total 11 marks)
5. A particle $P$ is moving in a straight line with simple harmonic motion on a smooth horizontal floor. The particle comes to instantaneous rest at points $A$ and $B$ where $AB$ is 0.5 m. The mid-point of $AB$ is $O$. The mid-point of $OA$ is $C$. The mid-point of $OB$ is $D$. The particle takes 0.2 s to travel directly from $C$ to $D$. At time $t = 0$, $P$ is moving through $O$ towards $A$.

(a) Show that the period of the motion is $\frac{6}{5}$ s. 

(b) Find the distance of $P$ from $B$ when $t = 2$ s. 

(c) Find the maximum magnitude of the acceleration of $P$. 

(d) Find the maximum speed of $P$. 

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Question 5 continued
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(Total 12 marks)
A smooth hollow cylinder of internal radius $a$ is fixed with its axis horizontal. A particle $P$ moves on the inner surface of the cylinder in a vertical circle with radius $a$ and centre $O$, where $O$ lies on the axis of the cylinder. The particle is projected vertically downwards with speed $u$ from point $A$ on the circle, where $OA$ is horizontal. The particle first loses contact with the cylinder at the point $B$, where $\angle AOB = 150^\circ$, as shown in Figure 3. Given that air resistance can be ignored,

(a) show that the speed of $P$ at $B$ is $\sqrt{\frac{ag}{2}}$, \hspace{1cm} (3)

(b) find $u$ in terms of $a$ and $g$. \hspace{1cm} (4)

After losing contact with the cylinder, $P$ crosses the diameter through $A$ at the point $D$. At $D$ the velocity of $P$ makes an angle $\theta^\circ$ with the horizontal.

(c) Find the value of $\theta$. \hspace{1cm} (7)
Question 6 continued
Question 6 continued
7. A particle $P$ of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity $\lambda$ newtons. The ends of the string are attached to two fixed points $A$ and $B$, where $AB$ is horizontal and $AB = 0.48$ m. Initially $P$ is held at rest at the mid-point, $M$, of the line $AB$ and the tension in the string is 240 N.

(a) Show that $\lambda = 400$ \hspace{1cm} (3)

The particle is now held at rest at the point $C$, where $C$ is 0.07 m vertically below $M$. The particle is released from rest at $C$.

(b) Find the magnitude of the initial acceleration of $P$. \hspace{1cm} (6)

(c) Find the speed of $P$ as it passes through $M$. \hspace{1cm} (6)
Question 7 continued

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Question 7 continued
Edexcel GCE
Statistics S1
Advanced/Advanced Subsidiary
Friday 18 January 2013 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. A teacher asked a random sample of 10 students to record the number of hours of television, \( t \), they watched in the week before their mock exam. She then calculated their grade, \( g \), in their mock exam. The results are summarised as follows.

\[
\begin{align*}
\sum t &= 258 & \sum t^2 &= 8702 & \sum g &= 63.6 & S_{gg} &= 7.864 & \sum gt &= 1550.2
\end{align*}
\]

(a) Find \( S_{tt} \) and \( S_{gt} \). 

(b) Calculate, to 3 significant figures, the product moment correlation coefficient between \( t \) and \( g \).

(c) Describe, giving a reason, the nature of the correlation you would expect to find between \( v \) and \( g \).
Question 1 continued
2. The discrete random variable $X$ can take only the values 1, 2 and 3. For these values the cumulative distribution function is defined by

$$F(x) = \frac{x^3 + k}{40} \quad x = 1, 2, 3$$

(a) Show that $k = 13$  

(b) Find the probability distribution of $X$.  

Given that $\text{Var}(X) = \frac{259}{320}$

(c) find the exact value of $\text{Var}(4X - 5)$.  


Question 2 continued
3. A biologist is comparing the intervals \( (m \text{ seconds}) \) between the mating calls of a certain species of tree frog and the surrounding temperature \( (t \degree \text{C}) \). The following results were obtained.

<table>
<thead>
<tr>
<th>( t \degree \text{C} )</th>
<th>8</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m \text{ secs} )</td>
<td>6.5</td>
<td>4.5</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

(You may use \( \sum tm = 469.5 \), \( S_n = 354 \), \( S_{mm} = 25.5 \))

(a) Show that \( S_m = -90.5 \) \( (4) \)

(b) Find the equation of the regression line of \( m \) on \( t \) giving your answer in the form \( m = a + bt \). \( (4) \)

(c) Use your regression line to estimate the time interval between mating calls when the surrounding temperature is 10 \( \degree \text{C} \). \( (1) \)

(d) Comment on the reliability of this estimate, giving a reason for your answer. \( (1) \)
Question 3 continued

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(Total 10 marks)
4. The length of time, \( L \) hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find \( P(L > 127) \). (3)

(b) Find the value of \( d \) such that \( P(L < d) = 0.10 \) (3)

Alice is about to go on a 6 hour journey.

Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed. (4)
5. A survey of 100 households gave the following results for weekly income £\(y\).

<table>
<thead>
<tr>
<th>Income (y) (£)</th>
<th>Mid-point</th>
<th>Frequency (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 \leq y &lt; 200)</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>(200 \leq y &lt; 240)</td>
<td>220</td>
<td>28</td>
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<tr>
<td>(240 \leq y &lt; 320)</td>
<td>280</td>
<td>22</td>
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<tr>
<td>(320 \leq y &lt; 400)</td>
<td>360</td>
<td>18</td>
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<tr>
<td>(400 \leq y &lt; 600)</td>
<td>500</td>
<td>12</td>
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<tr>
<td>(600 \leq y &lt; 800)</td>
<td>700</td>
<td>8</td>
</tr>
</tbody>
</table>

(You may use \(\sum fy^2 = 12\,452\,800\))

A histogram was drawn and the class \(200 \leq y < 240\) was represented by a rectangle of width 2 cm and height 7 cm.

(a) Calculate the width and the height of the rectangle representing the class \(320 \leq y < 400\)

(b) Use linear interpolation to estimate the median weekly income to the nearest pound.

(c) Estimate the mean and the standard deviation of the weekly income for these data.

One measure of skewness is \(\frac{3(\text{mean} - \text{median})}{\text{standard deviation}}\)

(d) Use this measure to calculate the skewness for these data and describe its value.

Katie suggests using the random variable \(X\) which has a normal distribution with mean 320 and standard deviation 150 to model the weekly income for these data.

(e) Find \(P(240 < X < 400)\).

(f) With reference to your calculations in parts (d) and (e) and the data in the table, comment on Katie’s suggestion.
Question 5 continued
Question 5 continued
6. A fair blue die has faces numbered 1, 1, 3, 3, 5 and 5. The random variable $B$ represents the score when the blue die is rolled.

(a) Write down the probability distribution for $B$. 

(b) State the name of this probability distribution.

(c) Write down the value of $E(B)$.

A second die is red and the random variable $R$ represents the score when the red die is rolled.

The probability distribution of $R$ is

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(d) Find $E(R)$.

(e) Find $\text{Var}(R)$.

(f) Tom invites Avisha to play a game with these dice.

Tom spins a fair coin with one side labelled 2 and the other side labelled 5. When Avisha sees the number showing on the coin she then chooses one of the dice and rolls it. If the number showing on the die is greater than the number showing on the coin, Avisha wins, otherwise Tom wins.

Avisha chooses the die which gives her the best chance of winning each time Tom spins the coin.

(f) Find the probability that Avisha wins the game, stating clearly which die she should use in each case.
Question 6 continued
Question 6 continued
7. Given that

\[ P(A) = 0.35, \quad P(B) = 0.45 \quad \text{and} \quad P(A \cap B) = 0.13 \]

find

(a) \( P(A \cup B) \) (2)

(b) \( P(A' \mid B') \) (2)

The event \( C \) has \( P(C) = 0.20 \)

The events \( A \) and \( C \) are mutually exclusive and the events \( B \) and \( C \) are independent.

(c) Find \( P(B \cap C) \) (2)

(d) Draw a Venn diagram to illustrate the events \( A, B \) and \( C \) and the probabilities for each region. (4)

(e) Find \( P(\overline{B \cup C}) \) (2)
Question 7 continued
Question 7 continued

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Edexcel GCE
Statistics S2
Advanced/Advanced Subsidiary
Friday 18 January 2013 – Afternoon
Time: 1 hour 30 minutes

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer to each question in the space following the question. Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet “Mathematical Formulae and Statistical Tables” is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 7 questions in this question paper. The total mark for this paper is 75. There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

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W850/R6684/57570 5/5/5/*P41482A0128*
1. (a) Write down the conditions under which the Poisson distribution can be used as an approximation to the binomial distribution.

The probability of any one letter being delivered to the wrong house is 0.01
On a randomly selected day Peter delivers 1000 letters.

(b) Using a Poisson approximation, find the probability that Peter delivers at least 4 letters to the wrong house.

Give your answer to 4 decimal places.
Question 1 continued

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(Total 5 marks)
2. In a village, power cuts occur randomly at a rate of 3 per year.

   (a) Find the probability that in any given year there will be
       
       (i) exactly 7 power cuts,
       
       (ii) at least 4 power cuts.

   (b) Use a suitable approximation to find the probability that in the next 10 years the
       number of power cuts will be less than 20
Question 2 continued
3. A random variable $X$ has the distribution $B(12, p)$.

(a) Given that $p = 0.25$ find

(i) $P(X < 5)$

(ii) $P(X \geq 7)$

(b) Given that $P(X = 0) = 0.05$, find the value of $p$ to 3 decimal places.

(c) Given that the variance of $X$ is 1.92, find the possible values of $p$. 


Question 3 continued

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4. The continuous random variable $X$ is uniformly distributed over the interval $[-4, 6]$.

(a) Write down the mean of $X$.  

(b) Find $P(X \leq 2.4)$  

(c) Find $P(-3 < X - 5 < 3)$  

The continuous random variable $Y$ is uniformly distributed over the interval $[a, 4a]$.  

(d) Use integration to show that $E(Y^2) = 7a^2$  

(e) Find $\text{Var}(Y)$.

(f) Given that $P(X < \frac{8}{3}) = P(Y < \frac{8}{3})$, find the value of $a$.  

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Question 4 continued

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Question 4 continued
Question 4 continued
5. The continuous random variable $T$ is used to model the number of days, $t$, a mosquito survives after hatching.

The probability that the mosquito survives for more than $t$ days is

$$
\frac{225}{(t+15)^2}, \quad t \geq 0
$$

(a) Show that the cumulative distribution function of $T$ is given by

$$
F(t) = \begin{cases} 
1 - \frac{225}{(t+15)^2} & t \geq 0 \\
0 & \text{otherwise}
\end{cases}
$$

(b) Find the probability that a randomly selected mosquito will die within 3 days of hatching.

(c) Given that a mosquito survives for 3 days, find the probability that it will survive for at least 5 more days.

(d) Find the number of days after which only 10% of these mosquitoes are expected to survive.
Question 5 continued
Question 5 continued
6. (a) Explain what you understand by a hypothesis. 

(b) Explain what you understand by a critical region. 

Mrs George claims that 45% of voters would vote for her.

In an opinion poll of 20 randomly selected voters it was found that 5 would vote for her.

(c) Test at the 5% level of significance whether or not the opinion poll provides evidence to support Mrs George’s claim.

In a second opinion poll of $n$ randomly selected people it was found that no one would vote for Mrs George.

(d) Using a 1% level of significance, find the smallest value of $n$ for which the hypothesis $H_0: p = 0.45$ will be rejected in favour of $H_1: p < 0.45$
Question 6 continued

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7. The continuous random variable $X$ has the following probability density function

$$f(x) = \begin{cases} a + bx & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

where $a$ and $b$ are constants.

(a) Show that $10a + 25b = 2$ (4)

Given that $E(X) = \frac{35}{12}$

(b) find a second equation in $a$ and $b$, (3)

(c) hence find the value of $a$ and the value of $b$. (3)

(d) Find, to 3 significant figures, the median of $X$. (3)

(e) Comment on the skewness. Give a reason for your answer. (2)
Question 7 continued
Question 7 continued
Edexcel GCE
Core Mathematics C1
Advanced Subsidiary
Monday 13 May 2013 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Calculators may NOT be used in this examination.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 11 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Total
1. Simplify

\[ \frac{7 + \sqrt{5}}{\sqrt{5} - 1} \]

giving your answer in the form \(a + b\sqrt{5}\), where \(a\) and \(b\) are integers.

(Total 4 marks)
2. Find

\[ \int \left( 10x^4 - 4x - \frac{3}{\sqrt{x}} \right) \, dx \]

giving each term in its simplest form.

(Total 4 marks)
3. (a) Find the value of $8^3$ 

(b) Simplify fully $\frac{(2x^{\frac{1}{2}})^3}{4x^2}$
4. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$

$$a_{n+1} = k(a_n + 2), \quad \text{for } n \geq 1$$

where $k$ is a constant.

(a) Find an expression for $a_2$ in terms of $k$. (1)

Given that $\sum_{i=1}^{3} a_i = 2$,

(b) find the two possible values of $k$. (6)
5. Find the set of values of \( x \) for which

(a) \( 2(3x + 4) > 1 - x \) \( \quad (2) \)

(b) \( 3x^2 + 8x - 3 < 0 \) \( \quad (4) \)
6. The straight line $L_1$ passes through the points $(-1, 3)$ and $(11, 12)$.

   (a) Find an equation for $L_1$ in the form $ax + by + c = 0$,
       where $a$, $b$ and $c$ are integers.

   The line $L_2$ has equation $3y + 4x - 30 = 0$.

   (b) Find the coordinates of the point of intersection of $L_1$ and $L_2$.
7. A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week \( N \).

(a) Find the value of \( N \).  

(b) The company then plans to continue to make 600 mobile phones each week.

(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.
Figure 1 shows a sketch of the curve with equation \( y = f(x) \) where

\[
f(x) = (x + 3)^2 (x - 1), \quad x \in \mathbb{R}.
\]

The curve crosses the \( x \)-axis at \( (1, 0) \), touches it at \( (-3, 0) \) and crosses the \( y \)-axis at \( (0, -9) \).

(a) In the space below, sketch the curve \( C \) with equation \( y = f(x + 2) \) and state the coordinates of the points where the curve \( C \) meets the \( x \)-axis.

(b) Write down an equation of the curve \( C \).

(c) Use your answer to part (b) to find the coordinates of the point where the curve \( C \) meets the \( y \)-axis.
Question 8 continued
Question 8 continued
9.

\[ f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0 \]

(a) Show that

\[ f'(x) = 9x^{-2} + A + Bx^2, \]

where \( A \) and \( B \) are constants to be found.

(b) Find \( f''(x) \).

Given that the point \((-3, 10)\) lies on the curve with equation \( y = f(x) \),

(c) find \( f(x) \).
Given the simultaneous equations

\[
\begin{align*}
2x + y &= 1 \\
x^2 - 4ky + 5k &= 0
\end{align*}
\]

where \( k \) is a non-zero constant,

(a) show that

\[
x^2 + 8kx + k = 0
\]

(2)

Given that \( x^2 + 8kx + k = 0 \) has equal roots,

(b) find the value of \( k \).

(3)

(c) For this value of \( k \), find the solution of the simultaneous equations.

(3)
Question 10 continued
Question 10 continued

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Question 10 continued
11. Figure 2

Figure 2 shows a sketch of the curve \( H \) with equation \( y = \frac{3}{x} + 4, \ x \neq 0 \).

(a) Give the coordinates of the point where \( H \) crosses the \( x \)-axis.

(b) Give the equations of the asymptotes to \( H \).

(c) Find an equation for the normal to \( H \) at the point \( P(-3, 3) \).

This normal crosses the \( x \)-axis at \( A \) and the \( y \)-axis at \( B \).

(d) Find the length of the line segment \( AB \). Give your answer as a surd.
Question 11 continued
Question 11 continued
Edexcel GCE
Core Mathematics C2
Advanced Subsidiary
Friday 24 May 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. The first three terms of a geometric series are

18, 12 and \( p \)

respectively, where \( p \) is a constant.

Find

(a) the value of the common ratio of the series,

(b) the value of \( p \),

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.
Question 1 continued

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(Total 4 marks)
2. (a) Use the binomial theorem to find all the terms of the expansion of

\[(2 + 3x)^4\]

Give each term in its simplest form.  

(b) Write down the expansion of

\[(2 - 3x)^4\]

in ascending powers of \(x\), giving each term in its simplest form.
3. \[ f(x) = 2x^3 - 5x^2 + ax + 18 \]

where \( a \) is a constant.

Given that \((x - 3)\) is a factor of \(f(x)\),

(a) show that \(a = -9\) \hspace{1cm} (2)

(b) factorise \(f(x)\) completely. \hspace{1cm} (4)

Given that

\[ g(y) = 2(3^y) - 5(3^y) - 9(3^y) + 18 \]

(c) find the values of \(y\) that satisfy \(g(y) = 0\), giving your answers to 2 decimal places where appropriate. \hspace{1cm} (3)
Question 3 continued
Question 3 continued

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Question 3 continued

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(Question 3 continued)

(Total 9 marks)
4. \( y = \frac{5}{(x^2 + 1)} \)

(a) Complete the table below, giving the missing value of \( y \) to 3 decimal places.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>4</td>
<td>2.5</td>
<td>1</td>
<td>0.690</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

(b) Use the trapezium rule, with all the values of \( y \) from your table, to find an approximate value for the area of \( R \).

(c) Use your answer to part (b) to find an approximate value for

\[
\int_{0}^{3} \left( 4 + \frac{5}{(x^2 + 1)} \right) \, dx
\]

giving your answer to 2 decimal places.
Question 4 continued

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(Total 7 marks)
Figure 2 shows a plan view of a garden. The plan of the garden $ABCEA$ consists of a triangle $ABE$ joined to a sector $BCDE$ of a circle with radius 12 m and centre $B$. The points $A$, $B$ and $C$ lie on a straight line with $AB = 23$ m and $BC = 12$ m.

Given that the size of angle $ABE$ is exactly 0.64 radians, find

(a) the area of the garden, giving your answer in m$^2$, to 1 decimal place, 

(b) the perimeter of the garden, giving your answer in metres, to 1 decimal place.
Question 5 continued
6.

Figure 3 shows a sketch of part of the curve $C$ with equation

$$y = x(x + 4)(x - 2)$$

The curve $C$ crosses the $x$-axis at the origin $O$ and at the points $A$ and $B$.

(a) Write down the $x$-coordinates of the points $A$ and $B$.  

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.
Question 6 continued

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7. (i) Find the exact value of $x$ for which

\[ \log_2(2x) = \log_2(5x + 4) - 3 \]

(ii) Given that

\[ \log_a y + 3\log_a 2 = 5 \]

express $y$ in terms of $a$.

Give your answer in its simplest form.
Question 7 continued

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(Total 7 marks)
8. (i) Solve, for $-180^\circ \leq x < 180^\circ$,

$$\tan(x - 40^\circ) = 1.5$$

giving your answers to 1 decimal place.  \hspace{1cm} (3)

(ii) (a) Show that the equation

$$\sin \theta \tan \theta = 3\cos \theta + 2$$

can be written in the form

$$4\cos^2 \theta + 2\cos \theta - 1 = 0$$\hspace{1cm} (3)

(b) Hence solve, for $0 \leq \theta < 360^\circ$,

$$\sin \theta \tan \theta = 3\cos \theta + 2$$

showing each stage of your working. \hspace{1cm} (5)
Question 8 continued

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Question 8 continued
9. The curve with equation

\[ y = x^2 - 32 \sqrt{x} + 20, \quad x > 0 \]

has a stationary point \( P \).

Use calculus

(a) to find the coordinates of \( P \),

(b) to determine the nature of the stationary point \( P \).
Question 9 continued
The circle $C$ has radius 5 and touches the $y$-axis at the point $(0, 9)$, as shown in Figure 4.

(a) Write down an equation for the circle $C$, that is shown in Figure 4.

(b) Find the length of $PT$. 

Figure 4
Examiner’s use only

Team Leader’s use only

Surname Initial(s)

Signature

Centre No.

Candidate No.

Paper Reference

Surname

Initial(s)

Signature

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Thursday 13 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

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Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
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Information for Candidates

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There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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Turn over

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1. Given that

\[
\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2
\]

find the values of the constants \(a, b, c, d\) and \(e\).
Question 1 continued
2. Given that \( f(x) = \ln x, \quad x > 0 \)

sketch on separate axes the graphs of

(i) \( y = f(x) \),

(ii) \( y = |f(x)| \),

(iii) \( y = -f(x - 4) \).

Show, on each diagram, the point where the graph meets or crosses the \( x \)-axis. In each case, state the equation of the asymptote.
Question 2 continued
Question 2 continued
Question 2 continued
3. Given that

$$2 \cos(x + 50) = \sin(x + 40)$$

(a) Show, without using a calculator, that

$$\tan x = \frac{1}{3} \tan 40$$

(b) Hence solve, for $0 \leq \theta < 360$,

$$2 \cos(2\theta + 50) = \sin(2\theta + 40)$$

giving your answers to 1 decimal place.
Question 3 continued
Question 3 continued
Question 3 continued

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(Total 8 marks)
4. \[ f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R} \]

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation \( y = f(x) \).

(b) Show that the equation \( f(x) = 0 \) can be written as \( x = \pm \frac{4}{5} e^{-x} \)

The equation \( f(x) = 0 \) has a root \( a \), where \( a = 0.5 \) to 1 decimal place.

(c) Starting with \( x_0 = 0.5 \), use the iteration formula

\[ x_{n+1} = \frac{4}{5} e^{x_n} \]

to calculate the values of \( x_1, x_2 \) and \( x_3 \), giving your answers to 3 decimal places.

(d) Give an accurate estimate for \( a \) to 2 decimal places, and justify your answer.
Question 4 continued

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Question 4 continued

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(Total 11 marks)
5. Given that
\[ x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6} \]

(a) find \( \frac{dx}{dy} \) in terms of \( y \).

(b) Hence show that
\[ \frac{dy}{dx} = \frac{1}{6x(x - 1)^{\frac{1}{2}}} \]

(c) Find an expression for \( \frac{d^2y}{dx^2} \) in terms of \( x \). Give your answer in its simplest form.
Question 5 continued
Question 5 continued
Question 5 continued

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(Total 10 marks)
6. Find algebraically the exact solutions to the equations

(a) \( \ln(4 - 2x) + \ln(9 - 3x) = 2 \ln(x + 1), \quad -1 < x < 2 \)  

(b) \( 2^x \cdot e^{3x+1} = 10 \)

Give your answer to (b) in the form \( \frac{a + \ln b}{c + \ln d} \) where \( a, b, c \) and \( d \) are integers.
Question 6 continued
Question 6 continued
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(Total 10 marks)
7. The function \( f \) has domain \(-2 \leq x \leq 6\) and is linear from \((-2, 10)\) to \((2, 0)\) and from \((2, 0)\) to \((6, 4)\). A sketch of the graph of \( y = f(x) \) is shown in Figure 1.

![Figure 1](image_url)

(a) Write down the range of \( f \). (1)

(b) Find \( f(f(0)) \). (2)

The function \( g \) is defined by

\[
g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5
\]

(c) Find \( g^{-1}(x) \). (3)

(d) Solve the equation \( gf(x) = 16 \). (5)
Question 7 continued
Question 7 continued

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(Total 11 marks)
Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s\(^{-1}\).
Kate is 24 m ahead of John when she starts to cross the road from the fixed point \(A\).
John passes her as she reaches the other side of the road at a variable point \(B\), as shown in Figure 2.
Kate’s speed is \(V\) m s\(^{-1}\) and she moves in a straight line, which makes an angle \(\theta\) with the edge of the road, as shown in Figure 2.

You may assume that \(V\) is given by the formula

\[
V = \frac{21}{24\sin\theta + 7\cos\theta}, \quad 0 < \theta < 150^\circ
\]

(a) Express \(24\sin\theta + 7\cos\theta\) in the form \(R\cos(\theta - \alpha)\), where \(R\) and \(\alpha\) are constants and where \(R > 0\) and \(0 < \alpha < 90^\circ\), giving the value of \(\alpha\) to 2 decimal places.

(b) find the minimum value of \(V\).

(c) find the distance \(AB\).

(d) find the two possible values of the angle \(\theta\), given that \(0 < \theta < 150^\circ\).
Question 8 continued
Question 8 continued
Question 8 continued

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Edexcel GCE
Core Mathematics C4
Advanced
Tuesday 18 June 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. (a) Find \( \int x^2e^x \, dx \). 

(b) Hence find the exact value of \( \int_0^1 x^2e^x \, dx \).
Question 1 continued

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(Total 7 marks)
2. (a) Use the binomial expansion to show that

\[
\sqrt{\frac{1 + x}{1 - x}} = 1 + x + \frac{1}{2}x^2, \quad |x| < 1
\]  

(b) Substitute \( x = \frac{1}{26} \) into

\[
\sqrt{\frac{1 + x}{1 - x}} = 1 + x + \frac{1}{2}x^2
\]

to obtain an approximation to \( \sqrt{3} \)

Give your answer in the form \( \frac{a}{b} \) where \( a \) and \( b \) are integers.
Question 2 continued
Question 2 continued

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(Total 9 marks)
3.

Figure 1 shows the finite region \( R \) bounded by the \( x \)-axis, the \( y \)-axis, the line \( x = \frac{\pi}{2} \) and the curve with equation

\[
y = \sec \left( \frac{1}{2} x \right), \quad 0 \leq x \leq \frac{\pi}{2}
\]

The table shows corresponding values of \( x \) and \( y \) for \( y = \sec \left( \frac{1}{2} x \right) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \frac{\pi}{6} )</th>
<th>( \frac{\pi}{3} )</th>
<th>( \frac{\pi}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>1.035276</td>
<td>1.414214</td>
<td>1.414214</td>
</tr>
</tbody>
</table>

(a) Complete the table above giving the missing value of \( y \) to 6 decimal places.

(b) Using the trapezium rule, with all of the values of \( y \) from the completed table, find an approximation for the area of \( R \), giving your answer to 4 decimal places.

Region \( R \) is rotated through \( 2\pi \) radians about the \( x \)-axis.

(c) Use calculus to find the exact volume of the solid formed.
Question 3 continued
Question 3 continued
Question 3 continued

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(Total 8 marks)
4. A curve $C$ has parametric equations

$$x = 2 \sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$

(b) Find a cartesian equation for $C$ in the form

$$y = f(x), \quad -k \leq x \leq k,$$

stating the value of the constant $k$.

(c) Write down the range of $f(x)$.  

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5. (a) Use the substitution \( x = u^2, \ u > 0, \) to show that

\[
\int \frac{1}{x(2\sqrt{x} - 1)} \, dx = \int \frac{2}{u(2u - 1)} \, du
\]

(b) Hence show that

\[
\int_1^9 \frac{1}{x(2\sqrt{x} - 1)} \, dx = 2\ln\left(\frac{a}{b}\right)
\]

where \( a \) and \( b \) are integers to be determined.
Question 5 continued
Question 5 continued

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Question 5 continued

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(Total 10 marks)
6. Water is being heated in a kettle. At time $t$ seconds, the temperature of the water is $\theta ^\circ C$.

The rate of increase of the temperature of the water at any time $t$ is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \quad \theta \leq 100$$

where $\lambda$ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches $100 ^\circ C$, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)
Question 6 continued
Question 6 continued
Question 6 continued

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(Total 11 marks)
7. A curve is described by the equation
\[ x^2 + 4xy + y^2 + 27 = 0 \]
(a) Find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).

A point \( Q \) lies on the curve.
The tangent to the curve at \( Q \) is parallel to the \( y \)-axis.
Given that the \( x \) coordinate of \( Q \) is negative,
(b) use your answer to part (a) to find the coordinates of \( Q \).
Question 7 continued
Question 7 continued
Question 7 continued

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(Total 12 marks)
8. With respect to a fixed origin $O$, the line $l$ has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix},$$

where $\lambda$ is a scalar parameter.

The point $A$ lies on $l$ and has coordinates $(3, -2, 6)$.

The point $P$ has position vector $(-p \mathbf{i} + 2p \mathbf{k})$ relative to $O$, where $p$ is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to $l$,

(a) find the value of $p$.

(4)

Given also that $B$ is a point on $l$ such that $\angle BPA = 45^\circ$,

(b) find the coordinates of the two possible positions of $B$.

(5)
Question 8 continued
Question 8 continued

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Figure 1 shows the possible allocations of six people, Alex (A), Ben (B), Harriet (H), Izzy (I), Leo (L) and Rowan (R), to six tasks, 1, 2, 3, 4, 5 and 6.

(a) Write down the technical name given to the type of diagram shown in Figure 1.

(b) Starting from the given initial matching, use the maximum matching algorithm to find a complete matching. You should list the alternating paths you use, and state your improved matching after each iteration.

(Total 7 marks)
2.  

0.6  1.5  1.6  0.2  0.4  0.5  0.7  0.1  0.9  0.3

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 2.  

(b) The list of numbers is to be sorted into **descending order**. Use a quick sort to obtain the sorted list. You must make your pivots clear.  

(c) Apply the first-fit decreasing bin packing algorithm to your ordered list to pack the numbers into bins of size 2.  

(d) Determine whether your answer to (c) uses the minimum number of bins. You must justify your answer.  

(Total 11 marks)
The table shows the times, in days, needed to repair the network of roads between six towns, A, B, C, D, E and F, following a flood.

(a) Use **Prim’s** algorithm, starting at A, to find the minimum connector for this network. You must list the **arcs** that form your tree in the order that you selected them.

(b) Draw your minimum connector using the vertices given in Diagram 1 in the answer book.

(c) Add arcs from D, E and F to Diagram 2 in the answer book, so that it shows the network of roads shown by the table.

(d) Use **Kruskal’s** algorithm to find the minimum connector. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum connector.

(e) State the minimum time needed, in days, to reconnect the six towns.

(Total 10 marks)
4.

Figure 3 represents a network of roads. The number on each arc represents the length, in miles, of the corresponding road. Liz wishes to travel from S to T.

(a) Use Dijkstra’s algorithm to find the shortest path from S to T. State your path and its length. 

(b) Find the shortest path from S to T that includes F, and state its length.

(Total 8 marks)
Figure 4 represents a railway network. The number on each arc represents the length, in miles, of that section of the railway.

Sophie needs to travel along each section to check that it is in good condition.

She must travel along each arc of the network at least once, and wants to find a route of minimum length. She will start and finish at A.

(a) Use the route inspection algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.

(b) Write down a possible shortest inspection route, giving its length.

Sophie now decides to start the inspection route at E. The route must still traverse each arc at least once but may finish at any vertex.

(c) Determine the finishing point so that the length of the route is minimised. You must give reasons for your answer and state the length of your route.

(Total 10 marks)
6. Harry wants to rent out boats at his local park. He can use linear programming to determine the number of each type of boat he should buy.

Let \( x \) be the number of 2-seater boats and \( y \) be the number of 4-seater boats.

One of the constraints is

\[
x + y \geq 90
\]

(a) Explain what this constraint means in the context of the question. 

Another constraint is

\[
2x \leq 3y
\]

(b) Explain what this constraint means in the context of the question.

A third constraint is

\[
y \leq x + 30
\]

(c) Represent these three constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region \( R \).

Each 2-seater boat costs £100 and each 4-seater boat costs £300 to buy. Harry wishes to minimise the total cost of buying the boats.

(d) Write down the objective function, \( C \), in terms of \( x \) and \( y \).

(e) Determine the number of each type of boat that Harry should buy. You must make your method clear and state the minimum cost.

(Total 12 marks)
A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and late event times.

(b) Calculate the total float for activity M. You must make the numbers you use in your calculation clear.

(c) For each of the situations below, explain the effect that the delay would have on the project completion date.

(i) A 2 day delay on the early start of activity P.

(ii) A 2 day delay on the early start of activity Q.

(d) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.
Diagram 2 in the answer book shows a partly completed cascade chart for this project.

(e) Complete the cascade chart. (4)

(f) Use your cascade chart to determine a second lower bound on the number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities. (2)

(g) State which of the two lower bounds found in (d) and (f) is better. Give a reason for your answer. (2)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END
Edexcel GCE
Decision Mathematics D1
Advanced/Advanced Subsidiary
Friday 17 May 2013 – Morning

Answer Book

Do not return the question paper with the answer book
Question 1 continued

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<td>R</td>
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<td>R</td>
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(Total 7 marks)
2. 
Question 2 continued
3. (a)

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<td>-</td>
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<tr>
<td>B</td>
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<td>-</td>
<td>12</td>
<td>-</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>12</td>
<td>-</td>
<td>7</td>
<td>10</td>
<td>-</td>
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<td>D</td>
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<td>E</td>
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<td>11</td>
<td>-</td>
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<td>F</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>17</td>
<td>5</td>
<td>-</td>
</tr>
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</table>

Order of arcs: ________________________________________________________

(b)

Diagram 1
Question 3 continued

(c) Diagram 2

(d) ___________________________________________________________________
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(e) Minimum time needed: ____________________________

(Total 10 marks)
4. (a)

Vertex Order of labelling

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Order of labelling</th>
<th>Final value</th>
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</thead>
<tbody>
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<td>B</td>
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<td>C</td>
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<td>D</td>
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<td>G</td>
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<td>H</td>
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<td>T</td>
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</table>

Working values (in order)
Question 4 continued

Shortest path from S to T: __________________________________________

Length of shortest path from S to T: __________________________________

(b) ______________________________________________________________________________________________________________

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________________________________________________________________________________________________________________

Shortest path from S to T via F: _____________________________________

Length of shortest path from S to T via F: ___________________________
Figure 4

[The total weight of the network is 344 miles]
Question 5 continued

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(Total 10 marks)
Question 6 continued

Diagram 1

(Total 12 marks)
Diagram 1

(b) 

(c) 

(d)
(e) represents the total float on each activity

Diagram 2

(f) 

(g) 

(Total 17 marks)
Write your answers in the D2 answer book for this paper.

1.

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<th>C</th>
<th>D</th>
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<tbody>
<tr>
<td>A</td>
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<td>15</td>
<td>19</td>
<td>25</td>
<td>20</td>
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<tr>
<td>B</td>
<td>15</td>
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<td>25</td>
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<td>C</td>
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<td>11</td>
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<td>D</td>
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<td>E</td>
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</tbody>
</table>

The table shows the least distances, in km, between five hiding places, A, B, C, D and E.

Agent Goodie has to leave a secret message in each of the hiding places. He will start and finish at A, and wishes to minimise the total distance travelled.

(a) Use Prim’s algorithm to find a minimum spanning tree for this network. Make your order of arc selection clear.

(2)

(b) Use your answer to part (a) to determine an initial upper bound for the length of Agent Goodie’s route.

(1)

(c) Show that there are two nearest neighbour routes which start from A. State these routes and their lengths.

(3)

(d) State the better upper bound from your answers to (b) and (c).

(1)

(e) Starting by deleting B, and all of its arcs, find a lower bound for the length of Agent Goodie’s route.

(4)

(f) Consider your answers to (d) and (e) and hence state an optimal route.

(1)

(Total 12 marks)
2. The table shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of three demand points, 1, 2 and 3. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

<table>
<thead>
<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>Supply</th>
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<tbody>
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<td>A</td>
<td>10</td>
<td>11</td>
<td>20</td>
<td>18</td>
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<td>15</td>
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<td>13</td>
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<td>C</td>
<td>24</td>
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<td>D</td>
<td>9</td>
<td>21</td>
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<td>Demand</td>
<td>27</td>
<td>18</td>
<td>20</td>
<td></td>
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</tbody>
</table>

(a) Use the north-west corner method to obtain an initial solution.  
(1)

(b) Taking D1 as the entering cell, use the stepping stone method to find an improved solution. Make your route clear.  
(2)

(c) Perform one further iteration of the stepping stone method to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, route, entering cell and exiting cell.  
(4)

(d) Determine whether your current solution is optimal, giving a reason for your answer.  
(3)

(Total 10 marks)
3.

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.  

(b) State the capacity of cut $C_1$.  

The labelling procedure has been used and the result drawn on Diagram 1 in the answer book.

(c) Use Diagram 1 to find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.  

(d) Draw a maximum flow pattern on Diagram 2 in your answer book.  

(e) Prove that the flow shown in (d) is maximal.  

(Total 10 marks)
4. A two-person zero-sum game is represented by the following pay-off matrix for player A.

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<tr>
<th></th>
<th>B plays 1</th>
<th>B plays 2</th>
<th>B plays 3</th>
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</thead>
<tbody>
<tr>
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<td>4</td>
<td>–6</td>
</tr>
<tr>
<td>A plays 2</td>
<td>–1</td>
<td>–2</td>
<td>3</td>
</tr>
<tr>
<td>A plays 3</td>
<td>1</td>
<td>–1</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Reduce the game so that player B has only two possible actions. (1)

(b) Write down the reduced pay-off matrix for player B. (2)

(c) Find the best strategy for player B and the value of the game to him. (8)

(Total 11 marks)
5. In solving a three-variable maximising linear programming problem, the following tableau was obtained after the first iteration.

<table>
<thead>
<tr>
<th>Basic variable</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>r</th>
<th>s</th>
<th>t</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>-1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>8</td>
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<tr>
<td>s</td>
<td>-1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>z</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>P</td>
<td>2</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>15</td>
</tr>
</tbody>
</table>

(a) State which variable was increased first, giving a reason for your answer.  

(b) Solve this linear programming problem. Make your method clear by stating the row operations you use.  

(c) State the final value of the objective function and the final values of each variable.  

(Total 11 marks)
6. Three workers, Harriet, Jason and Katherine, are to be assigned to three tasks, 1, 2 and 3. Each worker must be assigned to just one task and each task must be done by just one worker.

The amount each person would earn, in pounds, while assigned to each task is shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harriet</td>
<td>251</td>
<td>243</td>
<td>257</td>
</tr>
<tr>
<td>Jason</td>
<td>244</td>
<td>247</td>
<td>255</td>
</tr>
<tr>
<td>Katherine</td>
<td>249</td>
<td>252</td>
<td>246</td>
</tr>
</tbody>
</table>

The total income is to be maximised.

(a) Modify the table so it can be used to find the maximum income.  

(b) Formulate the above situation as a linear programming problem. You must define your decision variables and make your objective function and constraints clear.

(Total 8 marks)
7. Nigel has a business renting out his fleet of bicycles to tourists.

At the start of each year Nigel must decide on one of two actions:

- Keep his fleet of bicycles, incurring maintenance costs.
- Replace his fleet of bicycles.

The cost of keeping the fleet of bicycles, the cost of replacing the fleet of bicycles and the annual income are dependent on the age of the fleet of bicycles. Table 1 shows these amounts, in £1000s.

<table>
<thead>
<tr>
<th>Age of fleet of bicycles</th>
<th>new</th>
<th>1 year old</th>
<th>2 years old</th>
<th>3 years old</th>
<th>4 years old</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of keeping (£1000s)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Cost of replacing (£1000s)</td>
<td>–</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Income (£1000s)</td>
<td>11</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1

Nigel has a new fleet of bicycles now and wishes to maximise his total profit over the next four years.

He is planning to sell his business at the end of the fourth year.

The amount Nigel will receive will depend on the age of his fleet of bicycles.

These amounts, in £1000s, are shown in Table 2.

<table>
<thead>
<tr>
<th>Age of fleet of bicycles at end of 4th year</th>
<th>1 year old</th>
<th>2 years old</th>
<th>3 years old</th>
<th>4 years old</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount received at end of 4th year (£1000s)</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2

Complete the table in the answer book to determine Nigel’s best strategy to maximise his total profit over the next four years. You must state the action he should take each year (keep or replace) and his total profit.

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END
Edexcel GCE
Decision Mathematics D2
Advanced/Advanced Subsidiary
Thursday 6 June 2013 – Morning

Answer Book

Do not return the question paper with the answer book
1. (a) 

<table>
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<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<td>25</td>
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<td>E</td>
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(b) 

(c) 

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Question 1 continued

(d)

(e)

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(f)

(Total 12 marks)
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(a)

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<tr>
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<td>27</td>
<td>18</td>
<td>20</td>
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</table>

(Total 10 marks)
3. (a) Value of initial flow

(b) Capacity of cut

Diagram 1
Question 3 continued

(d) Diagram 2

(Total 10 marks)
<table>
<thead>
<tr>
<th></th>
<th>B plays 1</th>
<th>B plays 2</th>
<th>B plays 3</th>
</tr>
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<tbody>
<tr>
<td>A plays 1</td>
<td>5</td>
<td>4</td>
<td>-6</td>
</tr>
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<td>A plays 2</td>
<td>-1</td>
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<td>3</td>
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<td>A plays 3</td>
<td>1</td>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>
5. (a) ____________________________________________________________________________
___________________________________________________________________________

(b) You may not need to use all of these tableaux

<table>
<thead>
<tr>
<th>b.v.</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>r</th>
<th>s</th>
<th>t</th>
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<th>y</th>
<th>z</th>
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<th>s</th>
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<th>Value</th>
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<tr>
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Question 5 continued

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(c)

(Total 11 marks)
6.

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<th>Task 3</th>
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<tbody>
<tr>
<td>Harriet</td>
<td>251</td>
<td>243</td>
<td>257</td>
</tr>
<tr>
<td>Jason</td>
<td>244</td>
<td>247</td>
<td>255</td>
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<tr>
<td>Katherine</td>
<td>249</td>
<td>252</td>
<td>246</td>
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Question 6 continued

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(Question 6 continued (Total 8 marks))
7. *You may not need to use all the rows in this table*

<table>
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<tr>
<th>Stage</th>
<th>State (Age of fleet of bicycles at start of year)</th>
<th>Action (Keep or Replace)</th>
<th>Dest. (Age of fleet of bicycles at end of year)</th>
<th>Value (Profit)</th>
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<tbody>
<tr>
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<td>Replace</td>
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<td>$6 + 11 - 9 = 8*$</td>
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**Question 7 continued**

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<th>Action (Keep or Replace)</th>
<th>Dest. (Age of fleet of bicycles at end of year)</th>
<th>Value (Profit)</th>
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<td>Keep/Replace?</td>
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Total profit

(QTotal 13 marks)
Edexcel GCE
Further Pure Mathematics FP1
Advanced/Advanced Subsidiary
Monday 10 June 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
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Answer ALL the questions.
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Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. \[
M = \begin{pmatrix}
x & x - 2 \\
3x - 6 & 4x - 11
\end{pmatrix}
\]

Given that the matrix M is singular, find the possible values of x. (4)
Question 1 continued

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(Total 4 marks)
2. \[ f(x) = \cos(x^2) - x + 3, \quad 0 < x < \pi \]

(a) Show that the equation \( f(x) = 0 \) has a root \( \alpha \) in the interval \([2.5, 3]\). 

(2)

(b) Use linear interpolation once on the interval \([2.5, 3]\) to find an approximation for \( \alpha \), giving your answer to 2 decimal places. 

(3)
Question 2 continued
3. Given that \( x = \frac{1}{2} \) is a root of the equation

\[ 2x^3 - 9x^2 + kx - 13 = 0, \quad k \in \mathbb{R} \]

find

(a) the value of \( k \),

(b) the other 2 roots of the equation.
Question 3 continued
4. The rectangular hyperbola $H$ has Cartesian equation $xy = 4$

   The point $P\left(2t, \frac{2}{t}\right)$ lies on $H$, where $t \neq 0$

   (a) Show that an equation of the normal to $H$ at the point $P$ is

   $$ty - t^2x = 2 - 2t^4$$

   (5)

   The normal to $H$ at the point where $t = -\frac{1}{2}$ meets $H$ again at the point $Q$.

   (b) Find the coordinates of the point $Q$. (4)
Question 4 continued

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Q4

(Total 9 marks)
5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (r + 2)(r + 3) = \frac{1}{3} n(n^2 + 9n + 26)$$

for all positive integers $n$.

(b) Hence show that

$$\sum_{r=n+1}^{2n} (r + 2)(r + 3) = \frac{2}{3} n(an^2 + bn + c)$$

where $a$, $b$ and $c$ are integers to be found.
Question 5 continued

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(Total 10 marks)
6. A parabola $C$ has equation $y^2 = 4ax, \quad a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on $C$, where $p \neq 0, q \neq 0, p \neq q$.

(a) Show that an equation of the tangent to the parabola at $P$ is

$$py - x = ap^2$$

(b) Write down the equation of the tangent at $Q$.

The tangent at $P$ meets the tangent at $Q$ at the point $R$.

(c) Find, in terms of $p$ and $q$, the coordinates of $R$, giving your answers in their simplest form.

Given that $R$ lies on the directrix of $C$,

(d) find the value of $pq$. 

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Question 6 continued
7. \( z_1 = 2 + 3i, \; z_2 = 3 + 2i, \; z_3 = a + bi, \; a, b \in \mathbb{R} \)

(a) Find the exact value of \(|z_1 + z_2|\).

Given that \( w = \frac{z_1 z_3}{z_2} \),

(b) find \( w \) in terms of \( a \) and \( b \), giving your answer in the form \( x + iy \), \( x, y \in \mathbb{R} \)

Given also that \( w = \frac{17}{13} - \frac{7}{13}i \),

(c) find the value of \( a \) and the value of \( b \),

(d) find \( \arg w \), giving your answer in radians to 3 decimal places.
Question 7 continued
Question 7 continued
Question 7 continued
8.

\[
A = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}
\]

and \(I\) is the \(2 \times 2\) identity matrix.

(a) Prove that

\[A^2 = 7A + 2I\]

(b) Hence show that

\[A^{-1} = \frac{1}{2}(A - 7I)\]

The transformation represented by \(A\) maps the point \(P\) onto the point \(Q\).

Given that \(Q\) has coordinates \((2k + 8, -2k - 5)\), where \(k\) is a constant,

(c) find, in terms of \(k\), the coordinates of \(P\).
Question 8 continued
Question 8 continued
9. (a) A sequence of numbers is defined by

\[ u_1 = 8 \]
\[ u_{n+1} = 4u_n - 9n, \quad n \geq 1 \]

Prove by induction that, for \( n \in \mathbb{Z}^+ \),

\[ u_n = 4^n + 3n + 1 \]

(b) Prove by induction that, for \( m \in \mathbb{Z}^+ \),

\[
\begin{pmatrix}
3 & -4 \\
1 & -1
\end{pmatrix}^m =
\begin{pmatrix}
2m + 1 & -4m \\
m & 1 - 2m
\end{pmatrix}
\]

(5)
Question 9 continued

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(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END
Edexcel GCE
Further Pure Mathematics FP2
Advanced/Advanced Subsidiary
Friday 21 June 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

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Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. (a) Express \( \frac{2}{(2r + 1)(2r + 3)} \) in partial fractions. 

(b) Using your answer to (a), find, in terms of \( n \),

\[
\sum_{r=1}^{n} \frac{3}{(2r + 1)(2r + 3)}
\]

Give your answer as a single fraction in its simplest form.
2. \( z = 5\sqrt{3} - 5i \)

Find

(a) \( |z| \), \((1)\)

(b) \( \arg(z) \), in terms of \( \pi \). \((2)\)

\[ w = 2 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \]

Find

(c) \( \left| \frac{w}{z} \right| \), \((1)\)

(d) \( \arg \left( \frac{w}{z} \right) \), in terms of \( \pi \). \((2)\)
Question 2 continued

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(Total 6 marks)
3. \[
\frac{d^2y}{dx^2} + 4y - \sin x = 0
\]

Given that \( y = \frac{1}{2} \) and \( \frac{dy}{dx} = \frac{1}{8} \) at \( x = 0 \),

find a series expansion for \( y \) in terms of \( x \), up to and including the term in \( x^3 \).
Question 3 continued

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(Total 5 marks)
4. (a) Given that
\[ z = r (\cos \theta + i \sin \theta), \quad r \in \mathbb{R} \]
prove, by induction, that \[ z^n = r^n (\cos n\theta + i \sin n\theta), \quad n \in \mathbb{Z}^+ \]
\[(5)\]

(b) Find the exact value of \( w^5 \), giving your answer in the form \( a + ib \), where \( a, b \in \mathbb{R} \).
\[(2)\]
Question 4 continued
Question 4 continued
5. (a) Find the general solution of the differential equation

\[ x \frac{dy}{dx} + 2y = 4x^2 \]  

(b) Find the particular solution for which \( y = 5 \) at \( x = 1 \), giving your answer in the form \( y = f(x) \).

(c) (i) Find the exact values of the coordinates of the turning points of the curve with equation \( y = f(x) \), making your method clear.

(ii) Sketch the curve with equation \( y = f(x) \), showing the coordinates of the turning points.
Question 5 continued
6. (a) Use algebra to find the exact solutions of the equation

$$|2x^2 + 6x - 5| = 5 - 2x$$

(b) On the same diagram, sketch the curve with equation $y = |2x^2 + 6x - 5|$ and the line with equation $y = 5 - 2x$, showing the $x$-coordinates of the points where the line crosses the curve.

(c) Find the set of values of $x$ for which

$$|2x^2 + 6x - 5| > 5 - 2x$$
Question 6 continued
Question 6 continued

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(Total 12 marks)
7. (a) Show that the transformation $y = xv$ transforms the equation

$$4x^2 \frac{d^2 y}{dx^2} - 8x \frac{dy}{dx} + (8 + 4x^2)y = x^4$$

into the equation

$$4 \frac{d^3 v}{dx^3} + 4v = x$$

(b) Solve the differential equation (II) to find $v$ as a function of $x$.

(c) Hence state the general solution of the differential equation (I).
Question 7 continued
Question 7 continued
8.

Figure 1

Figure 1 shows a curve $C$ with polar equation $r = a \sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line $l$. The half-line $l$ meets $C$ at the pole $O$ and at the point $P$. The tangent to $C$ at $P$ is parallel to the initial line. The polar coordinates of $P$ are $(R, \phi)$.

(a) Show that $\cos \phi = \frac{1}{\sqrt{3}}$  \hspace{1cm} (6)

(b) Find the exact value of $R$.  \hspace{1cm} (2)

The region $S$, shown shaded in Figure 1, is bounded by $C$ and $l$.

(c) Use calculus to show that the exact area of $S$ is

$$\frac{1}{36} a^2 \left( 9 \arccos \left( \frac{1}{\sqrt{3}} \right) + \sqrt{2} \right)$$  \hspace{1cm} (7)

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Question 8 continued
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(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END
Edexcel GCE
Further Pure Mathematics FP3
Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon
Time: 1 hour 30 minutes

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
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There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. A hyperbola \( H \) has equation

\[
\frac{x^2}{a^2} - \frac{y^2}{25} = 1,
\]

where \( a \) is a positive constant.

The foci of \( H \) are at the points with coordinates (13, 0) and (–13, 0).

Find

(a) the value of the constant \( a \),

(b) the equations of the directrices of \( H \).
Question 1 continued
2. (a) Find
\[ \int \frac{1}{\sqrt{4x^2 + 9}} \, dx \]  
\[ (2) \]

(b) Use your answer to part (a) to find the exact value of
\[ \int_{-3}^{3} \frac{1}{\sqrt{4x^2 + 9}} \, dx \]
giving your answer in the form \( k \ln(a + b \sqrt{5}) \), where \( a \) and \( b \) are integers and \( k \) is a constant.  
\[ (3) \]
Question 2 continued
Question 2 continued
Question 2 continued

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(Total 5 marks)
3. The curve with parametric equations

\[ x = \cosh 2\theta, \quad y = 4 \sinh \theta, \quad 0 \leq \theta \leq 1 \]

is rotated through \(2\pi\) radians about the \(x\)-axis.

Show that the area of the surface generated is \(\lambda (\cosh^3 \alpha - 1)\), where \(\alpha = 1\) and \(\lambda\) is a constant to be found.

(7)
Question 3 continued

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(Total 7 marks)
Figure 1

Figure 1 shows part of the curve with equation

\[ y = 40 \text{arcosh} x - 9x, \quad x \geq 1 \]

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form \( \left( \frac{p}{q}, r \ln 3 + s \right) \), where \( p, q, r \) and \( s \) are integers.

(7)
Question 4 continued
5. The matrix \( M \) is given by
\[
M = \begin{pmatrix}
1 & 1 & a \\
2 & b & c \\
-1 & 0 & 1
\end{pmatrix}, \text{ where } a, b \text{ and } c \text{ are constants.}
\]

(a) Given that \( j + k \) and \( i - k \) are two of the eigenvectors of \( M \), find

(i) the values of \( a, b \) and \( c \),

(ii) the eigenvalues which correspond to the two given eigenvectors.

(b) The matrix \( P \) is given by
\[
P = \begin{pmatrix}
1 & 1 & 0 \\
2 & 1 & d \\
-1 & 0 & 1
\end{pmatrix}, \text{ where } d \text{ is constant, } d \neq -1
\]

Find

(i) the determinant of \( P \) in terms of \( d \),

(ii) the matrix \( P^{-1} \) in terms of \( d \).
Question 5 continued
Question 5 continued

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(Total 13 marks)
6. Given that

\[ I_n = \int_0^4 x^n \sqrt{16 - x^2} \, dx, \quad n \geq 0, \]

(a) prove that, for \( n \geq 2, \)

\[ (n + 2)I_n = 16(n - 1)I_{n-2} \]  

(b) Hence, showing each step of your working, find the exact value of \( I_5 \)
Question 6 continued

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Question 6 continued
7. The ellipse $E$ has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b > 0$$

The line $l$ is a normal to $E$ at a point $P(a \cos \theta, b \sin \theta)$, $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for $l$ is

$$ax \sin \theta - by \cos \theta = (a^2 - b^2) \sin \theta \cos \theta$$

(5)

The line $l$ meets the $x$-axis at $A$ and the $y$-axis at $B$.

(b) Show that the area of the triangle $OAB$, where $O$ is the origin, may be written as $k \sin 2\theta$, giving the value of the constant $k$ in terms of $a$ and $b$.

(4)

(c) Find, in terms of $a$ and $b$, the exact coordinates of the point $P$, for which the area of the triangle $OAB$ is a maximum.

(3)
Question 7 continued
Question 7 continued
8. The plane \( \Pi_1 \) has vector equation
\[ \mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5 \]

(a) Find the perpendicular distance from the point \((6, 2, 12)\) to the plane \( \Pi_1 \)

The plane \( \Pi_2 \) has vector equation
\[ \mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k}), \text{ where } \lambda \text{ and } \mu \text{ are scalar parameters.} \]

(b) Find the acute angle between \( \Pi_1 \) and \( \Pi_2 \) giving your answer to the nearest degree.

(c) Find an equation of the line of intersection of the two planes in the form \( \mathbf{r} \times \mathbf{a} = \mathbf{b}, \)
where \( \mathbf{a} \) and \( \mathbf{b} \) are constant vectors.
Question 8 continued

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Question 8 continued
Edexcel GCE
Mechanics M1
Advanced/Advanced Subsidiary
Monday 13 May 2013 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of \( g \) is required, take \( g = 9.8 \text{ m s}^{-2} \).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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*P41828A0128*
1. Particle $P$ has mass 3 kg and particle $Q$ has mass $m$ kg. The particles are moving in opposite directions along a smooth horizontal plane when they collide directly. Immediately before the collision, the speed of $P$ is 4 m s$^{-1}$ and the speed of $Q$ is 3 m s$^{-1}$. In the collision the direction of motion of $P$ is unchanged and the direction of motion of $Q$ is reversed. Immediately after the collision, the speed of $P$ is 1 m s$^{-1}$ and the speed of $Q$ is 1.5 m s$^{-1}$.

(a) Find the magnitude of the impulse exerted on $P$ in the collision. (3)

(b) Find the value of $m$. (3)
Question 1 continued

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(Total 6 marks)
2. A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 m s\(^{-2}\). By modelling the cable as being light and inextensible, find

(a) the tension in the cable, \((3)\)

(b) the magnitude of the force exerted on the woman by the floor of the lift. \((3)\)
Question 2 continued

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(Total 6 marks)
A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle $\alpha$, where $\tan \alpha = \frac{3}{4}$, and the plane is at an angle of $30^\circ$ to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\mu = \frac{1}{2}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

Figure 1

A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle $\alpha$, where $\tan \alpha = \frac{3}{4}$, and the plane is at an angle of $30^\circ$ to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{2}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

(8)
Question 3 continued
Question 3 continued
4. A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point \( A \) with speed \( u \) m s\(^{-1}\), \((u < 34)\), and 10 seconds later passes a point \( B \) with speed 34 m s\(^{-1}\). Given that \( AB = 240 \) m, find

(a) the value of \( u \),

(b) the time taken for the lorry to move from \( A \) to the mid-point of \( AB \).
Question 4 continued

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Question 4 continued
5. A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s\(^{-1}\). The car maintains this speed for \(T\) seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights.

(b) Find the value of \(T\).

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, \(a\) m s\(^{-2}\), and passes the car at the point \(A\) which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s\(^{-1}\).

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point \(A\).

(d) Find the value of \(a\).
Question 5 continued
Question 5 continued

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(Total 11 marks)
6. A beam $AB$ has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points $P$ and $Q$, where $AP = 2$ m and $QB = 3$ m. When a child of mass 50 kg stands on the beam at $A$, the beam remains in equilibrium and is on the point of tilting about $P$. When the same child of mass 50 kg stands on the beam at $B$, the beam remains in equilibrium and is on the point of tilting about $Q$. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from $A$.  

When the child stands at the point $X$ on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find $AX$.  

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7. [In this question, the horizontal unit vectors \( \mathbf{i} \) and \( \mathbf{j} \) are directed due east and due north respectively.]

The velocity, \( \mathbf{v} \) m s\(^{-1}\), of a particle \( P \) at time \( t \) seconds is given by

\[
\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}
\]

(a) Find the speed of \( P \) when \( t = 0 \)

(b) Find the bearing on which \( P \) is moving when \( t = 2 \)

(c) Find the value of \( t \) when \( P \) is moving

(i) parallel to \( \mathbf{j} \),

(ii) parallel to \( -\mathbf{i} - 3\mathbf{j} \).
Question 7 continued

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Two particles $A$ and $B$ have masses $2m$ and $3m$ respectively. The particles are attached to the ends of a light inextensible string. Particle $A$ is held at rest on a smooth horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Particle $B$ hangs at rest vertically below the pulley with the string taut, as shown in Figure 2. Particle $A$ is released from rest. Assuming that $A$ has not reached the pulley, find

(a) the acceleration of $B$, \hspace{1cm} (5)

(b) the tension in the string, \hspace{1cm} (1)

(c) the magnitude and direction of the force exerted on the pulley by the string. \hspace{1cm} (4)
Question 8 continued
Question 8 continued

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(Total 10 marks)
Edexcel GCE
Mechanics M2
Advanced/Advanced Subsidiary
Thursday 6 June 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer to each question in the space following the question. Whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 7 questions in this question paper. The total mark for this paper is 75. There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the examiner. Answers without working may not gain full credit.
1. A particle $P$ of mass 2 kg is moving with velocity $(i - 4j)$ m s$^{-1}$ when it receives an impulse of $(3i + 6j)$ N s.

Find the speed of $P$ immediately after the impulse is applied. (5)
Question 1 continued

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(Total 5 marks)
2. A particle $P$ of mass 3 kg moves from point $A$ to point $B$ up a line of greatest slope of a fixed rough plane. The plane is inclined at $20^\circ$ to the horizontal. The coefficient of friction between $P$ and the plane is 0.4

Given that $AB = 15$ m and that the speed of $P$ at $A$ is $20$ m s$^{-1}$, find

(a) the work done against friction as $P$ moves from $A$ to $B$, 

(b) the speed of $P$ at $B$. 

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(Question 2 continued)
3. A particle $P$ moves on the $x$-axis. At time $t$ seconds the velocity of $P$ is $v \text{ m s}^{-1}$ in the direction of $x$ increasing, where

$$v = 2t^2 - 14t + 20, \quad t \geq 0$$

Find

(a) the times when $P$ is instantaneously at rest, (3)

(b) the greatest speed of $P$ in the interval $0 \leq t \leq 4$ (5)

(c) the total distance travelled by $P$ in the interval $0 \leq t \leq 4$ (5)
Question 3 continued

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4.

The uniform lamina $ABCDEF$ is a regular hexagon with centre $O$ and sides of length 2 m, as shown in Figure 1.

![Figure 1](image1)

The triangles $OAF$ and $OEF$ are removed to form the uniform lamina $OABCDE$, shown in Figure 2.

![Figure 2](image2)

(a) Find the distance of the centre of mass of $OABCDE$ from $O$. 

(b) Find the size of the angle between $EO$ and the downward vertical.
Question 4 continued
Question 4 continued
Question 4 continued

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(Total 11 marks)
A uniform rod $AB$, of mass $m$ and length $2a$, is freely hinged to a fixed point $A$. A particle of mass $m$ is attached to the rod at $B$. The rod is held in equilibrium at an angle $\theta$ to the horizontal by a force of magnitude $F$ acting at the point $C$ on the rod, where $AC = b$, as shown in Figure 3. The force at $C$ acts at right angles to $AB$ and in the vertical plane containing $AB$.

(a) Show that $F = \frac{3amg \cos \theta}{b}$.

(b) Find, in terms of $a$, $b$, $g$, $m$ and $\theta$,

(i) the horizontal component of the force acting on the rod at $A$,

(ii) the vertical component of the force acting on the rod at $A$.

(c) find the value of $\frac{a}{b}$.
Question 5 continued
Question 5 continued

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(\textbf{Total 13 marks})
A ball is projected from a point $A$ which is 8 m above horizontal ground as shown in Figure 4. The ball is projected with speed $u \text{ m s}^{-1}$ at an angle $\theta^\circ$ above the horizontal. The ball moves freely under gravity and hits the ground at the point $B$. The speed of the ball immediately before it hits the ground is $2u \text{ m s}^{-1}$.

(a) By considering energy, find the value of $u$.  

The time taken for the ball to move from $A$ to $B$ is 2 seconds. Find

(b) the value of $\theta$,  

(c) the minimum speed of the ball on its path from $A$ to $B$.  

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Question 6 continued

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(Total 11 marks)
7. Three particles $P$, $Q$ and $R$ lie at rest in a straight line on a smooth horizontal table with $Q$ between $P$ and $R$. The particles $P$, $Q$ and $R$ have masses $2m$, $3m$ and $4m$ respectively. Particle $P$ is projected towards $Q$ with speed $u$ and collides directly with it. The coefficient of restitution between each pair of particles is $e$.

(a) Show that the speed of $Q$ immediately after the collision with $P$ is $\frac{2}{5} (1 + e)u$. (6)

After the collision between $P$ and $Q$ there is a direct collision between $Q$ and $R$.

Given that $e = \frac{3}{4}$, find

(b) (i) the speed of $Q$ after this collision,

(ii) the speed of $R$ after this collision. (6)

Immediately after the collision between $Q$ and $R$, the rate of increase of the distance between $P$ and $R$ is $V$.

(c) Find $V$ in terms of $u$. (3)
Question 7 continued
Question 7 continued

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(Total 15 marks)
Edexcel GCE
Mechanics M3
Advanced/Advanced Subsidiary
Monday 10 June 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

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Instructions to Candidates
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Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of \( g \) is required, take \( g = 9.8 \, \text{m s}^{-2} \).
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
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Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
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You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1.

A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre $O$. A particle $P$ rests on the disc at a distance 0.4 m from $O$, as shown in Figure 1. The coefficient of friction between $P$ and the disc is $\mu$. The particle $P$ is on the point of slipping.

Find the value of $\mu$.  

(6)
2. A particle $P$ of mass 0.5 kg is moving along the positive $x$-axis in the positive $x$-direction. The only force on $P$ is a force of magnitude $\left(2t + \frac{1}{2}\right)\text{N}$ acting in the direction of $x$ increasing, where $t$ seconds is the time after $P$ leaves the origin $O$. When $t = 0$, $P$ is at rest at $O$.

(a) Find an expression, in terms of $t$, for the velocity of $P$ at time $t$ seconds.

(b) The particle passes through the point $A$ with speed 6 m s$^{-1}$.

(b) Find the distance $OA$. 

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Question 2 continued
Two particles $P$ and $Q$, of mass $m$ and $2m$ respectively, are attached to the ends of a light inextensible string of length $6l$. The string passes through a small smooth fixed ring at the point $A$. The particle $Q$ is hanging freely at a distance $l$ vertically below $A$. The particle $P$ is moving in a horizontal circle with constant angular speed $\omega$. The centre $O$ of the circle is vertically below $A$. The particle $Q$ does not move and $AP$ makes a constant angle $\theta$ with the downward vertical, as shown in Figure 2.

Show that

(i) $\theta = 60^\circ$

(ii) $\omega = \sqrt{\left(\frac{2g}{5l}\right)}$

(8)
Question 3 continued
Question 3 continued

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4. A particle $P$ of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m. The other end of the string is attached to a fixed point $O$ on a rough horizontal plane. The coefficient of friction between $P$ and the plane is $\frac{2}{5}$. The particle is held at rest at a point $B$ on the plane, where $OB = 1.5$ m. When $P$ is at $B$, the tension in the string is 20 N. The particle is released from rest.

(a) Find the speed of $P$ when $OP = 1.2$ m.  

The particle comes to rest at the point $C$.

(b) Find the distance $BC$. 

(7) 

(2)
Question 4 continued
Question 4 continued
Question 4 continued

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(Total 9 marks)
5.

The shaded region \( R \) is bounded by the curve with equation \( y = (x + 1)^2 \), the \( x \)-axis, the \( y \)-axis and the line with equation \( x = 2 \), as shown in Figure 3. The region \( R \) is rotated through \( 2\pi \) radians about the \( x \)-axis to form a uniform solid \( S \).

(a) Use algebraic integration to find the \( x \) coordinate of the centre of mass of \( S \).  

(b) A uniform solid hemisphere is fixed to \( S \) to form a solid \( T \). The hemisphere has the same radius as the smaller plane face of \( S \) and its plane face coincides with the smaller plane face of \( S \), as shown in Figure 4. The mass per unit volume of the hemisphere is 10 times the mass per unit volume of \( S \). The centre of the circular plane face of \( T \) is \( A \). All lengths are measured in centimetres.

(b) Find the distance of the centre of mass of \( T \) from \( A \).
Question 5 continued
Question 5 continued
Question 5 continued

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Question 5 continued
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6. The points $A$ and $B$ are 3.75 m apart on a smooth horizontal floor. A particle $P$ has mass 0.8 kg. One end of a light elastic spring, of natural length 1.5 m and modulus of elasticity 24 N, is attached to $P$ and the other end is attached to $A$. The ends of another light elastic spring, of natural length 0.75 m and modulus of elasticity 18 N, are attached to $P$ and $B$. The particle $P$ rests in equilibrium at the point $O$, where $AOB$ is a straight line, as shown in Figure 5.

(a) Show that $AO = 2.4$ m. (4)

The point $C$ lies on the straight line $AOB$ between $O$ and $B$. The particle $P$ is held at $C$ and released from rest.

(b) Show that $P$ moves with simple harmonic motion. (5)

The maximum speed of $P$ is $\sqrt{2}$ m s$^{-1}$.

(c) Find the time taken by $P$ to travel 0.3 m from $C$. (5)
Question 6 continued
Question 6 continued

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(Total 14 marks)
A particle $P$ of mass $5m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle is held at the point $A$, where $OA = a$ and $OA$ is horizontal, as shown in Figure 6. The particle is projected vertically downwards with speed $\sqrt{\frac{9ag}{5}}$. When the string makes an angle $\theta$ with the downward vertical through $O$ and the string is still taut, the tension in the string is $T$.

(a) Show that $T = 3mg(5\cos \theta + 3)$.

At the instant when the particle reaches the point $B$ the string becomes slack.

(b) Find the speed of $P$ at $B$.

At time $t = 0$, $P$ is at $B$.

At time $t$, before the string becomes taut once more, the coordinates of $P$ are $(x, y)$ referred to horizontal and vertical axes with origin $O$. The $x$-axis is directed along $OA$ produced and the $y$-axis is vertically upward.

(c) Find

(i) $x$ in terms of $t$, $a$ and $g$,

(ii) $y$ in terms of $t$, $a$ and $g$.

Figure 6
Question 7 continued
Question 7 continued
Edexcel GCE
Mechanics M4
Advanced/Advanced Subsidiary
Tuesday 18 June 2013 – Morning
Time: 1 hour 30 minutes

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. A particle $P$ of mass 0.5 kg falls vertically from rest. After $t$ seconds it has speed $v$ m s$^{-1}$. A resisting force of magnitude $1.5v$ newtons acts on $P$ as it falls.

(a) Show that $3v = 9.8(1 - e^{-3t})$.  

(b) Find the distance that $P$ falls in the first two seconds of its motion.
Question 1 continued

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(Total 13 marks)
2. Figure 1

A river is 50 m wide and flows between two straight parallel banks. The river flows with a uniform speed of \( \frac{2}{3} \text{ m s}^{-1} \) parallel to the banks. The points \( A \) and \( B \) are on opposite banks of the river and \( AB \) is perpendicular to both banks of the river, as shown in Figure 1.

Keith and Ian decide to swim across the river. The speed relative to the water of both swimmers is \( \frac{10}{9} \text{ m s}^{-1} \).

Keith sets out from \( A \) and crosses the river in the least possible time, reaching the opposite bank at the point \( C \). Find

(a) the time taken by Keith to reach \( C \),

(b) the distance \( BC \).

Ian sets out from \( A \) and swims in a straight line so as to land on the opposite bank at \( B \). Find

(c) the time taken by Ian to reach \( B \).
Question 2 continued
Two smooth uniform spheres \( A \) and \( B \), of equal radius \( r \), have masses \( 3m \) and \( 2m \) respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision they are moving with speeds \( u \) and \( 2u \) respectively. The centres of the spheres are moving towards each other along parallel paths at a distance \( 1.6r \) apart, as shown in Figure 2.

The coefficient of restitution between the two spheres is \( \frac{1}{6} \).

Find, in terms of \( m \) and \( u \), the magnitude of the impulse received by \( B \) in the collision.
Question 3 continued
Question 3 continued
Question 3 continued

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(Total 10 marks)
4.

A small smooth peg $P$ is fixed at a distance $d$ from a fixed smooth vertical wire. A particle of mass $3m$ is attached to one end of a light inextensible string which passes over $P$. The particle hangs vertically below $P$. The other end of the string is attached to a small ring $R$ of mass $m$, which is threaded on the wire, as shown in Figure 3.

(a) Show that when $R$ is at a distance $x$ below the level of $P$ the potential energy of the system is

$$3mg \sqrt{(x^2 + d^2)} - mgx + \text{constant}$$

(b) Hence find $x$, in terms of $d$, when the system is in equilibrium.

(c) Determine the stability of the position of equilibrium.
Question 4 continued
5. A coastguard ship $C$ is due south of a ship $S$. Ship $S$ is moving at a constant speed of 12 km h$^{-1}$ on a bearing of 140°. Ship $C$ moves in a straight line with constant speed $V$ km h$^{-1}$ in order to intercept $S$.

(a) Find, giving your answer to 3 significant figures, the minimum possible value for $V$.  

(b) Find the bearing of the course that $C$ takes to intercept $S$.  

It is now given that $V = 14$
Question 5 continued
6. A particle $P$ of mass $m$ kg is attached to the end $A$ of a light elastic string $AB$, of natural length $a$ metres and modulus of elasticity $9ma$ newtons. Initially the particle and the string lie at rest on a smooth horizontal plane with $AB = a$ metres. At time $t = 0$ the end $B$ of the string is set in motion and moves at a constant speed $U$ m s$^{-1}$ in the direction $AB$. The air resistance acting on $P$ has magnitude $6mv$ newtons, where $v$ m s$^{-1}$ is the speed of $P$. At time $t$ seconds, the extension of the string is $x$ metres and the displacement of $P$ from its initial position is $y$ metres.

Show that, while the string is taut,

(a) $x + y = Ut$  \hspace{1cm} (2)

(b) $\frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 9x = 6U$  \hspace{1cm} (5)

You are given that the general solution of the differential equation in (b) is

$$x = (A + Bt)e^{-3t} + \frac{2U}{3}$$

where $A$ and $B$ are arbitrary constants.

(c) Find the value of $A$ and the value of $B$.  \hspace{1cm} (5)

(d) Find the speed of $P$ at time $t$ seconds.  \hspace{1cm} (2)
Question 6 continued
7. [In this question \( \mathbf{i} \) and \( \mathbf{j} \) are perpendicular unit vectors in a horizontal plane]

A small smooth ball of mass \( m \) kg is moving on a smooth horizontal plane and strikes a fixed smooth vertical wall. The plane and the wall intersect in a straight line which is parallel to the vector \( 2\mathbf{i} + \mathbf{j} \). The velocity of the ball immediately before the impact is \( b\mathbf{i} \text{ m s}^{-1} \), where \( b \) is positive. The velocity of the ball immediately after the impact is \( a(\mathbf{i} + \mathbf{j}) \text{ m s}^{-1} \), where \( a \) is positive.

(a) Show that the impulse received by the ball when it strikes the wall is parallel to \( (-\mathbf{i} + 2\mathbf{j}) \).

(b) Find the coefficient of restitution between the ball and the wall,

(c) the fraction of the kinetic energy of the ball that is lost due to the impact.
Question 7 continued
Question 7 continued
Question 7 continued

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(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END
Edexcel GCE
Mechanics M5
Advanced/Advanced Subsidiary
Monday 24 June 2013 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
1. Solve the differential equation

\[ \frac{dr}{dt} - 2r = 0 \]

given that when \( t = 0 \), \( \mathbf{r} \cdot \mathbf{j} = 0 \) and \( \mathbf{r} \times \mathbf{j} = \mathbf{i} + \mathbf{k} \). (7)
Question 1 continued
2. A uniform square lamina $S$ has side $2a$. The radius of gyration of $S$ about an axis through a vertex, perpendicular to $S$, is $k$.

(a) Show that $k^2 = \frac{8a^2}{3}$.  

(4)

The lamina $S$ is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to $S$ and passes through a vertex.

(b) By writing down an equation of rotational motion for $S$, find the period of small oscillations of $S$ about its position of stable equilibrium.  

(5)
Question 2 continued
3. A raindrop falls vertically under gravity through a stationary cloud. At time $t = 0$, the raindrop is at rest and has mass $m_0$. As the raindrop falls, water condenses onto it from the cloud so that the mass of the raindrop increases at a constant rate $c$. At time $t$, the mass of the raindrop is $m$ and the speed of the raindrop is $v$. The resistance to the motion of the raindrop has magnitude $mkv$, where $k$ is a constant. Show that

$$\frac{dv}{dt} + v\left(k + \frac{c}{m_0 + ct}\right) = g$$

(7)
Question 3 continued

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(Total 7 marks)
4. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act through the points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively.

$\mathbf{r}_1 = (-2i + 3j) \text{ m}, \quad \mathbf{F}_1 = (3i - 2j + k) \text{ N}$

$\mathbf{r}_2 = (3i + 2k) \text{ m}, \quad \mathbf{F}_2 = (-2i + j - k) \text{ N}$

Given that the system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is in equilibrium,

(a) find $\mathbf{F}_3$, \hspace{1cm} (2)

(b) find a vector equation of the line of action of $\mathbf{F}_3$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. \hspace{1cm} (5)

The force $\mathbf{F}_3$ is replaced by a force $\mathbf{F}_4$ acting through the point with position vector $(i - 2j + 3k) \text{ m}$. The system $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ is equivalent to a single force $(3i + j + k) \text{ N}$ acting through the point with position vector $(i + j + k) \text{ m}$ together with a couple.

(c) Find the magnitude of this couple. \hspace{1cm} (8)
Question 4 continued

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5.

A uniform triangular lamina $ABC$, of mass $M$, has $AB = AC$ and $BC = 2a$. The mid-point of $BC$ is $D$ and $AD = h$, as shown in Figure 1.

Show, using integration, that the moment of inertia of the lamina about an axis through $A$, perpendicular to the plane of the lamina, is

$$\frac{M}{6} (a^2 + 3h^2)$$

[You may assume without proof that the moment of inertia of a uniform rod, of length $2l$ and mass $m$, about an axis through its midpoint and perpendicular to the rod, is $\frac{1}{2}ml^2$.]

(10)
Question 5 continued
Question 5 continued

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(Total 10 marks)
A light inextensible string has a particle of mass \( m \) attached to one end and a particle of mass \( 4m \) attached to the other end. The string passes over a rough pulley which is modelled as a uniform circular disc of radius \( a \) and mass \( 2m \), as shown in Figure 2.

The pulley can rotate in a vertical plane about a fixed horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. As the pulley rotates, a frictional couple of constant magnitude \( 2mga \) acts on it.

The system is held with the string vertical and taut on each side of the pulley and released from rest. Given that the string does not slip on the pulley, find the initial angular acceleration of the pulley.
Question 6 continued

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7. A uniform circular disc, of radius $r$ and mass $m$, is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis is perpendicular to the plane of the disc and passes through a point $A$ on the circumference of the disc. The disc is held with $AB$ horizontal, where $AB$ is a diameter of the disc, and released from rest.

(a) Find the magnitude of

(i) the horizontal component,

(ii) the vertical component

of the force exerted on the disc by the axis immediately after the disc is released.

(b) Find the magnitude of the impulse.
Question 7 continued
Question 7 continued
Edexcel GCE
Statistics S1
Advanced/Advanced Subsidiary
Friday 17 May 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer to each question in the space following the question. Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.
1. A meteorologist believes that there is a relationship between the height above sea level, \( h \) m, and the air temperature, \( t \) °C. Data is collected at the same time from 9 different places on the same mountain. The data is summarised in the table below.

<table>
<thead>
<tr>
<th>( h )</th>
<th>1400</th>
<th>1100</th>
<th>260</th>
<th>840</th>
<th>900</th>
<th>550</th>
<th>1230</th>
<th>100</th>
<th>770</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>3</td>
<td>10</td>
<td>20</td>
<td>9</td>
<td>10</td>
<td>13</td>
<td>5</td>
<td>24</td>
<td>16</td>
</tr>
</tbody>
</table>

[You may assume that \( \sum h = 7150, \sum t = 110, \sum h^2 = 7171500, \sum t^2 = 1716, \sum th = 64980 \) and \( S_{tt} = 371.56 \)]

(a) Calculate \( S_{th} \) and \( S_{hh} \). Give your answers to 3 significant figures.

(b) Calculate the product moment correlation coefficient for this data.

(c) State whether or not your value supports the use of a regression equation to predict the air temperature at different heights on this mountain. Give a reason for your answer.

(d) Find the equation of the regression line of \( t \) on \( h \) giving your answer in the form \( t = a + bh \).

(e) Interpret the value of \( b \).

(f) Estimate the difference in air temperature between a height of 500 m and a height of 1000 m.
Question 1 continued

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2. The marks of a group of female students in a statistics test are summarised in Figure 1.

![Boxplot of female and male marks](image)

**Figure 1**

(a) Write down the mark which is exceeded by 75% of the female students.  

The marks of a group of male students in the same statistics test are summarised by the stem and leaf diagram below.

| Mark | (2|6 means 26) | Totals |
|------|--------------|--------|
| 1    | 4            | (1)    |
| 2    | 6            | (1)    |
| 3    | 4 4 7        | (3)    |
| 4    | 0 6 6 7 7 8  | (6)    |
| 5    | 0 0 1 1 1 3 6 7 7 | (9) |
| 6    | 2 2 3 3 3 8  | (6)    |
| 7    | 0 0 8        | (3)    |
| 8    | 5            | (1)    |
| 9    | 0            | (1)    |

(b) Find the median and interquartile range of the marks of the male students.

An outlier is a mark that is

either more than \(1.5 \times \text{interquartile range}\) above the upper quartile

or more than \(1.5 \times \text{interquartile range}\) below the lower quartile.
(c) In the space provided on Figure 1 draw a box plot to represent the marks of the male students, indicating clearly any outliers. (5)

(d) Compare and contrast the marks of the male and the female students. (2)
Question 2 continued

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3. In a company the 200 employees are classified as full-time workers, part-time workers or contractors. The table below shows the number of employees in each category and whether they walk to work or use some form of transport.

<table>
<thead>
<tr>
<th></th>
<th>Walk</th>
<th>Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-time worker</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Part-time worker</td>
<td>35</td>
<td>75</td>
</tr>
<tr>
<td>Contractor</td>
<td>30</td>
<td>50</td>
</tr>
</tbody>
</table>

The events $F$, $H$ and $C$ are that an employee is a full-time worker, part-time worker or contractor respectively. Let $W$ be the event that an employee walks to work.

An employee is selected at random.

Find

(a) $P(H)$

(b) $P([F \cap W]')$

(c) $P(W | C)$

Let $B$ be the event that an employee uses the bus. Given that 10% of full-time workers use the bus, 30% of part-time workers use the bus and 20% of contractors use the bus,

(d) draw a Venn diagram to represent the events $F$, $H$, $C$ and $B$,

(e) find the probability that a randomly selected employee uses the bus to travel to work.
Question 3 continued
Question 3 continued

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Question 3 continued
4. The following table summarises the times, \( t \) minutes to the nearest minute, recorded for a group of students to complete an exam.

<table>
<thead>
<tr>
<th>Time (minutes) ( t )</th>
<th>11 – 20</th>
<th>21 – 25</th>
<th>26 – 30</th>
<th>31 – 35</th>
<th>36 – 45</th>
<th>46 – 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students ( f )</td>
<td>62</td>
<td>88</td>
<td>16</td>
<td>13</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

[You may use \( \sum f t^2 = 134281.25 \)]

(a) Estimate the mean and standard deviation of these data. (5)

(b) Use linear interpolation to estimate the value of the median. (2)

(c) Show that the estimated value of the lower quartile is 18.6 to 3 significant figures. (1)

(d) Estimate the interquartile range of this distribution. (2)

(e) Give a reason why the mean and standard deviation are not the most appropriate summary statistics to use with these data. (1)

The person timing the exam made an error and each student actually took 5 minutes less than the times recorded above. The table below summarises the actual times.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students ( f )</td>
<td>62</td>
<td>88</td>
<td>16</td>
<td>13</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

(f) Without further calculations, explain the effect this would have on each of the estimates found in parts (a), (b), (c) and (d). (3)
Question 4 continued
Question 4 continued
5. A biased die with six faces is rolled. The discrete random variable $X$ represents the score
on the uppermost face. The probability distribution of $X$ is shown in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$a$</td>
<td>$a$</td>
<td>$a$</td>
<td>$b$</td>
<td>$b$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

(a) Given that $E(X) = 4.2$ find the value of $a$ and the value of $b$. (5)

(b) Show that $E(X^2) = 20.4$ (1)

(c) Find $\text{Var}(5 - 3X)$ (3)

A biased die with five faces is rolled. The discrete random variable $Y$ represents the score
which is uppermost. The cumulative distribution function of $Y$ is shown in the table
below.

<table>
<thead>
<tr>
<th>$y$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(y)$</td>
<td>$\frac{1}{10}$</td>
<td>$\frac{2}{10}$</td>
<td>$3k$</td>
<td>$4k$</td>
<td>$5k$</td>
</tr>
</tbody>
</table>

(d) Find the value of $k$. (1)

(e) Find the probability distribution of $Y$. (3)

Each die is rolled once. The scores on the two dice are independent.

(f) Find the probability that the sum of the two scores equals 2 (2)
Question 5 continued

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6. The weight, in grams, of beans in a tin is normally distributed with mean $\mu$ and standard deviation 7.8

Given that 10% of tins contain less than 200 g, find

(a) the value of $\mu$ 

(b) the percentage of tins that contain more than 225 g of beans.

The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with mean 205 and standard deviation $\sigma$.

(c) Given that 98% of tins contain between 200 g and 210 g find the value of $\sigma$. 

Question 6 continued

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Question 6 continued
Edexcel GCE
Statistics S2
Advanced/Advanced Subsidiary
Friday 24 May 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer to each question in the space following the question. Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 7 questions in this question paper. The total mark for this paper is 75. There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.
1. A bag contains a large number of 1p, 2p and 5p coins.

   50% are 1p coins
   20% are 2p coins
   30% are 5p coins

A random sample of 3 coins is chosen from the bag.

(a) List all the possible samples of size 3 with median 5p. (2)

(b) Find the probability that the median value of the sample is 5p. (4)

(c) Find the sampling distribution of the median of samples of size 3. (5)
Question 1 continued

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(Total 11 marks)
2. The number of defects per metre in a roll of cloth has a Poisson distribution with mean 0.25

Find the probability that

(a) a randomly chosen metre of cloth has 1 defect,

(b) the total number of defects in a randomly chosen 6 metre length of cloth is more than 2

A tailor buys 300 metres of cloth.

(c) Using a suitable approximation find the probability that the tailor’s cloth will contain less than 90 defects.
Question 2 continued

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(Total 10 marks)
3. An online shop sells a computer game at an average rate of 1 per day.

(a) Find the probability that the shop sells more than 10 games in a 7 day period. (3)

Once every 7 days the shop has games delivered before it opens.

(b) Find the least number of games the shop should have in stock immediately after a delivery so that the probability of running out of the game before the next delivery is less than 0.05 (3)

In an attempt to increase sales of the computer game, the price is reduced for six months. A random sample of 28 days is taken from these six months. In the sample of 28 days, 36 computer games are sold.

(c) Using a suitable approximation and a 5% level of significance, test whether or not the average rate of sales per day has increased during these six months. State your hypotheses clearly. (7)
Question 3 continued

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4. A continuous random variable $X$ is uniformly distributed over the interval $[b, 4b]$ where $b$ is a constant.

(a) Write down $E(X)$. (1)

(b) Use integration to show that $\text{Var}(X) = \frac{3b^2}{4}$. (3)

(c) Find $\text{Var}(3 - 2X)$. (2)

Given that $b = 1$ find

(d) the cumulative distribution function of $X$, $F(x)$, for all values of $x$, (2)

(e) the median of $X$. (1)
Question 4 continued
Question 4 continued
Question 4 continued

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5. The continuous random variable $X$ has a cumulative distribution function

$$F(x) = \begin{cases} 
0 & x < 1 \\
\frac{x^3}{10} + \frac{3x^2}{10} + ax + b & 1 \leq x \leq 2 \\
1 & x > 2
\end{cases}$$

where $a$ and $b$ are constants.

(a) Find the value of $a$ and the value of $b$.

(b) Show that $f(x) = \frac{3}{10} (x^2 + 2x - 2), \quad 1 \leq x \leq 2$

(c) Use integration to find $E(X)$.

(d) Show that the lower quartile of $X$ lies between $1.425$ and $1.435$
Question 5 continued
Question 5 continued

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(Total 12 marks)
6. In a manufacturing process 25% of articles are thought to be defective. Articles are produced in batches of 20

(a) A batch is selected at random. Using a 5% significance level, find the critical region for a two tailed test that the probability of an article chosen at random being defective is 0.25
You should state the probability in each tail which should be as close as possible to 0.025

(5)

The manufacturer changes the production process to try to reduce the number of defective articles. She then chooses a batch at random and discovers there are 3 defective articles.

(b) Test at the 5% level of significance whether or not there is evidence that the changes to the process have reduced the percentage of defective articles. State your hypotheses clearly.

(5)
Question 6 continued
7. A telesales operator is selling a magazine. Each day he chooses a number of people to telephone. The probability that each person he telephones buys the magazine is 0.1

(a) Suggest a suitable distribution to model the number of people who buy the magazine from the telesales operator each day.  

(b) On Monday, the telesales operator telephones 10 people. Find the probability that he sells at least 4 magazines.

(c) Calculate the least number of people he needs to telephone on Tuesday, so that the probability of selling at least 1 magazine, on that day, is greater than 0.95

A call centre also sells the magazine. The probability that a telephone call made by the call centre sells a magazine is 0.05  
The call centre telephones 100 people every hour.

(d) Using a suitable approximation, find the probability that more than 10 people telephoned by the call centre buy a magazine in a randomly chosen hour.
Question 7 continued
Edexcel GCE
Statistics S3
Advanced/Advanced Subsidiary
Thursday 13 June 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

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Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 7 questions in this question paper. The total mark for this paper is 75. There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

Turn over
1. A doctor takes a random sample of 100 patients and measures their intake of saturated fats in their food and the level of cholesterol in their blood. The results are summarised in the table below.

<table>
<thead>
<tr>
<th>Cholesterol level</th>
<th>Intake of saturated fats</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>High</td>
<td>12</td>
</tr>
<tr>
<td>Low</td>
<td>26</td>
</tr>
</tbody>
</table>

Using a 5% level of significance, test whether or not there is an association between cholesterol level and intake of saturated fats. State your hypotheses and show your working clearly.

(10)
Question 1 continued
2. The table below shows the number of students per member of staff and the student satisfaction scores for 7 universities.

<table>
<thead>
<tr>
<th>University</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students per member of staff</td>
<td>14.2</td>
<td>13.1</td>
<td>13.3</td>
<td>11.7</td>
<td>10.5</td>
<td>15.9</td>
<td>10.8</td>
</tr>
<tr>
<td>Student satisfaction score</td>
<td>4.1</td>
<td>4.2</td>
<td>3.8</td>
<td>4.0</td>
<td>3.9</td>
<td>4.3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

(a) Calculate Spearman’s rank correlation coefficient for these data. (5)

(b) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence of a correlation between the number of students per member of staff and the student satisfaction score. (3)
3. A college manager wants to survey students’ opinions of enrichment activities. She decides to survey the students on the courses summarised in the table below.

<table>
<thead>
<tr>
<th>Course</th>
<th>Number of students enrolled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure and Sport</td>
<td>420</td>
</tr>
<tr>
<td>Information Technology</td>
<td>337</td>
</tr>
<tr>
<td>Health and Social Care</td>
<td>200</td>
</tr>
<tr>
<td>Media Studies</td>
<td>43</td>
</tr>
</tbody>
</table>

Each student takes only one course.

The manager has access to the college’s information system that holds full details of each of the enrolled students including name, address, telephone number and their course of study. She wants to compare the opinions of students on each course and has a generous budget to pay for the cost of the survey.

(a) Give one advantage and one disadvantage of carrying out this survey using

(i) quota sampling,

(ii) stratified sampling.

The manager decides to take a stratified sample of 100 students.

(b) Calculate the number of students to be sampled from each course.

(c) Describe how to choose students for the stratified sample.
Question 3 continued

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(Total 7 marks)
4. Customers at a post office are timed to see how long they wait until being served at the counter. A random sample of 50 customers is chosen and their waiting times, \( x \) minutes, are summarised in Table 1.

<table>
<thead>
<tr>
<th>Waiting time in minutes (( x ))</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–3</td>
<td>8</td>
</tr>
<tr>
<td>3–5</td>
<td>12</td>
</tr>
<tr>
<td>5–6</td>
<td>13</td>
</tr>
<tr>
<td>6–8</td>
<td>9</td>
</tr>
<tr>
<td>8–12</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 1

(a) Show that an estimate of \( \bar{x} = 5.49 \) and an estimate of \( s_x^2 = 6.88 \)  

The post office manager believes that the customers’ waiting times can be modelled by a normal distribution. Assuming the data is normally distributed, she calculates the expected frequencies for these data and some of these frequencies are shown in Table 2.

<table>
<thead>
<tr>
<th>Waiting Time</th>
<th>( x &lt; 3 )</th>
<th>3–5</th>
<th>5–6</th>
<th>6–8</th>
<th>( x &gt; 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Frequency</td>
<td>8.56</td>
<td>12.73</td>
<td>7.56</td>
<td>( a )</td>
<td>( b )</td>
</tr>
</tbody>
</table>

Table 2

(b) Find the value of \( a \) and the value of \( b \).

(c) Test, at the 5% level of significance, the manager’s belief. State your hypotheses clearly.
Question 4 continued

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Question 4 continued
5. Blumen is a perfume sold in bottles. The amount of perfume in each bottle is normally distributed. The amount of perfume in a large bottle has mean 50ml and standard deviation 5ml. The amount of perfume in a small bottle has mean 15ml and standard deviation 3ml.

One large and 3 small bottles of Blumen are chosen at random.

(a) Find the probability that the amount in the large bottle is less than the total amount in the 3 small bottles.

(b) Find the probability that the large bottle contains more than 3 times the amount in the small bottle.
Question 5 continued
6. Fruit-n-Veg4U Market Gardens grow tomatoes. They want to improve their yield of tomatoes by at least 1 kg per plant by buying a new variety. The variance of the yield of the old variety of plant is 0.5 kg$^2$ and the variance of the yield for the new variety of plant is 0.75 kg$^2$. A random sample of 60 plants of the old variety has a mean yield of 5.5 kg. A random sample of 70 of the new variety has a mean yield of 7 kg.

(a) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence that the mean yield of the new variety is more than 1 kg greater than the mean yield of the old variety.

(b) Explain the relevance of the Central Limit Theorem to the test in part (a).
7. Lambs are born in a shed on Mill Farm. The birth weights, $x$ kg, of a random sample of 8 newborn lambs are given below.

4.12  5.12  4.84  4.65  3.55  3.65  3.96  3.40

(a) Calculate unbiased estimates of the mean and variance of the birth weight of lambs born on Mill Farm.

(3)

A further random sample of 32 lambs is chosen and the unbiased estimates of the mean and variance of the birth weight of lambs from this sample are 4.55 and 0.25 respectively.

(b) Treating the combined sample of 40 lambs as a single sample, estimate the standard error of the mean.

(7)

The owner of Mill Farm researches the breed of lamb and discovers that the population of birth weights is normally distributed with standard deviation 0.67 kg.

(c) Calculate a 95% confidence interval for the mean birth weight of this breed of lamb using your combined sample mean.

(3)

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Question 7 continued

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(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
General Principles for Core Mathematics Marking

Method mark for solving 3 term quadratic:
1. **Factorisation**
   
   \[(x^2 + bx + c) = (x + p)(x + q), \text{ where } pq = |c|, \text{ leading to } x = \ldots\]
   
   \[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } pq = |c| \text{ and } mn = |a|, \text{ leading to } x = \ldots\]

2. **Formula**
   
   Attempt to use correct formula (with values for \(a\), \(b\) and \(c\)).

3. **Completing the square**
   
   Solving \(x^2 + bx + c = 0\): \((x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0\), leading to \(x = \ldots\)

Method marks for differentiation and integration:
1. **Differentiation**
   
   Power of at least one term decreased by 1. \((x^n \rightarrow x^{n-1})\)

2. **Integration**
   
   Power of at least one term increased by 1. \((x^n \rightarrow x^{n+1})\)

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:
**Method mark** for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
### January 2013
6663 Core Mathematics C1
Mark Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1.              | \( x(1 - 4x^2) \)

Accept \( x(-4x^2 + 1) \) or \(-x(4x^2 - 1) \) or \(-x(-1 + 4x^2) \) or even \( 4x\left(\frac{1}{4} - x^2\right) \) or equivalent quadratic (or initial cubic) into two brackets

\( x(1 - 2x)(1 + 2x) \) or \(-x(2x - 1)(2x + 1) \) or \( x(2x - 1)(-2x - 1) \)

<table>
<thead>
<tr>
<th>B1</th>
<th>M1</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[3]</td>
</tr>
</tbody>
</table>

#### Notes

**B1**: Takes out a factor of \( x \) or \(-x \) or even \( 4x \). This line may be implied by correct final answer, but if this stage is shown it **must be correct**. So **B0** for \( x(1 + 4x^2) \)

**M1**: Factorises the quadratic resulting from their first factorisation using usual rules (see note 1 in General Principles). e.g. \( x(1 - 4x)(x - 1) \). Also allow attempts to factorise cubic such as \( (x - 2x^2)(1 + 2x) \) etc

N.B. Should not be completing the square here.

**A1**: Accept either \( (1 - 2x)(1 + 2x) \) or \(-x(2x - 1)(2x + 1) \) or \( x(2x - 1)(-2x - 1) \). (No fractions for this final answer)

#### Specific situations

**Note**: \( x(1 - 4x^2) \) followed by \( x(1 - 2x)^2 \) scores **B1M1A0** as factors follow quadratic factorisation criteria

**And**: \( x(1 - 4x^2) \) followed by \( x(1 - 4x)(1 + 4x) \) **B1M0A0**.

#### Answers with no working:
Correct answer gets all three marks **B1M1A1**

- \( x(2x - 1)(2x + 1) \) gets **B0M1A0** if **no working** as \( x(4x^2 - 1) \) would earn **B0**

#### Poor bracketing: e.g. \(-1 + 4x^2\) \(-x \) gets B0 unless subsequent work implies bracket round the \(-x \) in which case candidate may recover the mark by the following correct work.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>(8^{2x+3} = (2^3)^{2x+3} = 2^{3(2x+3)} ) or (2^{a+b} ) with (a = 6) or (b = 9) (\Rightarrow 2^{6x+9} \text{ or } 2^{3(2x+3)}) as final answer with no errors or ((y =) 6x + 9) or (3(2x + 3))</td>
</tr>
</tbody>
</table>

### Notes

**M1:** Uses \(8 = 2^3\), and multiplies powers \(3(2x + 3)\). Does not add powers. (Just \(8 = 2^3\) or \(8^2 = 2\) is M0)

**A1:** Either \(2^{6x+9} \text{ or } 2^{3(2x+3)}\) or \((y =) 6x + 9\) or \(3(2x + 3)\)

**Note:** Examples: \(6^{x+3}\) scores M1A0

| Special case: \(2^6 \cdot 2^9\) without seeing as single power M1A0 |

**Alternative method using logs:** \(8^{2x+3} = 2^{2x+3} \Rightarrow (2x + 3)\log 8 = y \log 2 \Rightarrow y = \frac{(2x + 3)\log 8}{\log 2}\)

So \((y =) 6x + 9\) or \(3(2x + 3)\) | A1 [2] |
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. (i)</td>
<td></td>
<td>[3]</td>
</tr>
<tr>
<td>((5 - \sqrt{8})(1 + \sqrt{2}))</td>
<td>(= 5 + 5\sqrt{2} - \sqrt{8} - 4)</td>
<td>(\sqrt{8} = 2\sqrt{2}), seen or implied at any point. 1 + 3(\sqrt{2}) or (a = 1) and (b = 3).</td>
</tr>
<tr>
<td></td>
<td>(= 5 + 3\sqrt{2})</td>
<td>M1</td>
</tr>
<tr>
<td>(i)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Method 1</td>
<td>Method 2</td>
</tr>
<tr>
<td>Either (\frac{\sqrt{80} + 30\sqrt{5}}{\sqrt{5}}) = 4(\sqrt{5}) + ...</td>
<td>Or (\left(\frac{\sqrt{400} + 30}{\sqrt{5}}\right)\sqrt{5})  = (\frac{\sqrt{80} + \sqrt{900}}{\sqrt{5}} = \sqrt{80} + \sqrt{180})</td>
<td>(= 4\sqrt{5} + ...)</td>
</tr>
<tr>
<td></td>
<td>(= 4\sqrt{5} + \sqrt{2} \sqrt{5})</td>
<td>= (\frac{50\sqrt{5}}{5}) = 10(\sqrt{5})</td>
</tr>
<tr>
<td>Alternative for (i)</td>
<td>((5 - 2\sqrt{2})(1 + \sqrt{2}))</td>
<td>This earns the B1 mark.</td>
</tr>
<tr>
<td></td>
<td>(= 5 + 5\sqrt{2} - 2\sqrt{2} - 2\sqrt{2}\sqrt{2})</td>
<td>Multiplies out correctly with (2\sqrt{2}). This may be seen or implied and may be simplified e.g. (5 + 3\sqrt{2} - 2\sqrt{4}) o.e.</td>
</tr>
<tr>
<td></td>
<td>(= 1 + 3\sqrt{2})</td>
<td>For earlier use of (2\sqrt{2}) 1 + 3(\sqrt{2}) or (a = 1) and (b = 3).</td>
</tr>
<tr>
<td>(ii)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1: Multiply out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) – can appear as table</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1: (\sqrt{8} = 2\sqrt{2}), seen or implied at any point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1: Fully and correctly simplified to (1 + 3\sqrt{2}) or (a = 1) and (b = 3).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1: Rationalises denominator i.e. Multiplies (\left(\frac{k}{\sqrt{5}}\right)) by (\frac{\sqrt{5}}{\sqrt{5}}) or (\frac{-\sqrt{5}}{\sqrt{5}}), seen or implied or uses Method 3 or similar e.g. (\left(\frac{30}{\sqrt{5}}\right) = \frac{6\times 5}{\sqrt{5}} = 6\sqrt{5})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1: (Independent mark) States (\sqrt{80} = 4\sqrt{5}) Or either (\sqrt{400} = 20) or (\sqrt{80\sqrt{5}} = 20) at any point if they use Method 2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1: (10\sqrt{5}) or (c = 10).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N.B There are other methods e.g. (\sqrt{80} = \frac{20}{\sqrt{5}}) (B1) then add (\frac{20}{\sqrt{5}} + \frac{30}{\sqrt{5}} = \frac{50}{\sqrt{5}}) then M1 A1 as before</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Those who multiply initial expression by (\sqrt{5}) to obtain (\sqrt{400} + 30 = 20 + 30 = 50) earn M0 B1 A0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u_2 = 9, \ u_{n+1} = 2u_n - 1, \ n \geq 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u_3 = 2u_2 - 1 = 2(9) - 1 = 17 )</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td>( u_4 = 2u_3 - 1 = 2(17) - 1 = 33 )</td>
<td>Can be implied by ( u_3 = 17 )</td>
<td>A1</td>
</tr>
<tr>
<td>Both ( u_3 = 17 ) and ( u_4 = 33 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td>[2]</td>
</tr>
<tr>
<td>( \sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (u_1) = 5 )</td>
<td>( (u_1) = 5 )</td>
<td>B1</td>
</tr>
<tr>
<td>( \sum_{r=1}^{4} u_r = &quot;5&quot; + 9 + &quot;17&quot; + &quot;33&quot; = 64 )</td>
<td>Adds their first four terms obtained legitimately (see notes below)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[3] 64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 marks</td>
</tr>
</tbody>
</table>

**Notes**

(a) \textbf{M1}: Substitutes 9 into RHS of iteration formula
\textbf{A1}: Needs both 17 and 33 (but allow if either or both seen in part (b) )

(b) \textbf{B1}: for \( u_1 = 5 \) (however obtained – may appear in (a)) May be called \( a = 5 \)
\textbf{M1}: Uses their \( u_1 \) found from \( u_2 = 2u_1 - 1 \) stated explicitly, or uses \( u_4 = 4 \) or \( 5 \frac{1}{2} \), and adds it to \( u_2 \), their \( u_3 \) and their \( u_4 \) only. (See special cases below).
There should be no fifth term included.
Use of sum of AP is irrelevant and scores M0
\textbf{A1}: 64
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>(a) Gradient of ( l_2 ) is ( \frac{1}{2} ) or 0.5 or ( -\frac{1}{2} )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>Either ( y - 6 = \frac{1}{2}(x - 5) ) or ( y = \frac{1}{2}x + c ) and ( 6 = \frac{1}{2}(5) + c \Rightarrow c = (\frac{7}{2}) )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( x - 2y + 7 = 0 ) or ( -x + 2y - 7 = 0 ) or ( k(x - 2y + 7) = 0 ) with ( k ) an integer</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>Puts ( x = 0 ), or ( y = 0 ) in their equation and solves to find appropriate co-ordinate</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( x )-coordinate of ( A ) is (-7) and ( y )-coordinate of ( B ) is ( \frac{7}{2} ).</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area ( OAB = \frac{1}{2} \left( \frac{7}{2} \right) \left( \frac{7}{2} \right) = \frac{49}{4} ) (units)²</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>Applies ( \pm \frac{1}{2} ) (base)(height)</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

(a) **B1**: Must have \( \frac{1}{2} \) or 0.5 or \( -\frac{1}{2} \) o.e. stated and stops, or used in their line equation

**M1**: Full method to obtain an equation of the line through \((5,6)\) with their “m”. So \( y - 6 = m(x - 5) \) with their gradient or uses \( y = mx + c \) with \((5, 6)\) and their gradient to find \( c \). Allow any numerical gradient here including \(-2\) or \(-1\) but not zero. (Allow \((6,5)\) as a slip if \( y - y_1 = m(x - x_1) \) is quoted first)

**A1**: Accept any multiple of the correct equation, provided that the coefficients are integers and equation = 0 e.g. \(-x + 2y - 7 = 0\) or \( k(x - 2y + 7) = 0 \) or even \( 2y - x - 7 = 0 \)

(b) **M1**: Either one of the \( x \) or \( y \) coordinates using their equation

**A1**: Needs both correct values. Accept any correct equivalent. Need not be written as co-ordinates. **Even just** \(-7\) and \(3.5\) with no indication which is which may be awarded the A1.

(c) **M1**: Any correct method for area of triangle \( OAB \), with their values for co-ordinates of \( A \) and \( B \) (may include negatives) **Method usually half base times height but determinants could be used**.

**A1**: Any exact equivalent to \( 49/4 \), e.g. \( 12.25 \). (negative final answer is A0 but replacing by positive is A1)

Do not need units.

C.S.O. implies if A0 is scored in (b) then A0 is scored in (c) as well. However if candidate has correct line equation in (a) of wrong form may score A0 in (a) and A1 in (b) and (c)

**Note: Special cases**: \( \frac{1}{2} \left( -7 \right) \left( \frac{7}{2} \right) = -\frac{49}{4} \) (units)² is M1 A0 but changing sign to area = \( +\frac{49}{4} \) gets M1 A1 (recovery)

**N.B.** Candidates making sign errors in (b) and obtaining \(+7\) and \(-\frac{7}{2}\). may also get \( \frac{49}{4} \) as their answer following previous errors. They should be awarded A0 as this answer is not ft and is for correct solution only

**Special Case**: In (a) and (b): Produces parallel line instead of perpendicular line: So uses \( m = -2 \) This is not treated as a misread as it simplifies the question. The marks will usually be B0 M1 A0, M1 A0, M1 A0 i.e. maximum of 3/7
Question Number | Scheme | Marks
--- | --- | ---
6. (a) | ![Graph](image) Check graph in question for possible answers and space below graph for answers to part (b) | M1

- $y = \frac{2}{x}$ is translated up or down. A1
- $y = \frac{2}{x} - 5$ is in the correct position. B1

Intersection with $x$-axis at $\left(\frac{2}{5}, \{0\}\right)$ only Independent mark. B1

- $y = 4x + 2$ : attempt at straight line, with positive gradient with positive $y$ intercept. B1

Intersection with $x$-axis at $\left(-\frac{1}{2}, \{0\}\right)$ and $y$-axis at $\{(0), 2\}$. [5]

(b) Asymptotes : $x = 0$ (or $y$-axis) and $y = -5$. (Lose second B mark for extra asymptotes)

(c) Method 1: $\frac{2}{x} - 5 = 4x + 2$

$4x^2 + 7x - 2 = 0 \Rightarrow x = x = -2, \frac{1}{4}$

When $x = -2, y = -6$, When $x = \frac{1}{4}, y = 3$

Method 2: $\frac{y - 2}{4} = \frac{2}{y + 5}$

$y^2 + 3y - 18 = 0 \rightarrow y = y = -6, 3$

When $y = -6, x = -2$ When $y = 3, x = \frac{1}{4}$. [5]

12 marks

Notes

(a) M1: Curve implies $y$ axis as asymptote and does not change shape significantly. Changed curve needs horizontal asymptote (roughly) Asymptote(s) need not be shown but shape of curve should be implying asymptote(s) parallel to $x$ axis. Curve should not remain where it was in the given figure. Both sections move in the same direction. There should be no reflection.

A1: Crosses positive $x$ axis. Hyperbola has moved down. Both sections move by almost same amount. See sheet on page 19 for guidance.

B1: Check diagram and text of answer. Accept 2/5 or 0.4 shown on $x$-axis or $y = 2/5$, or $(2/5, 0)$ stated clearly in text or on graph. This is independent of the graph. Accept (0, 2/5) if clearly on $x$ axis. Ignore any intersection points with $y$ axis. Do not credit work in table of values for this mark.

B1: Must be attempt at a straight line, with positive gradient & with positive $y$ intercept (need not cross $x$ axis)

B1: Accept $x = -1/2$, or $-0.5$ shown on $x$-axis or $(-1/2, 0)$ or $(-0.5, 0)$ in text or on graph and similarly accept 2 on $y$ axis or $y = 2$ or $(0, 2)$ in text or on graph. Need not cross curve and allow on separate axes.

(b) B1: For either correct asymptote equation. Second B1: For both correct (lose this if extras e.g. $x = \pm 1$ are given also). These asymptotes may follow correctly from equation after wrong graph in (a) Just $y = -5$ is B1 B0 This may be awarded if given on the graph. However for other B mark it must be clear that $x = 0$ (or the $y$-axis) is an asymptote. NB $x \neq 0, y \neq -5$ is B1B0

(c) M1: Either of these equations is enough for the method mark (May appear labelled as part (b))

dM1: Attempt to solve a 3 term quadratic by factorising, formula, completion of square or implied by correct answers. (see note 1) This mark depends on previous mark.

A1: Need both correct $x$ answers (Accept equivalents e.g. 0.25) or both correct $y$ values (Method 2)

M1: At least one attempt to find second variable (usually $y$) using their first variable (usually $x$) related to line meeting curve. Should not be substituting $x$ or $y$ values from part (a) or (b). This mark is independent of previous marks. Candidate may substitute in equation of line or equation of curve.

A1: Need both correct second variable answers Need not be written as co-ordinates (allow as in the scheme)

Note: Special case: Answer only with no working in part (c) can have 5 marks if completely correct, with both points found. If co-ordinates of just one of the points is correct – with no working – this earns M0 M0 A0 M1 A0 (i.e. 1 / 5)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>Lewis; arithmetic series, ( a = 140, \ d = 20. )</td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>( T_{20} = 140 + (20 - 1)(20); = 520 )</td>
<td>M1; A1</td>
</tr>
<tr>
<td></td>
<td>OR 120 + (20)(20)</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Either:</strong> Uses ( \frac{1}{2}n(2a + (n-1)d) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{20}{2}(2 \times 140 + (20 - 1)(20)) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 6600 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Or:</strong> lists 20 terms to get to 520</td>
<td>[2]</td>
</tr>
<tr>
<td>(b)</td>
<td>Sian; arithmetic series, ( a = 300, \ l = 700, \ S_n = 8500 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Either:</strong> Attempt to use ( 8500 = \frac{n}{2}(a + l) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 8500 = \frac{n}{2}(300 + 700) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow n = 17 )</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Sian; arithmetic series, ( a = 300, \ l = 700, \ S_n = 8500 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Either:</strong> May use both ( 8500 = \frac{1}{2}n(2a + (n-1)d) ) and ( l = a + (n-1)d ) and eliminate ( d )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( 8500 = \frac{n}{2}(600 +400) )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow n = 17 )</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Notes**

(a) **M1:** Attempt to use formula for 20th term of Arithmetic series with first term 140 and \( d = 20. \) **Normal formula rules** apply – see General principles at the start of the mark scheme re “Method Marks”

**Or:** uses 120 + 20n with \( n = 20 \)

**Or:** Listing method: Lists 140, 160, 180, 200, 220, 240, 260, 280, ... 520. M1A1 if correct M0A0 if wrong. (So 2 marks or zero)

A1: For 520

(b) **M1:** An attempt to apply \( \frac{1}{2}n(2a + (n-1)d) \) or \( \frac{1}{2}n(a + l) \) with their values for \( a, n, d \) and \( l \)

A1: Uses \( a = 140, d = 20, n = 20 \) in their formula (two alternatives given above) but \textbf{fit on their value of} \( l \) from (a) if they use Method 2.

A1: 6600 eao

**Or:** Listing method: Lists 140, 160, 180, 200, 220, 240, 260, 280, ... 520 and adds 6600 to get M1A1A1- any other answer gets M1 A0A0 provided there are 20 numbers, the first is 140 and the last is 520.

(c) **First method**

**M1:** Attempt to use \( S_n = \frac{n}{2}(a + l) \) with their values for \( a, \) and \( l \) and \( S=8500 \)

A1: Uses formula with correct values

A1: Finds exact value 17

**M1:** If both formulae \( 8500 = \frac{1}{2}n(2a + (n-1)d) \) and \( l = a + (n-1)d \) are used, then \( d \) \textbf{must be eliminated} before this mark is awarded by valid work. Should not be using \( d = 400. \) This would be M0.

A1: Correct equation in \( n \) only then A1 for 17 exactly

**Trial and error methods:** Finds \( d = 25 \) and \( n = 17 \) and list from 300 to 700 with total checked – 3/3
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td>$\frac{dy}{dx} = -x^3 + \frac{2}{2}x^{-2} - \frac{\left(\frac{5}{2}\right)}{n}x^{-3}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$(y =) \quad -\frac{1}{4}x^4 + \frac{2}{2}x^{-1} - \frac{\left(\frac{5}{2}\right)}{n}x^{-2} + c$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$(y =) \quad -\frac{1}{4}x^4 + \frac{2}{2}x^{-1} - \frac{5}{2}x^{-2} + c$</td>
<td>A1ft</td>
</tr>
<tr>
<td></td>
<td>Raises power correctly on any one term.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Any two follow through terms correct.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>This is not follow through – must be correct</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Given that $y = 7$, at $x = 1$, then $7 = -\frac{1}{4} - 2 + \frac{5}{2} + c \Rightarrow c =$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>So, $(y =) \quad -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + c, \quad c = 8$ or $(y =) \quad -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + 8$</td>
<td>A1</td>
</tr>
</tbody>
</table>

**6 marks**

**Notes**

**M1:** Expresses as three term polynomial with powers 3, -2 and -3. Allow slips in coefficients. This may be implied by later integration having all three powers 4, -1 and -2.

**M1:** An attempt to integrate at least one term so $x^n \rightarrow x^{n+1}$ (not a term in the numerator or denominator)

**A1ft:** Any two integrations are correct – coefficients may be unsimplified (follow through errors in coefficients only here) so should have two of the powers 4, -1 and -2 after integration – depends on 2nd method mark only. There should be a maximum of three terms here.

**A1:** Correct three terms – coefficients may be unsimplified- do not need constant for this mark Depends on both Method marks

**M1:** Need constant for this mark. Uses $y = 7$ and $x = 1$ in their changed expression in order to find $c$, and attempt to find $c$. *This mark is available even after there is suggestion of differentiation.*

**A1:** Need all four correct terms to be simplified and need $c = 8$ here.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 9. (a) | **Method 1:** Attempts \( b^2 - 4ac \) for \( a = (k + 3) \), \( b = 6 \) and their \( c \). \( c \neq k \)  
\( b^2 - 4ac = 6^2 - 4(k + 3)(k - 5) \)  
\( (b^2 - 4ac =) -4k^2 + 8k + 96 \) or \( -(b^2 - 4ac =) 4k^2 - 8k - 96 \)  
As \( b^2 - 4ac > 0 \), then \( -4k^2 + 8k + 96 > 0 \) and so, \( k^2 - 2k - 24 < 0 \) | M1  
A1  
B1  
A1 * |
| **Method 2:** Considers \( b^2 > 4ac \) for \( a = (k + 3) \), \( b = 6 \) and their \( c \). \( c \neq k \)  
\( 6^2 > 4(k + 3)(k - 5) \)  
\( 4k^2 - 8k - 96 < 0 \) or \( -4k^2 + 8k + 96 > 0 \) or \( 9 > (k + 3)(k - 5) \) \( (with \ no \ prior \ algebraic \ errors) \)  
and so, \( k^2 - 2k - 24 < 0 \) following correct work | M1  
A1  
B1  
A1 * |
| (b) Attempts to solve \( k^2 - 2k - 24 = 0 \) to give \( k = \) \( (\Rightarrow \text{Critical values, } k = 6, -4.) \) | M1  
M1 A1 |

**Notes**

(a) **Method 1:** M1: Attempts \( b^2 - 4ac \) for \( a = (k + 3) \), \( b = 6 \) and their \( c \). \( c \neq k \) or uses quadratic formula and has this expression under square root. (ignore \( > 0, < 0 \) or \( = 0 \) for first 3 marks)  
A1: Correct expression for \( b^2 - 4ac \) - need not be simplified (may be under root sign)  
B1: Uses algebra to manipulate result **without error** into **one of these** three term quadratics. Again may be under root sign in quadratic formula. If inequality is used early in “proof” may see  
\( 4k^2 - 8k - 96 < 0 \) and B1 would be given for \( 4k^2 - 8k - 96 \) correctly stated.  
A1: Applies \( b^2 - 4ac > 0 \) correctly (or writes \( b^2 - 4ac > 0 \)) to achieve the result given in the question.  
No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by \(-1\), or taking every term to other side of inequality. **Need conclusion i.e. printed answer.**

**Method 2:** M1: Allow \( b^2 > 4ac \), \( b^2 < 4ac \) or \( b^2 = 4ac \) for \( a = (k + 3) \), \( b = 6 \) and their \( c \). \( c \neq k \)  
A1: Correct expressions on either side (ignore \( >, < \) or \( =\)).  
B1: Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 again **without error**  
A1: Produces result with no errors seen from initial consideration of \( b^2 > 4ac \).

(b) **M1:** Uses factorisation, formula, completion of square method to find two values for \( k \), or finds two **correct** answers with no obvious method  
M1: Their Lower Limit \( < k < Their \ Upper \ Limit \)  
Allow the M mark mark for \( \leq \) . (Allow \( k < \) upper and \( k > \) lower)  
A1: \( -4 < k < 6 \) Lose this mark for \( \leq \) Allow \( (-4, 6) \) [not square brackets] or \( k > -4 \) and \( k < 6 \) (must be and not or) Can also use intersection symbol \( \bigcap \) **NOT** \( k > -4, k < 6 \) (M1A0)  
**Special case:** In part (a) uses \( c = k \) instead of \( k - 5 \) - scores 0. Allow \( k + 5 \) for method marks  
**Special Case:** In part (b) Obtaining \( -6 < k < 4 \) This is a common wrong answer. Give M1 M1 A0 special case.  
**Special Case:** In part (b) Use of \( x \) instead of \( k - M1M1A0 \)  
**Special Case:** \(-4 < k < 6 \) and \( k < -4, k > 6 \) both given is M0A0 for last two marks. Do not treat as isw.
10. (a) This may be done by completion of square or by expansion and comparing coefficients

\[ a = 4 \]
\[ b = 1 \]

All three of \( a = 4 \), \( b = 1 \) and \( c = -1 \)

(b) U shaped quadratic graph.

The curve is correctly positioned with the minimum in the third quadrant. It crosses \( x \) axis twice on negative \( x \) axis and \( y \) axis once on positive \( y \) axis.

Curve cuts \( y \)-axis at \((0, 3)\). only

\[ \text{Curve cuts } x\text{-axis at } \left( -\frac{3}{2}, 0 \right) \text{ and } \left( -\frac{1}{2}, 0 \right). \]

Notes

(a) B1: States \( a = 4 \) or obtains \( 4(x + b)^2 + c \),
B1: States \( b = 1 \) or obtains \( a(x + 1)^2 + c \),
B1: States \( a = 4 \), \( b = 1 \) and \( c = -1 \) or \( 4(x + 1)^2 - 1 \)  (Needs all 3 correct for final mark)

Special cases: If answer is left as \( (2x + 2)^2 - 1 \) treat as misread B1B0B0
or as  \( 2(x + 1)^2 - 1 \) then the mark is B0B1B0 from scheme

(b) M1: Any position provided U shaped (be generous in interpretation of U shape but V shape is M0)
A1: The curve is correctly positioned with the minimum in the third quadrant. It crosses \( x \) axis twice on negative \( x \) axis and \( y \) axis once on positive \( y \) axis.
B1: Allow 3 on \( y \) axis and allow either \( y = 3 \) or \( (0, 3) \) if given in text Curve does not need to pass through this point and this mark may be given even if there is no curve at all or if it is drawn as a line.
B1: Allow \(-3/2 \) and \(-1/2 \) if given on \( x \) axis – need co-ordinates if given in text or \( x = -3/2, x = -1/2 \). Accept decimal equivalents. Curve does not need to pass through these points and this mark may be given even if there is no curve. Ignore third point of intersection and allow touching instead of cutting. So even a cubic curve might get M0A0 B1 B1.
A V shape with two ruled lines for example might get M0A0B1B1
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>11.</strong> C: ( y = 2x - 8 \sqrt{x} + 5, \quad x \geq 0 )</td>
<td>( \frac{dy}{dx} = 2 - 4x^{\frac{1}{2}} + {0} \quad (x &gt; 0) )</td>
</tr>
<tr>
<td>(a) So, ( y = 2x - 8x^{\frac{1}{2}} + 5 )</td>
<td>M1 A1 A1</td>
</tr>
<tr>
<td>(b) (When ( x = \frac{1}{4}, y = 2(\frac{1}{4}) - 8 \sqrt{\frac{1}{4}} + 5 ) so) ( y = \frac{3}{2} )</td>
<td>B1</td>
</tr>
<tr>
<td>( \text{gradient} = \frac{dy}{dx} = 2 - \frac{4}{\sqrt{\frac{1}{4}}} {= -6} )</td>
<td>M1</td>
</tr>
<tr>
<td>( \text{Either:}\quad y - \frac{3}{2} = -6(x - \frac{1}{4}) )</td>
<td>dM1</td>
</tr>
<tr>
<td>( \text{or:} \quad y = -6x + c \quad \Rightarrow \quad c = 3 )</td>
<td>A1</td>
</tr>
<tr>
<td>( y = -6x + 3 )</td>
<td>[4]</td>
</tr>
<tr>
<td>(c) Tangent at ( Q ) is parallel to ( 2x - 3y + 18 = 0 )</td>
<td>Sets their gradient function = their numerical gradient.</td>
</tr>
<tr>
<td>( (y = \frac{2}{3}x + 6 \quad \Rightarrow \quad \text{Gradient} = \frac{2}{3} ) so tangent gradient is ( \frac{2}{3} )</td>
<td>M1</td>
</tr>
<tr>
<td>( \Rightarrow \frac{2}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9 )</td>
<td>Ignore extra answer ( x = -9 )</td>
</tr>
<tr>
<td>When ( x = 9, y = 2(9) - 8 \sqrt{9} + 5 = -1 )</td>
<td>dM1</td>
</tr>
<tr>
<td>( y = -1. )</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Notes**

(a) **M1:** Evidence of differentiation, so \( x^n \rightarrow x^{n-1} \) at least once so \( x^1 \rightarrow 1 \) or \( x^0 \) or \( x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}} \) not just \( 5 \rightarrow 0 \)

**A1:** Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified.

**A1:** \( 2 - 4x^{\frac{1}{2}} \) Both terms correct, and simplified. Do not need to include domain \( x > 0 \)

(b) **B1:** Obtaining \( y = 3/2 \) or fractional or decimal equivalent (no working need be seen)

**M1:** An attempt to substitute \( x = \frac{1}{4} \) into \( \frac{dy}{dx} \) to establish gradient. This may be implied by \( -6 \) or \( m = -6 \) but not \( y = -6 \). Can earn this M mark if they go on to use \( m = \frac{1}{6} \) or use their numerical value of \( \frac{dy}{dx} \).

**dM1:** This depends on previous method mark. Complete method for obtaining the equation of the tangent, using their tangent gradient and their value for \( y_1 \) (obtained from \( x = \frac{1}{4} \), allow slip) i.e.

\[ y - y_1 = m_1(x - \frac{1}{4}) \text{with their tangent gradient and their } y_1 \]

or uses \( y = mx + c \) with \( (\frac{1}{4}, \text{their } y_1) \) and their tangent gradient.

**A1:** \( y = -6x + 3 \) or \( y = 3 - 6x \) or \( a = -6 \) and \( b = 3 \)

(c) **B1:** For the value 2/3 not 2/3 x not -3/2

**M1:** Sets their gradient function \( dy/dx = \) their numerical gradient

**A1:** Obtains \( x = 9 \)

**dM1:** Substitutes their \( x \) (from gradient equation) into original equation of curve \( C \) i.e. original expression \( y = A1: (9, -1) \) or \( x = 9, y = -1 \), or just \( y = -1 \)

**Special Cases:** In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4

In (c) Uses perpendicular instead of parallel then award B0 M1 A0 M1 A0 i.e max 2/5 – see over
Mark Scheme (Results)

January 2013

GCE Core Mathematics C2 (6664/01)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

   In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \checkmark \) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp  decimal places
   - sf significant figures
   - \* The answer is printed on the paper
   - \□ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
• If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
• If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question/item is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>aA</td>
<td>●</td>
<td></td>
</tr>
<tr>
<td>bM1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA1</td>
<td>●</td>
<td></td>
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<tr>
<td>bB</td>
<td></td>
<td>●</td>
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<tr>
<td>bM2</td>
<td>●</td>
<td></td>
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<tr>
<td>bA2</td>
<td></td>
<td>●</td>
</tr>
</tbody>
</table>
## Question 1

$$ (2 - 5x)^6 $$

1. **$(2^6 =) 64$**

   Award this when first seen (not $64x^6$)  
   
   B1

2. $$ +6 \times (2)^5 (\pm 5x) + \frac{6 \times 5}{2} (2)^3 (-5x)^2 $$

   Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $$ \binom{6}{p} \times (2)^{6-p} (-5x)^p $$

   with $p = 1$ or $p = 2$ consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients e.g. $\binom{6}{1}$ or even $\binom{6}{1}$

   M1

   $$ -960x $$

   Not $+ 960x$  
   
   A1 (first)

   $$ (+)6000x^2 $$

   A1 (Second)

3. **Way 2**  

   64 $(1 \pm \ldots \ldots)$  

   64 and $(1 \pm \ldots -)$ Award when first seen.  
   
   B1

   $$ \left(1 - \frac{5x}{2}\right)^6 = 1 - \frac{6 \times 5x}{2} + \frac{6 \times 5}{2} \left(-\frac{5x}{2}\right)^2 $$

   Correct structure for at least one of the underlined terms. E.g. a term of the form: $$ \binom{6}{p} \times (kx)^p $$ with $p = 1$ or $p = 2$

   consistently and $k \neq \pm 5$

   Condone sign errors. Condoned missing brackets if later work implies correct structure but it must be an expansion of $$ \left(1 - kx\right)^6 $$ where $k \neq \pm 5$

   M1

   $$ -960x $$

   Not $+ 960x$  
   
   A1

   $$ (+)6000x^2 $$

   A1

   (4)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(a)</strong> f(1) = a + b - 4 - 3 = 0 or a + b - 7 = 0</td>
<td>Attempt f(±1) M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a + b = 7 * Must be f(1) and = 0 needs to be seen</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(b)</strong> f(-2) = a(-2)^3 + b(-2)^2 - 4(-2) - 3 = 9</td>
<td>Attempt f(±2) and uses f(±2) = 9 M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-8a + 4b + 8 - 3 = 9 Correct equation with exponents of (-2) removed</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(-8a + 4b = 4) Solves the given equation from part (a) and their equation in a and b from part (b) as far as a =... or b =...</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>a = 2 and b = 5 Both correct</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Long Division**

| (a) | \( (ax^3 + bx^2 - 4x - 3) \div (x - 1) = ax^2 + px + q \) where \( p \) and \( q \) are in terms of \( a \) or \( b \) or both and sets their remainder = 0 NB Quotient = \( ax^2 + (a + b)x + (a + b - 4) \) a + b = 7 * | A1 |
| (b) | \( (ax^3 + bx^2 - 4x - 3) \div (x + 2) = ax^2 + px + q \) where \( p \) and \( q \) are in terms of \( a \) or \( b \) or both and sets their remainder = 9 NB Quotient = \( ax^2 + (b - 2a)x + (4a - 4 - 2b) \) 4b - 8a + 5 = 9 | A1 |

Follow scheme for final 2 marks
<p>| 3. | (a) $120000 \times (1.05)^3 = 138915 \times \text{B1}$ &lt;br&gt; Or $120000 \times 1.05 \times 1.05 \times 1.05 = 138915$ &lt;br&gt; Or $a = 120000$ and $a \times (1.05)^3 = 138915$ | (1) |
| (b) $120000 \times (1.05)^{n-1} &gt; 200000$ &lt;br&gt; Allow $n$ or $n - 1$ and “&gt;”, “&lt;”, or “=” etc. | M1 &lt;br&gt; Takes logs correctly &lt;br&gt; Allow $n$ or $n - 1$ and “&gt;”, “&lt;”, or “=” etc. | M1 &lt;br&gt; Takes logs correctly &lt;br&gt; Allow $n$ or $n - 1$ and “&gt;”, “&lt;”, or “=” etc. Allow $1.6$ or awrt $1.67$ for $5/3$. | A1 |
| $2024$ | M1: Identifies a calendar year using their value of $n$ or $n - 1$ &lt;br&gt; A1: 2024 | M1A1 |
| (c) $a(1 - r^n) = \frac{120000 \times (1 - 1.05^{11})}{1 - 1.05}$ &lt;br&gt; M1: Correct sum formula with $n = 10, 11$ or $12$ &lt;br&gt; A1: Correct numerical expression with $n = 11$ | M1A1 &lt;br&gt; 1704814 &lt;br&gt; Cao (Allow 1704814.00) | A1 |
| Listing or trial/improvement in (b) &lt;br&gt; $U_{10} = 186,159.39$, $U_{11} = 195,467.36$, $U_{12} = 205,240.72$ | Attempt to find at least the $10^{th}$ or $11^{th}$ or $12^{th}$ terms correctly using a common ratio of $1.05$ (all the terms need not be listed) | M1 &lt;br&gt; Forms the geometric progression correctly to reach a term $&gt; 200,000$ | M1 &lt;br&gt; Obtains an “$11^{th}$ term of awrt $195,500$ and a “$12^{th}$” term of awrt $205,200$ | A1 &lt;br&gt; Uses their number of terms to identify a calendar year | M1 &lt;br&gt; 2024 | A1 |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td>( \cos^{-1}(-0.4) = 113.58 \ (\alpha) )</td>
<td>Awrt 114</td>
</tr>
<tr>
<td></td>
<td>( 3x - 10 = \alpha \Rightarrow x = \frac{\alpha + 10}{3} )</td>
<td>Uses their ( \alpha ) to find ( x ). Allow ( x = \frac{\alpha \pm 10}{3} ) not ( \frac{\alpha}{3} \pm 10 )</td>
</tr>
<tr>
<td></td>
<td>( x = 41.2 )</td>
<td>Awrt</td>
</tr>
<tr>
<td></td>
<td>( (3x - 10 =) 360 - \alpha \ (246.4\ldots) )</td>
<td>360 - ( \alpha ) (can be implied by 246.4\ldots)</td>
</tr>
<tr>
<td></td>
<td>( x = 85.5 )</td>
<td>Awrt</td>
</tr>
<tr>
<td></td>
<td>( (3x - 10 =) 360 + \alpha \ (=473.57\ldots) )</td>
<td>360 + ( \alpha ) (Can be implied by 473.57\ldots)</td>
</tr>
<tr>
<td></td>
<td>( x = 161.2 )</td>
<td>Awrt</td>
</tr>
</tbody>
</table>
5.

(a)

(i) The centre is at (10, 12)  
B1: \( x = 10 \)  
B1: \( y = 12 \)

(ii) Uses \((x-10)^2 + (y-12)^2 = -195 + 100 + 144 \Rightarrow r = \ldots\)  
Completes the square for both \( x \) and \( y \) in an attempt to find \( r \).  
\((x \pm 10)^2 \pm a \) and \((y \pm 12)^2 \pm b \) and \(+ 195 = 0, (a, b \neq 0)\)  
Allow errors in obtaining their \( r^2 \) but must find square root

\[
r = \sqrt{10^2 + 12^2 - 195}
\]
A correct numerical expression for \( r \) including the square root and can implied by a correct value for \( r \)  
A1

\( r = 7 \)  
Not \( r = \pm 7 \) unless \( -7 \) is rejected  
A1

(a) **Way 2**

Compared the given equation with \( x^2 + y^2 + 2gx + 2fy + c = 0 \) to write down centre \((-g, -f)\) i.e. \((10, 12)\)

Uses \( r = \sqrt{(\pm 10)^2 + (\pm 12)^2 - c} \)  
M1

\[
r = \sqrt{10^2 + 12^2 - 195}
\]
A correct numerical expression for \( r \)  
A1

\( r = 7 \)  
A1

(b) \( MN = \sqrt{(25 - 10)^2 + (32 - 12)^2} \)  
Correct use of Pythagoras  
M1

\[
MN \left( = \sqrt{625} \right) = 25
\]
A1

(2)

(c) \( NP = \sqrt{(25 - n)^2 - 7n^2} \)  
\( NP = \sqrt{(MN^2 - r^2)} \)  
M1

\[
NP \left( = \sqrt{576} \right) = 24
\]
A1

(2)

(c) **Way 2**

\[
\cos\left( NMP = \frac{7}{25} \right) \Rightarrow NP = 25 \sin\left( NMP \right)
\]
Correct strategy for finding \( NP \)  
M1

\( NP = 24 \)  
A1

(2)

[9]
6.

(a) \[ 2 \log(x+15) = \log(x+15)^2 \]

\[
\log(x+15)^2 - \log x = \log \left( \frac{(x+15)^2}{x} \right)
\]

Correct use of \( \log a - \log b = \log \frac{a}{b} \)  

M1

\[ 2^k = 64 \text{ or } \log_2 64 = 6 \]

64 used in the correct context  

B1

\[ \log_2 \left( \frac{(x+15)^2}{x} \right) = 6 \implies \frac{(x+15)^2}{x} = 64 \]

Removes logs correctly  

M1

\[ \Rightarrow x^2 + 30x + 225 = 64x \]

or \[ x + 30 + 225x^{-1} = 64 \]

Must see expansion of \((x+15)^2\) to score the final mark.  

A1

(b) \[ (x-25)(x-9) = 0 \Rightarrow x = 25 \text{ or } x = 9 \]

M1: Correct attempt to solve the given quadratic as far as \( x = \ldots \)  

M1 A1

A1: Both 25 and 9  

(2)
7. (a) \[ 9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha = \ldots. \]

\[
\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} = -\frac{29}{48} = -0.604.. \]

\[ \alpha = 2.22^\circ \quad * \]

Cso (2.22 must be seen here) A1

(NB \( \alpha = 2.219516005 \)) (2)

(a) Way 2 \[ XY^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos 2.22 \Rightarrow XY^2 = \ldots \]

\[ XY^2 = 81.01.. \]

\[ XY = 9.00.. \quad A1 \]

(b) \[ 2\pi - 2.22(= 4.06366......) \]

\[ 2\pi - 2.22 \quad \text{or} \quad \text{awrt} \ 4.06 \quad B1 \]

\[ \frac{1}{2} \times 4^2 \times "4.06" \quad \text{Correct method for major sector area.} \quad \text{M1} \]

32.5 \[ \quad \text{Awrt 32.5} \quad A1 \]

(b) Way 2 \text{ Circle – Minor sector} \[
\pi \times 4^2 \quad \text{Correct expression for circle area} \quad \text{B1} \]

\[ \pi \times 4^2 - \frac{1}{2} \times 4^2 \times 2.22 = 32.5 \]

Correct method for circle - minor sector area M1

= 32.5 \[ \quad \text{Awrt 32.5} \quad A1 \]

(c) Area of triangle = \[ \frac{1}{2} \times 4 \times 6 \times \sin 2.22 (= 9.56) \]

Correct expression for the area of triangle XYZ B1

So area required = “9.56” + “32.5” Their Triangle XYZ + (part (b) answer or correct attempt at major sector) M1

= 42.1 cm\(^2\) or 42.0 cm\(^2\) Awrt 42.1 or 42.0 (Or just 42) A1

(d) Arc length = \[ 4 \times 4.06 (= 16.24) \]

Or \[ 8\pi - 4 \times 2.22 \]

M1: \[ 4 \times \text{their} (2\pi - 2.22) \]

Or circumference – minor arc \[ 9 + 2 + \text{Any Arc} \quad \text{M1} \]

Perimeter = \[ ZY + WY + \text{Arc Length} \quad \text{A1: Correct ft expression} \]

Perimeter = 27.2 or 27.3 Awrt 27.2 or awrt 27.3 A1
8. \[ y = 6 - 3x - \frac{4}{x^4} \]

(a) \[
\frac{dy}{dx} = -3 + \frac{12}{x^4} \quad \text{or} \quad -3 + 12x^{-4}
\]

M1: \( x^n \rightarrow x^{-n} \)

\((x^i \rightarrow x^{-i} \text{ or } x^i \rightarrow -x^{-i} \text{ or } 6 \rightarrow 0)\)

A1: Correct derivative

\[
\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = \ldots \quad \text{or}
\]

\[
\frac{dy}{dx} = -3 + \frac{12}{\sqrt[4]{x^4}}
\]

\( y' = 0 \) and attempt to solve for \( x \)

May be implied by

\[
\frac{dy}{dx} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = \ldots \quad \text{or}
\]

Substitutes \( x = \sqrt[4]{2} \) into their \( y' \)

So \( x^4 = 4 \) and \( x = \sqrt[4]{2} \) or

\[
\frac{dy}{dx} = -3 + \frac{12}{\left(\sqrt[4]{2}\right)^4} \quad \text{or} \quad -3 + 12\left(\sqrt[4]{2}\right)^{-4} = 0
\]

Correct completion to answer with no errors by solving their \( y' = 0 \) or substituting \( x = \sqrt[4]{2} \) into their \( y' \)

A1

(b) \( x = -\sqrt{2} \)

Awrt -1.41

B1

(c) \[
\frac{d^2y}{dx^2} = \frac{-48}{x^5} \quad \text{or} \quad -48x^{-5}
\]

Follow through their first derivative from part (a)

B1ft

(d) An appreciation that either

\( y'' > 0 \Rightarrow \) a minimum

or \( y'' < 0 \Rightarrow \) a maximum

Maximum at \( P \) as \( y'' < 0 \)

CSo

B1

Need a fully correct solution for this mark. \( y'' \) need not be evaluated but must be correct and there must be reference to \( P \) or to \( \sqrt[4]{2} \) and negative or \( < 0 \) and maximum. There must be no incorrect or contradictory statements (NB allow \( y'' = \) awrt 8 or -9)

Minimum at \( Q \) as \( y'' > 0 \)

CSo

B1

Need a fully correct solution for this mark. \( y'' \) need not be evaluated but must be correct and part (b) must be correct and there must be reference to \( P \) or to \( -\sqrt[4]{2} \) and positive or \( > 0 \) and minimum. There must be no incorrect or contradictory statements (NB allow \( y'' = \) awrt 8 or 9)

Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of \( y \) or \( dy/dx \) either side of \( \sqrt[4]{2} \) or their \( x \) at \( Q \) and the second and third B1’s for fully correct solutions to identify the maximum/minimum.
<table>
<thead>
<tr>
<th></th>
<th>[ y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(6.272, 3.634)</td>
</tr>
<tr>
<td>(b)</td>
<td>(\frac{1}{2} \times \frac{1}{2} \text{ or } \frac{1}{4})</td>
</tr>
<tr>
<td></td>
<td>(...{0 + 0} + 2(5.866 + &quot;6.272&quot; + 5.210 + &quot;3.634&quot; + 1.856)} \text{ Need } {} \text{ or implied later for A1} )</td>
</tr>
<tr>
<td></td>
<td>(\frac{1}{2} \times 0.5{0 + 0} + 2(5.866 + &quot;6.272&quot; + 5.210 + &quot;3.634&quot; + 1.856)} \text{ Need } {} \text{ or implied later for A1} )</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{4} \times 45.676)</td>
</tr>
<tr>
<td></td>
<td>(= 11.42)</td>
</tr>
<tr>
<td>(c)</td>
<td>[\int y , dx = 27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1} + c]</td>
</tr>
<tr>
<td></td>
<td>(M1: x'' \rightarrow x'^{\ast1} \text{ on any term})</td>
</tr>
<tr>
<td></td>
<td>(A1: 27x - x^2)</td>
</tr>
<tr>
<td></td>
<td>(A1: -6x^{\frac{3}{2}})</td>
</tr>
<tr>
<td></td>
<td>(A1: +16x^{-1})</td>
</tr>
<tr>
<td></td>
<td>(\left(27(4) - (4)^2 - 6(4)^{\frac{3}{2}} + 16(4)^{-1}\right)) Attempt to subtract either way round using the limits 4 and 1. Dependent on the previous M1</td>
</tr>
<tr>
<td></td>
<td>(-\left(27(1) - (1)^2 - 6(1)^{\frac{3}{2}} + 16(1)^{-1}\right))</td>
</tr>
<tr>
<td></td>
<td>(= (48 - 36))</td>
</tr>
<tr>
<td></td>
<td>(12) Cao</td>
</tr>
</tbody>
</table>

[12]
Mark Scheme (Results)

January 2013

GCE Core Mathematics – C3 (6665/01)
Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

   In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \sqrt{ } \) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - \* The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question/item is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>aA</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bM1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA1</td>
<td></td>
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<tr>
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</table>
## January 2013
### 6665 Core Mathematics C3
#### Mark Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a)</td>
<td>$-32 = (2w - 3)^5 \Rightarrow w = \frac{1}{2}$ oe</td>
<td>M1A1 (2)</td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = 5 \times (2x - 3)^4 \times 2$ or $10(2x - 3)^4$</td>
<td>M1A1</td>
</tr>
<tr>
<td>(b)</td>
<td>When $x = \frac{1}{2}$, Gradient = 160</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Equation of tangent is $'160' = \frac{y - (-32)}{x - \frac{1}{2}}$ oe</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$y = 160x - 112$ cso</td>
<td>A1 (5)</td>
</tr>
</tbody>
</table>

(a) M1 Substitute $y = -32$ into $y = (2w - 3)^5$ and proceed to $w = \ldots$ [Accept positive sign used of $y$, ie $y = +32$] A1 Obtains $w$ or $x = \frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5. Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y = (2x - 3)^4$ using the chain rule. Sight of $\pm A(2x - 3)^4$ where $A$ is a non-zero constant is sufficient for the method mark. A1 A correct (un simplified) form of the differential. Accept $\frac{dy}{dx} = 5 \times (2x - 3)^4 \times 2$ or $\frac{dy}{dx} = 10(2x - 3)^4$ M1 This is awarded for an attempt to find the gradient of the tangent to the curve at $P$. Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent dM1 Award for a correct method to find an equation of the tangent to the curve at $P$. It is dependent upon the previous M mark being awarded. 

Award for 'their 160' = \[\frac{y - (-32)}{x - \text{their } \frac{1}{2}}\]

If they use $y = mx + c$ it must be a full method, using $m = \text{'their 160'}, \text{their } \frac{1}{2}$, and -32. An attempt must be seen to find $c = \ldots$
The question is specific and requires the answer in this form. You may isw in this question after a correct answer.
2. (a) \(0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6 - x) + 1\)  
M1 A1*  
(2)  
(b) Sub \(x_0 = 2\) into \(x_{n+1} = \ln(6 - x_n) + 1 \Rightarrow x_1 = 2.3863\) 
AWRT 4 dp. \(x_2 = 2.2847 \quad x_3 = 2.3125\)  
M1, A1  
A1  
(3)  
(c) Chooses interval \([2.3065, 2.3075]\) 
\(g(2.3065) = -0.0002(7), \ g(2.3075) = 0.004(4)\)  
Sign change, hence root (correct to 3 dp)  
M1  
dM1  
A1  
(3)  
(8 marks)  

(a) M1 Sets \(g(x) = 0\), and using correct \(\ln\) work, makes the \(x\) of the \(e^{x-1}\) term the subject of the formula. Look for \(e^{x-1} + x - 6 = 0 \Rightarrow e^{x-1} = \pm(\pm x) \Rightarrow x = \ln(\pm x) \pm 1\)  
Do not accept \(e^{x-1} = 6 - x\) without firstly seeing \(e^{x-1} + x - 6 = 0\) or a statement that \(g(x) = 0 \Rightarrow cso. x = \ln(6 - x) + 1\) Note that this is a given answer (and a proof).  
A1*  

‘Invisible’ brackets are allowed for the M but not the A  
Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.  
\(0 = e^{x-1} + x - 6 \Rightarrow 0 = x - 1 + \ln(x - 6) \Rightarrow x = \ln(6 - x) + 1\)  
(b) M1 Sub \(x_0 = 2\) into \(x_{n+1} = \ln(6 - x_n) + 1\) to produce a numerical value for \(x_i\).  
Evidence for the award could be any of \(\ln(6 - 2) + 1\), \(\ln 4 + 1\), \(2.3\ldots\) or awrt 2.4  
A1  
Answer correct to 4 dp \(x_1 = 2.3863\).  
The subscript is not important. Mark as the first value given/found.  
A1  
Awrt 4 dp. \(x_2 = 2.2847\) and \(x_3 = 2.3125\)  
The subscripts are not important. Mark as the second and third values given/found  

(c) M1 Chooses the interval \([2.3065, 2.3075]\) or smaller containing the root 2.306558641  
dM1 Calculates \(g(2.3065)\) and \(g(2.3075)\) with at least one of these correct to 1sf.  
The answers can be rounded or truncated  
\(g(2.3065) = -0.0003\) rounded, \(g(2.3065) = -0.0002\) truncated  
\(g(2.3075) = (+) 0.004\) rounded and truncated  
A1  
Both values correct (rounded or truncated),  
A reason which could include change of sign, \(>0 <0, \ g(2.3065) \times g(2.3075) <0\)  
AND a minimal conclusion such as hence root, \(\alpha = 2.307\) or □  
Do not accept continued iteration as question demands an interval to be chosen.  

Alternative solution to (a) working backwards  
M1 Proceeds from \(x = \ln(6 - x) + 1\) using correct exp work to ……. = 0  
A1 Arrives correctly at \(e^{x-1} + x - 6 = 0\) and makes a statement to the effect that this is \(g(x) = 0\)  

Alternative solution to (c) using \(f(x) = \ln(6 - x) + 1 - x\) \{Similarly \(h(x) = x - 1 - \ln(6 - x)\)\}  
M1 Chooses the interval \([2.3065, 2.3075]\) or smaller containing the root 2.306558641  
dM1 Calculates \(f(2.3065)\) and \(f(2.3075)\) with at least 1 correct rounded or truncated \(f(2.3065) = 0.000074\). Accept 0.00007 rounded or truncated. Also accept 0.0001
f(2.3075) = - 0.0011. Accept -0.001 rounded or truncated
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<tr>
<td>3.</td>
<td>(a) $ff(-3) = f(0) = 2$</td>
<td>M1,A1 (2)</td>
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<tr>
<td></td>
<td>(b) <img src="https://via.placeholder.com/150" alt="Graph" /></td>
<td>Shape B1 (0,-3) and (2,0) B1 (2)</td>
</tr>
<tr>
<td></td>
<td>(c) <img src="https://via.placeholder.com/150" alt="Graph" /></td>
<td>Shape B1 (0,0) B1 (2)</td>
</tr>
<tr>
<td></td>
<td>(d) <img src="https://via.placeholder.com/150" alt="Graph" /></td>
<td>Shape B1 (-6,0) or (0,4) B1 (-6,0) and (0,4) B1 (3)</td>
</tr>
</tbody>
</table>

(a) **M1** A full method of finding $ff(-3)$. $f(0)$ is acceptable but $f(-3) = 0$ is not.
   Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x + 4)$, which only passes through one of the points.
   **A1** Cao $ff(-3)=2$. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b) **B1** For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only.
   Do not award if the curve bends back on itself or has a clear minimum
   **B1** This is independent to the first mark and for the graph passing through (0,-3) and (2,0)
Accept -3 and 2 marked on the correct axes.
Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes
Accept P’=(0,-3), Q’=(2,0) stated elsewhere as long as P’ and Q’ are marked in the correct place on the graph

There must be a graph for this to be awarded

(c )
B1 Award for a correct shape ‘roughly’ symmetrical about the y- axis. It must have a cusp and a gradient that ‘decreases’ either side of the cusp. Do not award if the graph has a clear maximum
B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked

(d) B1 Shape. The position is not important. The gradient should be always positive but decreasing
B1 The graph passes through (0,4) or (-6,0). See part (b) for allowed variations
B1 The graph passes through (0,4) and (-6,0). See part (b) for allowed variations
4. (a) \[ R^2 = 6^2 + 8^2 \implies R = 10 \]

\[ \tan \alpha = \frac{8}{6} \implies \alpha = \text{awrt 0.927} \]

M1A1

(b)(i) \[ p(x) = \frac{4}{12 + 10 \cos(\theta - 0.927)} \]

\[ p(x) = \frac{4}{12 - 10} \]

Maximum = 2

M1

A1

(b)(ii) \[ \theta - \text{'their } \alpha' = \pi \]

\[ \theta = \text{awrt 4.07} \]

M1

A1

(8 marks)

(a) M1 Using Pythagoras’ Theorem with 6 and 8 to find \( R \). Accept \( R^2 = 6^2 + 8^2 \)

If \( \alpha \) has been found first accept \( R = \pm \frac{8}{\sin' \alpha} \) or \( R = \pm \frac{6}{\cos' \alpha} \)

A1 \( R = 10 \). Many candidates will just write this down which is fine for the 2 marks. Accept \( \pm 10 \) but not -10

M1 For \( \tan \alpha = \pm \frac{8}{6} \) or \( \tan \alpha = \pm \frac{6}{8} \)

If \( R \) is used then only accept \( \sin \alpha = \pm \frac{8}{R} \) or \( \cos \alpha = \pm \frac{6}{R} \)

A1 \( \alpha = \text{awrt 0.927} \). Note that 53.1° is A0

(b) Note that (b)(i) and (b)(ii) can be marked together

(i) M1 Award for \( p(x) = \frac{4}{12 - 'R'} \).

A1 Cao \( p(x)_{\text{max}} = 2 \).

The answer is acceptable for both marks as long as no incorrect working is seen

(ii) M1 For setting \( \theta - \text{'their } \alpha' = \pi \) and proceeding to \( \theta =.. \)

If working exclusively in degrees accept \( \theta - \text{'their } \alpha' = 180 \)

Do not accept mixed units

A1 \( \theta = \text{awrt 4.07} \). If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1
5. (i)(a) \[
\frac{dy}{dx} = 3x^2 \ln 2x + x^3 \times \frac{1}{2x} \\
= 3x^2 \ln 2x + x^2
\]

M1A1A1

(i)(b) \[
\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2 \cos 2x)
\]

B1M1A1

(ii) \[
\frac{dx}{dy} = -\csc^2 y \\
\frac{dy}{dx} = -\frac{1}{\csc^2 y}
\]

M1A1

M1

Uses \( \csc^2 y = 1 + \cot^2 y \) and \( x = \cot y \) in \( \frac{dy}{dx} \) or \( \frac{dx}{dy} \) to get an expression in \( x \)

\[
\frac{dy}{dx} = -\frac{1}{\csc^2 y} = -\frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}
\]

cso M1, A1*

(11 marks)

(i)(a) M1 Applies the product rule \( vu' + uv' \) to \( x^3 \ln 2x \).

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out \( u=\ldots, u'=\ldots, v=\ldots, v'=\ldots \) followed by their \( vu'+uv' \)) then only accept answers of the form \( Ax^2 \ln 2x + x^3 \times \frac{B}{x} \) where \( A, B \) are constants \( \neq 0 \)

A1 One term correct, either \( 3x^2 \ln 2x \) or \( x^3 \times \frac{1}{2x} \times 2 \)

A1 Cao. \( \frac{dy}{dx} = 3x^2 \ln 2x + x^3 \times \frac{1}{2x} \). The answer does not need to be simplified.

For reference the simplified answer is \( \frac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2 (3 \ln 2x + 1) \)

(i)(b) B1 Sight of \( (x + \sin 2x)^2 \)

M1 For applying the chain rule to \( (x + \sin 2x)^3 \). If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of \( C(x + \sin 2x)^2 \times (1 \pm D \cos 2x) \) where \( C \) and \( D \) are non-zero constants.

Alternatively accept \( u = x + \sin 2x \), \( u' = \) followed by \( Cu^2 \times \) their \( u' \)

Do not accept \( C(x + \sin 2x)^2 \times 2 \cos 2x \) unless you have evidence that this is their \( u' \)

Allow ‘invisible’ brackets for this mark, ie. \( C(x + \sin 2x)^2 \times 1 \pm D \cos 2x \)

A1 Cao \( \frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2 \cos 2x) \). There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)
(ii) M1 Writing the derivative of \( \cot y \) as \( -\csc^2 y \). It must be in terms of \( y \).
A1 \( \frac{dx}{dy} = -\csc^2 y \) or \( 1 = -\csc^2 y \frac{dy}{dx} \). Both lhs and rhs must be correct.
M1 Using \( \frac{dy}{dx} = -\frac{1}{dy/dx} \)
M1 Using \( \csc^2 y = 1 + \cot^2 y \) and \( x = \cot y \) to get \( \frac{dy}{dx} \) or \( \frac{dx}{dy} \) just in terms of \( x \).
A1 \( \csc \frac{dy}{dx} = -\frac{1}{1 + x^2} \)

Alternative to (a)(i) when \( \ln(2x) \) is written \( \ln x + \ln 2 \)
M1 Writes \( x^3 \ln 2x \) as \( x^3 \ln 2 + x^3 \ln x \).
Achieves \( A\times x^2 \) for differential of \( x^3 \ln 2 \) and applies the product rule \( vu' + uv' \) to \( x^3 \ln x \).
A1 Either \( 3x^2 \times \ln 2 + 3x^2 \ln x \) or \( x^3 \times \frac{1}{x} \)
A1 A correct (un simplified) answer. Eg \( 3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x} \)

Alternative to 5(ii) using quotient rule
M1 Writes \( \cot y \) as \( \frac{\cos y}{\sin y} \) and applies the quotient rule, a form of which appears in the
formula book. If the rule is quoted it must be correct. There must have been some attempt
to differentiate both terms. If the rule is not quoted (nor implied by their working,
meaning terms are written out \( u=\ldots,u'=\ldots,v=\ldots,v'=\ldots \) followed by their \( \frac{vu'-uv'}{v^2} \))
only accept answers of the form \( \frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2} \)
A1 Correct un simplified answer with both lhs and rhs correct.
\( \frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \{ -1 - \cot^2 y \} \)
M1 Using \( \frac{dy}{dx} = \frac{1}{\dy/dx} \)
M1 Using \( \sin^2 y + \cos^2 y = 1 \), \( \frac{1}{\sin^2 y} = \csc^2 y \) and \( \cosec^2 y = 1 + \cot^2 y \) to get \( \frac{dy}{dx} \) or \( \frac{dx}{dy} \) in \( x \)
A1 \( \csc \frac{dy}{dx} = -\frac{1}{1 + x^2} \)

Alternative to 5(ii) using the chain rule, first two marks
M1  Writes cot $y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).
Accept answers of the form $- (\tan y)^{-2} \times \sec^2 y$

A1  Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = - (\tan y)^{-2} \times \sec^2 y$$

Alternative to 5(ii) using a triangle – last M1

M1  Uses triangle with $\tan y = \frac{1}{x}$ to find $\sin y$
and get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of $x$

\[ x = \cot y \Rightarrow \tan y = \frac{1}{x} \]
\[
\begin{align*}
\text{sin } y &= \frac{1}{\sqrt{1+x^2}} \\
\triangle &
\end{align*}
\]
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| 6.              | (i) \((\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + \cos^2 22.5 + \ldots \)
|                 | \(= \sin^2 22.5 + \cos^2 22.5 + 2 \sin 22.5 \cos 22.5 \)
|                 | States or uses \(\sin^2 22.5 + \cos^2 22.5 = 1 \)
|                 | Uses \(2 \sin x \cos x = \sin 2x \Rightarrow 2 \sin 22.5 \cos 22.5 = \sin 45 \)
|                 | \((\sin 22.5 + \cos 22.5)^2 = 1 + \sin 45 \)
|                 | \(= 1 + \frac{\sqrt{2}}{2} \) or \(1 + \frac{1}{\sqrt{2}} \)
|                 | (5) |
|                 | (ii) (a) \(\cos 2\theta + \sin \theta = 1 \Rightarrow 1 - 2 \sin^2 \theta + \sin \theta = 1 \)
|                 | \(\sin \theta - 2 \sin^2 \theta = 0 \)
|                 | \(2 \sin^2 \theta - \sin \theta = 0 \) or \(k = 2 \)
|                 | (2) |
|                 | (b) \(\sin \theta(2 \sin \theta - 1) = 0 \)
|                 | \(\sin \theta = 0, \quad \sin \theta = \frac{1}{2} \)
|                 | Any two of 0, 30, 150, 180
|                 | All four answers 0, 30, 150, 180
|                 | (11 marks) |

(i) M1 Attempts to expand \((\sin 22.5 + \cos 22.5)^2 \). Award if you see \(\sin^2 22.5 + \cos^2 22.5 + \ldots \)

There must be \(>2\) terms. Condone missing brackets ie \(\sin 22.5^2 + \cos 22.5^2 + \ldots \)

B1 Stating or using \(\sin^2 22.5 + \cos^2 22.5 = 1 \). Accept \(\sin 22.5^2 + \cos 22.5^2 = 1 \) as the intention is clear.

Note that this may also come from using the double angle formula

\(\sin 22.5 + \cos 22.5 = \left( \frac{1-\cos 45}{2} \right) + \left( \frac{1+\cos 45}{2} \right) = 1 \)

M1 Uses \(2 \sin x \cos x = \sin 2x \) to write \(2 \sin 22.5 \cos 22.5 \) as \(\sin 45 \) or \(\sin(2\times22.5) \)

A1 Reaching the intermediate answer \(1 + \sin 45 \)

A1 Cso \(1 + \frac{\sqrt{2}}{2} \) or \(1 + \frac{1}{\sqrt{2}} \). Be aware that both 1.707 and \(\frac{2 + \sqrt{2}}{2} \) can be found by using a calculator for \(1 + \sin 45 \). Neither can be accepted on their own without firstly seeing one of the two answers given above. **Each stage should be shown as required by the mark scheme.**

Note that if the candidates use \((\sin \theta + \cos \theta)^2 \) they can pick up the first M and B marks, but no others until they use \(\theta = 22.5 \). All other marks then become available.

(iiia) M1 Substitutes \(\cos 2\theta = 1 - 2 \sin^2 \theta \) in \(\cos 2\theta + \sin \theta = 1 \) to produce an equation in \(\sin \theta \) only.

It is acceptable to use \(\cos 2\theta = 2 \cos^2 \theta - 1 \) or \(\cos^2 \theta - \sin^2 \theta \) as long as the \(\cos^2 \theta \) is subsequently replaced by \(1 - \sin^2 \theta \)

A1* Obtains the correct simplified equation in \(\sin \theta \)

\(\sin \theta - 2 \sin^2 \theta = 0 \) or \(\sin \theta = 2 \sin^2 \theta \) must be written in the form \(2 \sin^2 \theta - \sin \theta = 0 \) as required by the question. Also accept \(k = 2 \) as long as no incorrect working is seen.

(iiib) M1 Factorises or divides by \(\sin \theta \). For this mark \(1 = k \sin \theta \) is acceptable. If they have a 3 TQ in \(\sin \theta \) this can be scored for correct factorisation

A1 **Both** \(\sin \theta = 0, \) and \(\sin \theta = \frac{1}{2} \)

B1 Any two answers from 0, 30, 150, 180.

A1 All four answers 0, 30, 150, 180 with no extra solutions inside the range. Ignore solutions outside the range.
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| **6.alt 1**     | (i) $(\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + \cos^2 22.5 + \ldots$  
                     = $\sin^2 22.5 + \cos^2 22.5 + 2 \sin 22.5 \cos 22.5$  
                     States or uses $\sin^2 22.5 + \cos^2 22.5 = 1$  
                     Uses $2 \sin x \cos x = 2 \sqrt{\frac{1 - \cos 2x}{2}} \sqrt{\frac{\cos 2x + 1}{2}} \Rightarrow \sqrt{1 - \cos 45} \sqrt{1 + \cos 45}$  
                     $= \sqrt{1 - \cos^2 45}$  
                     Hence $(\sin 22.5 + \cos 22.5)^2 = 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$  
                           | M1     |       |
|                 |        | B1    |
|                 |        | A1    |

(5)

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| **6.alt 2**     | (i) Uses Factor Formula $(\sin 22.5 + \sin 67.5)^2 = (2 \sin 45 \cos 22.5)^2$  
                     Reaching the stage $= 2 \cos^2 22.5$  
                     Uses the double angle formula $= 2 \cos^2 22.5 = 1 + \cos 45$  
                     $= 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$  
                           | M1,A1  |       |
|                 |        | B1    |
|                 |        | M1    |
|                 |        | A1    |

(5)

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| **6.alt 3**     | (i) Uses Factor Formula $(\cos 67.5 + \cos 22.5)^2 = (2 \cos 45 \cos 22.5)^2$  
                     Reaching the stage $= 2 \cos^2 22.5$  
                     Uses the double angle formula $= 2 \cos^2 22.5 = 1 + \cos 45$  
                     $= 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$  
                           | M1,A1  |       |
|                 |        | B1    |
|                 |        | M1    |
|                 |        | A1    |

(5)
7. (a) \[ \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x+2)(x^2+5)} = \frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)} \]

\[ = \frac{2x(x+2)}{(x+2)(x^2+5)} \]

\[ = \frac{2x}{x^2+5} \]

M1A1

(b) \[ h'(x) = \frac{(x^2+5)\times 2 - 2x\times 2x}{(x^2+5)^2} \]

\[ h'(x) = \frac{10-2x^2}{(x^2+5)^2} \]

cso A1

M1A1

(c) Maximum occurs when \( h'(x) = 0 \Rightarrow 10 - 2x^2 = 0 \Rightarrow x = \ldots \)

\[ \Rightarrow x = \sqrt{5} \]

M1

When \( x = \sqrt{5} \Rightarrow h(x) = \frac{\sqrt{5}}{5} \)

M1,A1

Range of \( h(x) \) is \( 0 \leq h(x) \leq \frac{\sqrt{5}}{5} \)

A1ft

(12 marks)

(a) M1 Combines the three fractions to form a single fraction with a common denominator. Allow errors on the numerator but at least one must have been adapted. Condone ‘invisible’ brackets for this mark. Accept three separate fractions with the same denominator. Amongst possible options allowed for this method are

\[ \frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)} \]

Eg 1 An example of ‘invisible’ brackets

\[ \frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)} \]

Eg 2 An example of an error (on middle term), 1st term has been adapted

\[ \frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2} \]

Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

\[ \frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)} \]

Accept if there are three separate fractions with the correct (lowest) common denominator.

Eg \[ \frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4(x+2)}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)} \]
Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator. There must be a single denominator. Terms must be collected on the numerator. A factor of \((x+2)\) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term ‘disappears’.

\[\frac{2x}{(x^2 + 5)}\] This is a given solution and this mark should be withheld if there are any errors.

(b) M1 Applies the quotient rule to \(\frac{2x}{(x^2 + 5)}\), a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out \(u=\ldots,u'=\ldots,v=\ldots,v'=\ldots\) followed by their \(\frac{vu'-uv'}{v^2}\)) then only accept answers of the form

\[
\frac{(x^2 + 5) \times A - 2x \times Bx}{(x^2 + 5)^2}
\]

where \(A, B > 0\)

A1 Correct unsimplified answer \(h'(x) = \frac{(x^2 + 5) \times 2 - 2x \times 2x}{(x^2 + 5)^2}\). The correct simplified answer. Accept \(\frac{2(5-x^2)}{(x^2 + 5)^2}\), \(\frac{-2(5-x^2)}{(x^2 + 5)^2}\), \(\frac{10-x^2}{(x^2 + 10x^2 + 25)}\)

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their \(h'(x)=0\) and proceeds with a correct method to find \(x\). There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from ‘unsolvable’ equations.

A1 Finds the correct \(x\) value of the maximum point \(x=\sqrt{5}\). Ignore the solution \(x=-\sqrt{5}\) but withhold this mark if other positive values found.

M1 Substitutes their answer into their \(h'(x)=0\) in \(h(x)\) to determine the maximum value

A1 Cso-the maximum value of \(h(x) = \frac{\sqrt{5}}{5}\). Accept equivalents such as \(\frac{2\sqrt{5}}{10}\) but not \(0.447\)

A1ft Range of \(h(x)\) is \(0 \leq h(x) \leq \frac{\sqrt{5}}{5}\). Follow through on their maximum value if the M’s have been scored. Allow \(0 \leq y \leq \frac{\sqrt{5}}{5}\), \(0 \leq \text{Range} \leq \frac{\sqrt{5}}{5}\), \(\left[0, \frac{\sqrt{5}}{5}\right]\) but not \(0 \leq x \leq \frac{\sqrt{5}}{5}\), \(\left(0, \frac{\sqrt{5}}{5}\right)\)

If a candidate attempts to work out \(h^{-1}(x)\) in (b) and does all that is required for (b) in (c), then allow. Do not allow \(h^{-1}(x)\) to be used for \(h'(x)\) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets \(h(x) = 2x(x^2 + 5)^{-1}\) and applies the product rule \(vu'+uv'\) with terms being \(2x\) and \((x^2+5)^{-1}\)

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out \(u=\ldots,u'=\ldots,v=\ldots,v'=\ldots\) followed by their \(vu'+uv'\)) then only accept answers of the form

\[(x^2 + 5)^{-1} \times A + 2x \times \pm Bx(x^2 + 5)^{-2}\]

A1 Correct unsimplified answer \((x^2 + 5)^{-1} \times 2 + 2x \times -2x(x^2 + 5)^{-2}\)

A1 The question asks for \(h'(x)\) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.
For a correct simplified answer accept
\[
h'(x) = \frac{10 - 2x^2}{(x^2 + 5)^2} = \frac{2(5 - x^2)}{(x^2 + 5)^2} = \frac{-2(x^2 - 5)}{(x^2 + 5)^2} = \frac{(10 - 2x^2)(x^2 + 5)^{-2}}{}
\]
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
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</thead>
<tbody>
<tr>
<td>8.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) (£) 19500</td>
<td>B1</td>
<td>(1)</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500 ]</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>[ 17e^{-0.25t} + 2e^{-0.5t} = 9 ]</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>[(xe^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t} ]</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>[ 0 = 9e^{0.5t} - 17e^{0.25t} - 2 ]</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>[ 0 = (9e^{0.25t} + 1)(e^{0.25t} - 2) ]</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>[ e^{0.25t} = 2 ]</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>[ t = 4\ln(2) \text{ oe} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td>[ \left( \frac{dV}{dt} \right) = -4250e^{-0.25t} - 1000e^{-0.5t} ]</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>When ( t=8 ) Decrease = 593 (£/year)</td>
<td>M1A1</td>
<td>(4)</td>
</tr>
</tbody>
</table>

(a) B1 19500. The £ sign is not important for this mark

(b) M1 Substitute \( V=9500 \), collect terms and set on 1 side of an equation =0. Indices must be correct
Accept 17000e^{-0.25t} + 2000e^{-0.5t} - 9000 = 0 and 17000x + 2000x^2 - 9000 = 0 where \( x = e^{-0.25t} \)

M1 Factorise the quadratic in \( e^{0.25t} \) or \( e^{-0.25t} \)
For your information the factorised quadratic in \( e^{-0.25t} \) is \( (2e^{-0.25t} - 1)(e^{-0.25t} + 9) = 0 \)
Alternatively let \('x' = e^{0.25t}\) or otherwise and factorise a quadratic equation in \( x \)

A1 Correct solution of the quadratic. Either \( e^{0.25t} = 2 \) or \( e^{-0.25t} = \frac{1}{2} \text{ oe.} \)

A1 Correct exact value of \( t \). Accept variations of \( 4\ln(2) \), such as \( \ln(16) \), \( \frac{\ln(2)}{-0.25} \), \( \frac{\ln(2)}{0.25} \), \(-4\ln(\frac{1}{2})\)

(c) M1 Differentiates \( V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500 \) by the chain rule.
Accept answers of the form \( \left( \frac{dV}{dt} \right) = \pm Ae^{-0.25t} \pm Be^{-0.5t} \) \( A, B \) are constants \( \neq 0 \)

A1 Correct derivative \( \left( \frac{dV}{dt} \right) = -4250e^{-0.25t} - 1000e^{-0.5t} \).
There is no need for it to be simplified so accept
\( \left( \frac{dV}{dt} \right) = 17000 \times -0.25e^{-0.25t} + 2000 \times -0.5e^{-0.5t} \text{ oe} \)

M1 Substitute \( t=8 \) into their \( \left( \frac{dV}{dt} \right) \).
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept \( t=8 \) in \( V \)
±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. This would not be isw. Be aware that sub t=8 into V first and then differentiating can achieve 593. This is M0A0M0A0.
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General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

   In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes .
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \rightarrow \) will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - \( * \) or AG: The answer is printed on the paper
   - dM1 denotes a method mark which is dependent upon the award of the previous method mark.
   - ddM1 denotes a method mark which is dependent upon the award of the previous 2 method marks.
   - dM1* denotes a method mark which is dependent upon the award of the M1* mark.

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.
**Use of a formula**
Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is *not* quoted, the method mark can be gained by implication from *correct* working with values, but may be lost if there is any mistake in the working.

**Exact answers**
Examiners’ reports have emphasised that where, for example, an *exact* answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**
The rubric says that these *may* not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.

**Misreads**
A misread must be consistent for the *whole question* to be interpreted as such.
These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.
### Question 1

\[(2 + 3x)^3 = \left(\frac{1}{8}\right)^3 \left(1 + \frac{3x}{2}\right)^3 = \frac{1}{8} \left(1 + \frac{3x}{2}\right)^3\]

\[= \frac{1}{8} \left[ 1 + (-3)(kx) + \frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3 + ... \right]\]

\[= \frac{1}{8} \left[ 1 + \frac{3x}{2} \right] + \frac{(-3)(-4)}{2!} \left(\frac{3x}{2}\right)^2 + \frac{(-3)(-4)(-5)}{3!} \left(\frac{3x}{2}\right)^3 + ...\]

\[= \frac{1}{8} \left[ 1 - \frac{9}{2}x + \frac{27}{2}x^2 - \frac{135}{4}x^3 + ... \right]\]

**Scheme**

\[= \frac{1}{8} - \frac{9}{16}x + \frac{27}{16}x^2 - \frac{135}{32}x^3 + ...\]

#### Marking Scheme

<table>
<thead>
<tr>
<th>Marking Scheme</th>
<th>Marks</th>
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<tbody>
<tr>
<td><strong>B1</strong>: ( (2)^{-3} ) or ( \frac{1}{8} ) outside brackets or ( \frac{1}{8} ) as constant term in the binomial expansion.</td>
<td>5</td>
</tr>
<tr>
<td><strong>M1</strong>: Expands ((...+kx)^{-3}) to give any 2 terms out of 4 terms simplified or un-simplified,</td>
<td></td>
</tr>
<tr>
<td>Eg: (1 + (-3)(kx)) or (-3)(kx) + (\frac{(-3)(-4)}{2!}(kx)^2) or (1 + ...... + \frac{(-3)(-4)}{2!}(kx)^2)</td>
<td></td>
</tr>
<tr>
<td>or (\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3) where (k \neq 1) are ok for M1.</td>
<td></td>
</tr>
<tr>
<td><strong>A1</strong>: A correct simplified or un-simplified (1 + (-3)(kx) + \frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3) expansion with consistent ((kx)) where (k \neq 1).</td>
<td></td>
</tr>
</tbody>
</table>

**Incorrect bracketing** \(\frac{1}{8} \left[ 1 + (-3)(\frac{3x}{2}) + \frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2 + \frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3 + ... \right]\) is M1A0 unless recovered.

**A1**: For \(\frac{1}{8} - \frac{9}{16}x\) (simplified fractions) or also allow \(0.125 - 0.5625x\).

**Allow Special Case A1 for either** SC: \(\frac{1}{8} \left[ 1 - \frac{9}{2}x; ... \right]\) or SC: \(K \left[ 1 - \frac{9}{2}x + \frac{27}{2}x^2 - \frac{135}{4}x^3 + ... \right]\)

(where \(K\) can be 1 or omitted), with each term in the \([\ldots]\) either a simplified fraction or a decimal.

**A1**: Accept only \(\frac{27}{16}x^2 - \frac{135}{32}x^3\) or \(\frac{11}{16}x^2 - \frac{7}{32}x^3\) or \(1.6875x^2 - 4.21875x^3\)
Candidates who write 
\[
\frac{1}{8} \left[ 1 + (-3) \left( -\frac{3x}{2} \right) + \frac{(-3)(-4)}{2!} \left( -\frac{3x}{2} \right)^2 + \frac{(-3)(-4)(-5)}{3!} \left( -\frac{3x}{2} \right)^3 + \ldots \right]
\]
where 
\[k = -\frac{3}{2}\] and not \[\frac{3}{2}\] and achieve 
\[
\frac{1}{8} + \frac{9}{16} x + \frac{27}{16} x^2 + \frac{135}{32} x^3 + \ldots \] will get B1M1A1A0A0.

**Alternative method:** Candidates can apply an alternative form of the binomial expansion.

\[
(2 + 3x)^{-3} = \left(2^{-3} + (-3)(2)^{-4}(3x) + \frac{(-3)(-4)}{2!}(2)^{-5}(3x)^2 + \frac{(-3)(-4)(-5)}{3!}(2)^{-6}(3x)^3 \right)
\]

**B1:** \[\frac{1}{8}\] or \[(2)^{-3}\]

**M1:** Any two of four (un-simplified) terms correct.

**A1:** All four (un-simplified) terms correct.

\[
\frac{1}{8} - \frac{9}{16} x
\]

**A1:** \[\frac{27}{16} x^2 \] \[\frac{-135}{32} x^3\]

**Note:** The terms in C need to be evaluated, so \[-3C_0(2)^{-3} + (-3)C_1(2)^{-4}(3x) + (-3)C_2(2)^{-5}(3x)^2 + (-3)C_3(2)^{-6}(3x)^3\] without further working is B0M0A0.
<table>
<thead>
<tr>
<th>Question Number</th>
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</table>
| 2. (a) | \[
\int \frac{1}{x^2} \ln x \, dx, \quad \begin{cases} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ dv = -\frac{1}{x^2} \, dx & \Rightarrow v = -\frac{1}{2x} \end{cases}
\]
In the form \( \pm \frac{1}{x^2} \ln x \pm \int \mu \frac{1}{x^2} \cdot \frac{1}{x} \) \( \text{M1} \)

\[= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx \quad \text{\( \pm \frac{1}{2x^2} \ln x \) simplified or un-simplified. \( \text{A1} \)}

\[= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^2} \, dx \quad \text{\( \pm \int \mu \frac{1}{x^2} \cdot \frac{1}{x} \rightarrow \pm \beta x^{-2} \). \( \text{A1} \)}

\[= -\frac{1}{2x^2} \ln x + \frac{1}{2} \left( -\frac{1}{2} \right) \{ + c \} \quad \text{Correct answer, with/without } + c \]

\[\{ \left[ -\frac{1}{2x^2} \ln x - \frac{1}{4x^2} \right]^2 \} = \left( -\frac{1}{2(2)^2} \ln 2 - \frac{1}{4(2)^2} \right) - \left( -\frac{1}{2(1)^2} \ln 1 - \frac{1}{4(1)^2} \right) \quad \text{Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round. \( \text{M1} \)}

\[= \frac{3}{16} - \frac{1}{8} \ln 2 \quad \text{or} \quad \frac{3}{16} - \ln 2^\frac{1}{8} \quad \text{or} \quad \frac{1}{16} (3-2\ln 2), \text{ etc., or awrt 0.1 or equivalent. \( \text{A1} \)}

(b) \[\int \frac{1}{x^2} \ln x \, dx = \frac{3}{16} - \frac{1}{8} \ln 2 \quad \text{or} \quad \frac{3}{16} - \ln 2^\frac{1}{8} \quad \text{or} \quad \frac{1}{16} (3-2\ln 2) \quad \text{or} \quad \text{awrt 0.1 or equivalent.} \]

\[= \frac{3}{16} - \frac{1}{8} \ln 2 \quad \text{or} \quad \frac{3}{16} - \ln 2^\frac{1}{8} \quad \text{or} \quad \frac{1}{16} (3-2\ln 2) \quad \text{or} \quad \text{awrt 0.1 or equivalent.} \]

(a) \[\text{M1: Integration by parts is applied in the form } \pm \frac{1}{x^2} \ln x \pm \int \mu \frac{1}{x^2} \cdot \frac{1}{x} \text{ or equivalent.} \]

\[\text{A1: } -\frac{1}{2x} \ln x \text{ simplified or un-simplified.} \]

\[\text{A1: } -\int -\frac{1}{2x^2} \cdot \frac{1}{x} \quad \text{or equivalent. You can ignore the } dx. \]

\[\text{dM1: Depends on the previous M1. } \pm \int \mu \frac{1}{x^2} \cdot \frac{1}{x} \rightarrow \pm \beta x^{-2}. \]

\[\text{A1: } -\frac{1}{2x^2} \ln x + \frac{1}{2} \left( -\frac{1}{2x^2} \right) \{ + c \} \quad \text{or} \quad -\frac{1}{2x^2} \ln x - \frac{1}{4x^2} \{ + c \} \quad \text{or} \quad \frac{x^{-2}}{2} \ln x - \frac{x^{-2}}{4} \{ + c \} \]

\[\text{or} \quad \frac{1-2\ln x}{4x^2} \{ + c \} \text{ or equivalent.} \]

\[\text{You can ignore subsequent working after a correct stated answer.} \]

(b) \[\text{M1: Some evidence of applying limits of 2 and 1 to their part (a) answer and subtracts the correct way round. }\]

\[\text{A1: Two term exact answer of either } \frac{3}{16} - \frac{1}{8} \ln 2 \quad \text{or} \quad \frac{3}{16} - \ln 2^\frac{1}{8} \quad \text{or} \quad \frac{1}{16} (3-2\ln 2) \quad \text{or} \quad \frac{\ln(\frac{1}{2}) + 3}{16} \]

\[\text{or } 0.1875 - 0.125 \ln 2. \text{ Also allow awrt 0.1. Also note the fraction terms must be combined.} \]

\[\text{Note: Award the final A0 in part (b) for a candidate who achieves awrt 0.1 in part (b), when their answer to part (a) is incorrect.} \]
2. (b) ctd  

Note: Decimal answer is 0.100856... in part (b).

Alternative Solution

\[
\int \frac{1}{x^3} \ln x \, dx, \quad \begin{cases} 
  u = x^{-3} & \Rightarrow \frac{du}{dx} = -3x^{-4} \\
  \frac{dv}{dx} = \ln x & \Rightarrow v = x \ln x - x
\end{cases}
\]

\[
\int \frac{1}{x^3} \ln x \, dx = \frac{1}{x^3} (x \ln x - x) - \int (x \ln x - x) \frac{-3}{x^3} \, dx
\]

\[
-2 \int \frac{1}{x^3} \ln x \, dx = \frac{1}{x^3} (x \ln x - x) - \int \frac{3}{x^3} \, dx
\]

\[
-2 \int \frac{1}{x^3} \ln x \, dx = \frac{1}{x^3} (x \ln x - x) + \frac{3}{2x^2} \{ + c \}
\]

\[
\int \frac{1}{x^3} \ln x \, dx = -\frac{1}{2x^2} (x \ln x - x) - \frac{3}{4x^2} \{ + c \}
\]

\[
= -\frac{1}{2x^2} \ln x - \frac{1}{4x^2} \{ + c \}
\]

\[
k \int \frac{1}{x^3} \ln x \, dx = \frac{1}{x^3} (x \ln x - x) \pm \int \frac{\lambda}{x^3} \, dx
\]

where \( k \neq 1 \)

Any one of \( \frac{1}{x^3} (x \ln x - x) \) or \( -\int \frac{3}{x^3} \, dx \)

\[
= \frac{1}{x^3} (x \ln x - x) - \int \frac{3}{x^3} \, dx \quad \text{and} \quad k = -2
\]

\[
\pm \int \mu \frac{1}{x^3} \to \pm \beta x^{-2}
\]

\[
-\frac{1}{2x^2} (x \ln x - x) - \frac{3}{4x^2} \quad \text{or equivalent}
\]

\[
\text{with/without} \quad + c.
\]
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
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</tr>
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<tbody>
<tr>
<td>3.</td>
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<td></td>
</tr>
<tr>
<td><strong>Method 1: Using one identity</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9x^2 + 20x - 10) &amp; (\equiv A + \frac{B}{x+2} + \frac{C}{3x-1})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A = 3) &amp; (\text{their constant term} = 3)</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>(9x^2 + 20x - 10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x^2: 9 = 3A, x: 20 = 5A + 3B + C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attempts to find the value of either one of their B or their C from their identity.</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>or (x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Method 2: Long Division</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9x^2 + 20x - 10) &amp; (\equiv 3 + \frac{5x - 4}{(x+2)(3x-1)})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B = \frac{5x - 4}{(x+2)(3x-1)}) &amp; (C = \frac{B}{(x+2)})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5x - 4 \equiv B(3x-1) + C(x+2)) &amp; (\text{their constant term} = 3)</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>Attempts to find the value of either one of their B or their C from their identity.</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>or (x = -2 \Rightarrow -10 - 4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x = \frac{1}{3} \Rightarrow \frac{5}{3} - 4 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correct values for their B and their C, which are found using (5x - 4 \equiv B(3x-1) + C(x+2))</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(9x^2 + 20x - 10) &amp; (\equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1\(^{st}\) B1: Their constant term must be equal to 3 for this mark.

2\(^{nd}\) B1 (M1 open): Forming a correct identity. This can be implied by later working.

M1 (A1 open): Attempts to find the value of either one of their B or their C from their identity. This can be achieved by *either* substituting values into their identity *or* comparing coefficients and solving the resulting equations simultaneously.

A1: Correct values for their B and their C, which are found using a correct identity.

Note: \(9x^2 + 20x - 10 \equiv \frac{A}{x+2} + \frac{B}{3x-1}\), leading to \(9x^2 + 20x - 10 \equiv A(3x-1) + B(x+2)\), leading to \(A = 2\) and \(B = -1\) will gain a maximum of B0B0M1A0.
Note: You can imply the \( 2^{nd} \) \( B_1 \) from either
\[
\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} = \frac{A(x + 2)(3x - 1) + B(3x - 1) + C(x + 2)}{(x + 2)(3x - 1)}
\]
or
\[
\frac{5x - 4}{(x + 2)(3x - 1)} = \frac{B(3x - 1) + C(x + 2)}{(x + 2)(3x - 1)}
\]

**Alternative Method 1: Initially dividing by \((x + 2)\)**
\[
\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \equiv \frac{9x + 2}{(3x - 1)} - \frac{14}{(x + 2)(3x - 1)}
\]
\[
= 3 + \frac{5}{(3x - 1)} - \frac{14}{(x + 2)(3x - 1)}
\]

So,
\[
\frac{-14}{(x + 2)(3x - 1)} \equiv \frac{B}{(x + 2)} + \frac{C}{(3x - 1)}
\]
\[
-14 \equiv B(3x - 1) + C(x + 2)
\]
\[
\Rightarrow B = 2, \ C = -6
\]
So,
\[
\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \equiv 3 + \frac{5}{(3x - 1)} + \frac{2}{(x + 2)} - \frac{6}{(3x - 1)}
\]
and
\[
\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \equiv 3 + \frac{2}{(x + 2)} - \frac{1}{(3x - 1)}
\]

**Alternative Method 2: Initially dividing by \((3x - 1)\)**
\[
\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \equiv \frac{3x + \frac{7}{3}}{(x + 2)} - \frac{\frac{7}{3}}{(3x - 1)}
\]
\[
= 3 + \frac{\frac{7}{3}}{(x + 2)} - \frac{\frac{7}{3}}{(3x - 1)}
\]

So,
\[
\frac{-\frac{7}{3}}{(x + 2)(3x - 1)} \equiv \frac{B}{(x + 2)} + \frac{C}{(3x - 1)}
\]
\[
-\frac{7}{3} \equiv B(3x - 1) + C(x + 2)
\]
\[
\Rightarrow B = \frac{1}{3}, \ C = -1
\]
So,
\[
\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \equiv 3 + \frac{\frac{7}{3}}{(x + 2)} + \frac{\frac{1}{3}}{(x + 2)} - \frac{1}{(3x - 1)}
\]
and
\[
\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} \equiv 3 + \frac{2}{(x + 2)} - \frac{1}{(3x - 1)}
\]

**B1:** their constant term = 3

**M1:** Attempts to find either one of their \( B \) or their \( C \) from their identity.

**A1:** Correct answer in partial fractions.
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<td>4. (a)</td>
<td>1.0981</td>
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</table>
| (b)             | Area \( \approx \frac{1}{2} \times 1 \times \left[ 0.5 + 2(0.8284 + \text{their } 1.0981) + 1.3333 \right] \)  
|                 | \( = \frac{1}{2} \times 5.6863 = 2.84315 = 2.843 \) (3 dp)         |       |
| (c)             | \( \{ \frac{u}{1 + \sqrt{x}} \} \Rightarrow \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \) or \( \frac{dx}{du} = 2(u - 1) \)  
|                 | \( \left\{ \int \frac{x}{1 + \sqrt{x}} \, dx \right\} = \int \frac{(u - 1)^2}{u} \cdot 2(u - 1) \, du \)  
|                 | \( = 2 \int \frac{(u - 1)^2}{u} \, du = \left\{ 2 \right\} \int \frac{(u^3 - 3u^2 + 3u - 1)}{u} \, du \)  
|                 | \( = \left\{ 2 \right\} \left( \frac{u^3}{3} - 3 \frac{u^2}{2} + 3u - \ln u \right) \)  
|                 | Area \( (R) = \left[ \frac{2u^3}{3} - 3u^2 + 6u - 2 \ln u \right] \) \)  
|                 | \( = \left( \frac{2(3)^3}{3} - 3(3)^2 + 6(3) - 2 \ln 3 \right) - \left( \frac{2(2)^3}{3} - 3(2)^2 + 6(2) - 2 \ln 2 \right) \)  
|                 | \( = \frac{11}{3} + 2 \ln 2 - 2 \ln 3 \) or \( \frac{11}{3} + 2 \ln \left( \frac{2}{3} \right) \) or \( \frac{11}{3} - \ln \left( \frac{9}{4} \right) \), etc. |       |

**Notes:**
- **B1 cao:** 1.0981 correct answer only. Look for this on the table or in the candidate’s working.
- **B1:** Outside brackets \( \frac{1}{2} \times 1 \) or \( \frac{1}{2} \)
- **M1:** For structure of trapezium rule \( \ldots \ldots \) .
- **A1:** anything that rounds to 2.843
- **Note:** Working must be seen to demonstrate the use of the trapezium rule. **Note:** actual area is 2.85573645…
- **Note:** Award B1M1 A1 for \( \frac{1}{2} \cdot (0.5 + 1.3333) + (0.8284 + \text{their } 1.0981) = 2.84315 \)
- **Bracketing mistake:** Unless the final answer implies that the calculation has been done correctly
- Award B1M0A0 for \( \frac{1}{2} \times 1 + 0.5 + 2(0.8284 + \text{their } 1.0981) + 1.3333 \) (nb: answer of 6.1863).
- Award B1M0A0 for \( \frac{1}{2} \times 1 (0.5 + 1.3333) + 2(0.8284 + \text{their } 1.0981) \) (nb: answer of 4.76965).
Alternative method for part (b): Adding individual trapezia

Area \( \approx 1 \times \left[ \frac{0.5 + 0.8284}{2} + \frac{0.8284 + 1.0981}{2} + \frac{1.0981 + 1.3333}{2} \right] = 2.84315 \)

**B1:** 1 and a divisor of 2 on all terms inside brackets.

**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.

**A1:** anything that rounds to 2.843

\[
\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \quad \text{or} \quad du = \frac{1}{2 \sqrt{x}} \, dx \quad \text{or} \quad 2\sqrt{x} \, du = dx \quad \text{or} \quad dx = 2(u-1)du \quad \text{or} \quad \frac{dx}{du} = 2(u-1) \quad \text{oe.}
\]

\[1^\text{st} \, M1: \quad \frac{x}{1 + \sqrt{x}} \quad \text{becoming} \quad \frac{(u-1)^2}{u} \quad \text{(Ignore integral sign)}.
\]

\[1^\text{st} \, A1 \, \text{(B1 on open)}: \quad \frac{dx}{1 + \sqrt{x}} \quad \text{becoming} \quad \frac{(u-1)^2}{u} \cdot 2(u-1)\{du\} \quad \text{or} \quad \frac{(u-1)^2}{u} \cdot \frac{2}{(u-1)^2}\{du\}.
\]

You can ignore the integral sign and the \( du \).

\[2^\text{nd} \, M1: \quad \text{Expands to give a “four term” cubic in} \, u, \quad \pm Au^3 \pm Bu^2 \pm Cu \pm D\]

\[\text{where} \quad A \neq 0, B \neq 0, C \neq 0 \quad \text{and} \quad D \neq 0 \quad \text{The cubic does not need to be simplified for this mark.}
\]

\[3^\text{rd} \, M1: \quad \text{An attempt to divide at least three terms in their cubic by} \, u.
\]

\[\text{Ie.} \quad \frac{(u^3 - 3u^2 + 3u - 1)}{u} \quad \to \quad u^2 - 3u + 3 - \frac{1}{u}
\]

\[2^\text{nd} \, A1: \quad \int \frac{(u-1)^3}{u} \, du \quad \to \quad \left( \frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u \right)
\]

\[4^\text{th} \, M1: \quad \text{Some evidence of limits of 3 and 2 in} \, u \quad \text{and subtracting either way round.}
\]

\[3^\text{rd} \, A1: \quad \text{Exact answer of} \quad \frac{11}{3} + 2 \ln 2 - 2 \ln 3 \quad \text{or} \quad \frac{11}{3} + 2 \ln \left( \frac{2}{3} \right) \quad \text{or} \quad \frac{11}{3} - \ln \left( \frac{9}{4} \right) \quad \text{or} \quad 2\left( \frac{11}{6} + \ln 2 - \ln 3 \right)
\]

\[\text{or} \quad \frac{22}{6} + 2 \ln \left( \frac{2}{3} \right), \text{etc.} \quad \text{Note:} \quad \text{that fractions must be combined to give either} \quad \frac{11}{3} \quad \text{or} \quad \frac{22}{6} \quad \text{or} \quad \frac{32}{3}.
\]

**Alternative method for 2\text{nd} \, M1 \quad \text{and} \quad 3\text{rd} \, M1 \, \text{mark}

\[
\{2\} \int \frac{(u-1)^2}{u} \cdot (u-1) \, du = \{2\} \int \frac{(u^2 - 2u + 1)}{u} \cdot (u-1) \, du
\]

\[
= \{2\} \int \left( u - 2 + \frac{1}{u} \right) \cdot (u-1) \, du = \{2\} \int \left( u^2 - \ldots \right) \, du
\]

\[
= \{2\} \int \left( u^2 - 2u + 1 - u + 2 - \frac{1}{u} \right) \, du
\]

\[
= \{2\} \int \left( u^2 - 3u + 3 - \frac{1}{u} \right) \, du
\]

An attempt to expand \((u-1)^2\), then divide the result by \(u\) and then go on to multiply by \((u-1)\).

\[2^\text{nd} \, M1 \]

to give three out of four of \(\pm Au^3, \pm Bu^2, \pm C \text{ or} \pm \frac{D}{u}\)

\[3^\text{rd} \, M1 \]
Final two marks in part (c): $u = 1 + \sqrt{x}$

$$\text{Area}(R) = \left[ \frac{2(1 + \sqrt{x})^3}{3} - 3(1 + \sqrt{x})^2 + 6(1 + \sqrt{x}) - 2\ln(1 + \sqrt{x}) \right]_1^4$$

$$= \left( \frac{2(1 + \sqrt{4})^3}{3} - 3(1 + \sqrt{4})^2 + 6(1 + \sqrt{4}) - 2\ln(1 + \sqrt{4}) \right)$$

$$- \left( \frac{2(1 + \sqrt{1})^3}{3} - 3(1 + \sqrt{1})^2 + 6(1 + \sqrt{1}) - 2\ln(1 + \sqrt{1}) \right)$$

$$= (18 - 27 + 18 - 2\ln 3) - \left( \frac{16}{3} - 12 + 12 - 2\ln 2 \right)$$

$$= \frac{11}{3} + 2\ln 2 - 2\ln 3 \quad \text{or} \quad \frac{11}{3} + 2\ln \left( \frac{2}{3} \right) \quad \text{or} \quad \frac{11}{3} - \ln \left( \frac{9}{4} \right), \text{ etc}$$

A1: Correct exact answer or equivalent.

Alternative method for the final 5 marks in part (b)

$$\int \frac{(u - 1)^3}{u} \, du$$

\[ u = u^{-1} \quad \Rightarrow \quad \frac{dv}{dx} = -u^{-2} \]

\[ \frac{dv}{dx} = (u - 1)^3 \quad \Rightarrow \quad v = \frac{(u - 1)^4}{4} \]

\[ \int \frac{(u - 1)^3}{u} \, du = \frac{(u - 1)^4}{4u} - \frac{1}{4} \int \frac{(u - 1)^4}{u^2} \, du \]

\[ = \frac{(u - 1)^4}{4u} + \frac{1}{4} \int \frac{u^4 - 4u^3 + 6u^2 - 4u + 1}{u^2} \, du \]

\[ = \frac{(u - 1)^4}{4u} + \frac{1}{4} \int u^2 - 4u + 6 - \frac{4}{u} + \frac{1}{u^2} \, du \]

\[ = \frac{(u - 1)^4}{4u} + \frac{1}{4} \left( \frac{u^3}{3} - 2u^2 + 6u - 4\ln u - \frac{1}{u} \right) \]

\[ \int \left. \frac{(u - 1)^3}{u} \, du \right|_2^3 = \left[ \frac{(u - 1)^4}{4u} + \frac{u^3}{12} - \frac{u^2}{2} + \frac{3u}{2} - \ln u - \frac{1}{4u} \right]_2^3 \]

\[ = \left( \frac{16}{12} + \frac{27}{12} - \frac{9}{2} + \frac{9}{2} - \ln 3 - \frac{1}{12} \right) - \left( \frac{1}{8} + \frac{8}{12} - \frac{4}{2} + \frac{6}{2} - \ln 2 - \frac{1}{8} \right) \]

\[ = (7 - \ln 3) - \left( \frac{5}{3} - \ln 2 \right) \]

\[ = \frac{11}{6} + \ln \left( \frac{2}{3} \right) \]

A1
### Question 5

**Working parametrically:**

\[ x = 1 - \frac{1}{2} t, \quad y = 21^{\frac{t}{2}} - 1 \]

(a) \(\{x = 0 \Rightarrow 0 = 1 - \frac{1}{2} t \Rightarrow t = 2\)

When \(t = 2\), \(y = 2^2 - 1 = 3\)

(b) \(\{y = 0 \Rightarrow 0 = 2^2 - 1 \Rightarrow t = 0\)

When \(t = 0\), \(x = 1 - \frac{1}{2}(0) = 1\)

(c) \(\frac{dx}{dt} = -\frac{1}{2}\) and either \(\frac{dy}{dt} = 2t \ln 2\) or \(\frac{dy}{dt} = e^{t/2} \ln 2\)

\(\frac{dy}{dx} = \frac{2t \ln 2}{-\frac{1}{2}}\)

Attempts their \(\frac{dy}{dt}\) divided by their \(\frac{dx}{dt}\).

At \(A\), \(t = "2"\), so \(m(T) = -8 \ln 2 \Rightarrow m(N) = \frac{1}{8 \ln 2}\)

\[ y - 3 = \frac{1}{8 \ln 2} (x - 0) \quad \text{or} \quad y = 3 + \frac{1}{8 \ln 2} x \quad \text{or equivalent.} \]

(d) Area\((R) = \int (2^t - 1) \left(-\frac{1}{2}\right) dt\)

\[ x = -1 \rightarrow t = 4 \quad \text{and} \quad x = 1 \rightarrow t = 0 \]

\[ \left. \left(\frac{-1}{2} \left[ \frac{2'-t}{\ln 2} \right] \right) \right|_{4}^{0} = \frac{15}{2 \ln 2} - 2 \]

Either \(2' \rightarrow \frac{2'}{\ln 2}\)

or \((2' - 1) \rightarrow \frac{(2')}{\pm \alpha (\ln 2)} - t\)

or \((2' - 1) \rightarrow \pm \alpha (\ln 2)(2') - t\)

(2' - 1) \rightarrow \frac{2'}{\ln 2} - t

Depends on the previous method mark.

Substitutes their changed limits in \(t\) and subtracts either way round.

\[ \frac{15}{2 \ln 2} - 2 \quad \text{or equivalent.} \]
### Question 5

#### (a)

**M1:** Applies $x = 0$ and obtains a value of $t$.

**A1:** For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$

**Alternative Solution 1:**

**M1:** For substituting $t = 2$ into either $x$ or $y$.

**A1:**

$$x = 1 - \frac{1}{2}(2) = 0 \quad \text{and} \quad y = 2^2 - 1 = 3$$

**Alternative Solution 2:**

**M1:** Applies $y = 3$ and obtains a value of $t$.

**A1:** For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$.

**Alternative Solution 3:**

**M1:** Applies $y = 3$ or $x = 0$ and obtains a value of $t$.

**A1:** Shows that $t = 2$ for both $y = 3$ and $x = 0$.

**Note:** Award M1A1 for $x = 1$.

#### (b)

**M1:** For $\frac{1}{2}(2) = 0^2$ or $4 - 1 = 3$

**A1:**

$$x = 1 - \frac{1}{2}(2) = 0$$

**Alternative Solution 2:**

**M1:** Applies $y = 0$ and obtains a value of $t$.

**A1:** For $y = 1 - \frac{1}{2}(2) = 0$ or $y = 1 - 1 = 0$.

**Note:** Working must be seen in part (b).

**Note:** Award B1A1 for $x = 1$.

#### (c)

**B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. This mark can be implied by later working.

**M1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$. **Note:** their $\frac{dy}{dt}$ must be a function of $t$.

**M1:** Uses their value of $t$ found in part (a) and applies $m(N) = \frac{-1}{m(T)}$.

**M1:** $y - 3 = (\text{their normal gradient})x$ or $y = (\text{their normal gradient})x + 3$ or equivalent.

**A1:**

$$y - 3 = \frac{1}{8\ln 2} (x - 0) \quad \text{or} \quad y = 3 + \frac{1}{8\ln 2} x \quad \text{or} \quad y - 3 = \frac{1}{\ln 256} (x - 0) \quad \text{or} \quad (8\ln 2)y - 24\ln 2 = x$$

or

$$\frac{y - 3}{(x - 0)} = \frac{1}{8\ln 2}. \quad \text{You can apply isw here.}$$

**Working in decimals** is ok for the three method marks. B1, A1 require exact values.

#### (d)

**M1:** Complete substitution for both $y$ and $dx$. So candidate should write down $\int (2^t - 1) \left( \frac{dx}{dt} \right)$

**B1:** Changes limits from $x \to t$. $x = -1 \to t = 4$ and $x = 1 \to t = 0$. Note $t = 4$ and $t = 0$ seen is B1.

**M1\*: Integrates $2^t$ correctly to give $\frac{2^t}{\ln 2}$**

$$\int (2^t - 1) \quad \text{to give either} \quad \frac{(2^t)}{\pm \alpha (\ln 2)} - t \quad \text{or} \quad \pm \alpha (\ln 2)(2^t - t).$$

**A1:** Correct integration of $(2^t - 1)$ with respect to $t$ to give $\frac{2^t}{\ln 2} - t$.

**dM1\*: Depends upon the previous method mark.**

Substitutes their limits in $t$ and subtracts either way round.

**A1:** Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.
### Question Number 5

**Alternative: Converting to a Cartesian equation:**

\[ t = 2 - 2x \Rightarrow y = 2^{2-2x} - 1 \]

(a) \( \{ x = 0 \Rightarrow \} \ y = 2^2 - 1 \)

\[ y = 3 \]

(b) \( \{ y = 0 \Rightarrow \} 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2 - 2x \Rightarrow x = ... \)

\[ x = 1 \]

(c) \( \frac{dy}{dx} = -2\left(2^{2-2x}\right)\ln 2 \)

At \( A, \ x = 0, \) so \( m(T) = -8\ln 2 \Rightarrow m(N) = \frac{1}{8\ln 2} \)

\[ y - 3 = \frac{1}{8\ln 2} (x - 0) \quad \text{or} \quad y = 3 + \frac{1}{8\ln 2} x \quad \text{or equivalent.} \]

(d) \( \text{Area}(R) = \int (2^{2-2x} - 1) \, dx \)

\[ = \int_{-1}^{1} (2^{2-2x} - 1) \, dx \]

\[ = \left(\frac{2^{2-2x}}{-2\ln 2} - x\right) \]

\[ \left[ \frac{2^{2-2x}}{-2\ln 2} - x \right]_{-1}^{1} = \left(\left(\frac{1}{-2\ln 2} - 1\right) - \left(\frac{16}{-2\ln 2} + 1\right)\right) \]

\[ = \frac{15}{2\ln 2} - 2 \]

**Alternative method:** In Cartesian and applying \( u = 2 - 2x \)

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**Marks**

- **M1:** Applies \( x = 0 \) in their Cartesian equation...
- **A1:** ... to arrive at a correct answer of 3.
- **M1:** Applies \( y = 0 \) to obtain a value for \( x \).
- **A1:** (Must be seen in part (b)).
- **M1:** Applies \( x = 0 \) and \( m(N) = \frac{-1}{m(T)} \)
- **M1:** As in the original scheme.
- **B1:** Form the integral of their Cartesian equation of \( C \).
- **M1:** For \( 2^{2-2x} - 1 \) with limits of \( x = -1 \) and \( x = 1 \). Ie. \( \int_{-1}^{1} (2^{2-2x} - 1) \)
- **M1:** Either \( 2^{2-2x} \rightarrow \frac{2^{2-2x}}{-2\ln 2} \)
- **M1:** or \( (2^{2-2x} - 1) \rightarrow \frac{2^{2-2x}}{\pm\alpha(\ln 2)} - x \)
- **M1:** or \( (2^{2-2x} - 1) \rightarrow \pm\alpha(\ln 2)(2^{2-2x}) - x \)
- **A1:** Depends on the previous method mark.
- **dM1:** Substitutes limits of -1 and their \( x \) and subtracts either way round.
- **A1:** \( \frac{15}{2\ln 2} - 2 \) or equivalent.
Area(R) = \int \left(2^u - 1\right)\{dx\} 
\hspace{1cm} \text{, where } u = 2 - 2x
\hspace{1cm} = \int_{1}^{0} (2^u - 1)\left(-\frac{1}{2}\right)\{du\}

**M0:** Unless a candidate \textit{writes} \(\int (2^{2x} - 1)\{dx\}\)

Then apply the “working parametrically” mark scheme.
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<tr>
<td><strong>Alternative method: For substitution</strong> $u = 2^t$</td>
<td></td>
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<tr>
<td>Area($R$) = $\int (2^t - 1) \cdot \left(-\frac{1}{2}\right) dt$</td>
<td>Complete substitution for both $y$ and $dx$</td>
<td>M1</td>
</tr>
<tr>
<td>where $u = 2^t$ (\Rightarrow) $\frac{du}{dt} = 2^t \ln 2$ (\Rightarrow) $\frac{du}{dt} = u \ln 2$</td>
<td>Both correct limits in $t$ or both correct limits in $u$.</td>
<td>B1</td>
</tr>
<tr>
<td>$x = -1 \rightarrow t = 4 \rightarrow u = 16$ and $x = 1 \rightarrow t = 0 \rightarrow u = 1$</td>
<td>If not awarded above, you can award M1 for this integral</td>
<td></td>
</tr>
<tr>
<td>So area($R$) = $-\frac{1}{2} \int \frac{u - 1}{u \ln 2} du$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= -\frac{1}{2} \int \left(\frac{1}{\ln 2} - \frac{1}{u \ln 2}\right) du$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\left{ -\frac{1}{2} \left[ \frac{u}{\ln 2} - \frac{\ln u}{\ln 2} \right] \right}_{16}$</td>
<td>Either $2^t \rightarrow \frac{u}{\ln 2}$ or $(2^t - 1) \rightarrow \frac{u}{\ln 2} - \frac{\ln u}{\ln 2}$</td>
<td>M1*</td>
</tr>
<tr>
<td>$= -\frac{1}{2} \left( \frac{16}{\ln 2} - \frac{\ln 16}{\ln 2} \right)$</td>
<td>or $(2^t - 1) \rightarrow \pm \alpha (\ln 2)(u) - \frac{\ln u}{\ln 2}$</td>
<td>A1</td>
</tr>
<tr>
<td>$= \frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$</td>
<td>Depends on the previous method mark.</td>
<td>dM1*</td>
</tr>
<tr>
<td></td>
<td>Substitutes their changed limits in $u$ and subtracts either way round.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$</td>
<td>A1</td>
</tr>
<tr>
<td>Question Number</td>
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</tr>
<tr>
<td>6. (a)</td>
<td>(y = 0 \Rightarrow 1 - 2 \cos x = 0)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3})</td>
<td>A1 cs0</td>
</tr>
<tr>
<td></td>
<td>(1 - 2 \cos x = 0, \text{ seen or implied.})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>At least one correct value of (x). (See notes).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Both (\frac{\pi}{3}) and (\frac{5\pi}{3})</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>(V = \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos x)^2 , dx)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>({ \int (1 - 2 \cos x)^2 , dx } = \int (1 - 4 \cos x + 4 \cos^2 x) , dx)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(= \int 1 - 4 \cos x + 4 \left( \frac{1 + \cos 2x}{2} \right) , dx)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= \int (3 - 4 \cos x + 2 \cos 2x) , dx)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= 3x - 4 \sin x + \frac{2 \sin 2x}{2})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Attempts (\int y^2) to give any two of</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\pm A \rightarrow \pm Ax, \pm B \cos x \rightarrow \pm B \sin x) or</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(\pm \lambda \cos 2x \rightarrow \pm \mu \sin 2x).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correct integration.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Applying limits the correct way round. Ignore (\pi).</td>
<td>ddM1</td>
</tr>
<tr>
<td></td>
<td>(= \pi \left( \left( \frac{5\pi + 2 \sqrt{3} - \sqrt{3}}{2} \right) - \left( \frac{\pi - 2 \sqrt{3} + \sqrt{3}}{2} \right) \right))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= \pi \left( (18.3060...) - (0.5435...) \right) = 17.7625\pi = 55.80)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= \pi \left( 4\pi + 3 \sqrt{3} \right) \text{ or } 4\pi^2 + 3\pi \sqrt{3})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Two term exact answer.</td>
<td>A1</td>
</tr>
</tbody>
</table>
6. (a)  

**M1:** \( 1 - 2 \cos x = 0 \).

This can be implied by either \( \cos x = \frac{1}{2} \) or any one of the correct values for \( x \) in radians or in degrees.

1st A1: Any one of either \( \frac{\pi}{3} \) or \( \frac{5\pi}{3} \) or 60 or 300 or awrt 1.05 or 5.23 or awrt 5.24.

2nd A1: Both \( \frac{\pi}{3} \) and \( \frac{5\pi}{3} \).

(b)  

**B1:** (M1 on open) For \( \pi \int (1 - 2 \cos x)^2 \). Ignore limits and \( \text{d}x \).

1st M1: Any correct form of \( \cos 2x = 2 \cos^2 x - 1 \) used or written down in the same variable.

This can be implied by \( \cos^2 x = \frac{1 + \cos 2x}{2} \) or \( 4 \cos^2 x \rightarrow 2 + 2 \cos 2x \) or \( \cos 2A = 2 \cos^2 A - 1 \).

2nd M1: Attempts \( \int y^2 \) to give any two of \( \pm A \rightarrow \pm A x \), \( \pm B \cos x \rightarrow \pm B \sin x \) or \( \pm \lambda \cos 2x \rightarrow \pm \mu \sin 2x \).

Do not worry about the signs when integrating \( \cos x \) or \( \cos 2x \) for this mark.

Note: \( \int (1 - 2 \cos x)^2 = \int 1 + 4 \cos^2 x \) is ok for an attempt at \( \int y^2 \).

1st A1: Correct integration. Eg. \( 3x - 4 \sin x + \frac{2 \sin 2x}{2} \) or \( x - 4 \sin x + \frac{2 \sin 2x}{2} + 2x \) oe.

3rd ddM1: Depends on both of the two previous method marks. (Ignore \( \pi \)).

Some evidence of substituting their \( x = \frac{5\pi}{3} \) and their \( x = \frac{\pi}{3} \) and subtracting the correct way round.

You will need to use your calculator to check for correct substitution of their limits into their integrand if a candidate does not explicitly give some evidence.

Note: For correct integral and limits decimals gives: \( \pi((18.3060...) - (0.5435...)) = 17.7625\pi = 55.80 \)

2nd A1: Two term exact answer of either \( \pi \left(4 \pi + 3 \sqrt{3}\right) \) or \( 4\pi^2 + 3\pi \sqrt{3} \) or equivalent.

Note: The \( \pi \) in the volume formula is only required for the B1 mark and the final A1 mark.

Note: Decimal answer of 58.802... without correct exact answer is A0.

Note: Applying \( \int (1 - 2 \cos x) \text{d}x \) will usually be given no marks in this part.
7. (a)  
\[ \begin{align*}
\mathbf{i}: & \quad 9 + \lambda = 2 + 2\mu \\
\mathbf{j}: & \quad 13 + 4\lambda = -1 + \mu \\
\mathbf{k}: & \quad -3 - 2\lambda = 1 + \mu
\end{align*} \]

Any two equations. (Allow one slip). M1

\[ \text{Eg: (2)} - (3): 16 + 6\lambda = -2 \quad \text{or} \]
\[ (2) - 4(1): -23 = -9 - 7\mu \]

Leading to \( \lambda = -3 \) or \( \mu = 2 \)

An attempt to eliminate one of the parameters. dM1

Either \( \lambda = -3 \) or \( \mu = 2 \) A1

Realisation that the dot product is required between \( \pm A\mathbf{d}_1 \) and \( \pm B\mathbf{d}_2 \). M1

Correct equation. A1

\[ \text{awrt 69.1} \quad \text{A1} \]

(b)  
\[ \mathbf{d}_1 = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \]

\[ \Rightarrow \quad \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 1 \]

Correct equation. A1

\[ \text{awrt 69.1} \quad \text{A1} \]

(c)  
\[ \overrightarrow{OA} = \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix}, \quad \overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 + \lambda \\ 13 + 4\lambda \\ -3 - 2\lambda \end{pmatrix} \]

\[ \overrightarrow{AP} = \begin{pmatrix} 9 + \lambda \\ 13 + 4\lambda \\ -3 - 2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ -3 - 2\lambda \end{pmatrix} = \begin{pmatrix} \lambda + 5 \\ 4\lambda - 3 \\ -2\lambda \end{pmatrix} \]

\[ \overrightarrow{AP} \cdot \mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} \lambda + 5 \\ 4\lambda - 3 \\ -2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \lambda + 5 + 16\lambda - 12 + 4\lambda = 0 \]

leading to \( 21\lambda - 7 = 0 \Rightarrow \lambda = \frac{1}{3} \)

\[ \overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \frac{1}{3} \\ 14 \frac{1}{3} \\ -\frac{2}{3} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 28 \\ 43 \\ 11 \end{pmatrix} \]

Position vector \( \overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 28 \\ 43 \\ 11 \end{pmatrix} \)

ddM1 A1

\[ \text{[6]} \quad \text{[6]} \quad \text{[6]} \]

\[ \text{[14]} \quad \text{[14]} \quad \text{[14]} \]
7. (a) **M1:** Writes down any two equations. Allow one slip.

**dM1:** Attempts to eliminate either $\lambda$ or $\mu$ to form an equation in one parameter only.

**A1:** For either $\lambda = -3$ or $\mu = 2$. **Note:** candidates only need to find one of the parameters.

**ddM1:** For either substituting their value of $\lambda$ into $l_1$ or their $\mu$ into $l_2$.

2nd **A1:** For either $\begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$ or $6i + j + 3k$ or $(6 \ 1 \ 3)$.

**Note:** Each of the method marks in this part are dependent upon the previous method marks.

**M1:** Realisation that the dot product is required between $\pm A d_1$ and $\pm B d_2$. Allow one slip in $d_i = i + 4j - 2k$.

**A1:** Correct application of the dot product formula $\cos \theta = \frac{d_1 \cdot d_2}{|d_1||d_2|}$ or $\cos \theta = \frac{d_1 \cdot d_2}{|d_1||d_2|}$.

The dot product must be correctly applied and the square roots although they can be un-simplified must be correctly applied.

**A1:** awrt 69.1. This can be also be achieved by $180 - 110.876 = \text{awrt} 69.1$. $\theta = 1.2064...^\circ$ is $A0$.

**Common response:** $\cos \theta = \frac{-12 - 24 + 12}{\sqrt{(-3)^2 + (-12)^2 + (6)^2} \cdot \sqrt{(4)^2 + (2)^2 + (2)^2}} = \frac{-24}{189.24}$ is M1A1...

*Alternative Method: Vector Cross Product*

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$\mathbf{d}_1 \times \mathbf{d}_2 = \begin{vmatrix} i & j & k \\ 1 & 4 & -2 \\ -2 & 1 & 1 \end{vmatrix} = \begin{pmatrix} 6i - 5j - 7k \end{pmatrix}$

$\sin \theta = \frac{\sqrt{(6)^2 + (5)^2 + (-7)^2}}{\sqrt{(1)^2 + (4)^2 + (-2)^2} \cdot \sqrt{(2)^2 + (1)^2 + (1)^2}}$

$\sin \theta = \frac{\sqrt{110}}{\sqrt{21} \cdot \sqrt{6}} \Rightarrow \theta = 69.1238974... = 69.1 \ (1 \ dp)$

**A1:** awrt 69.1

(b) **M1:** Attempts to find $\mathbf{AP}$ in terms of the parameter by subtracting the components of $\overrightarrow{OP}$ from $l_1$ and $\overrightarrow{OA}$. Ignore the direction of subtraction and ignore any confusion between $\overrightarrow{OP}$ and $\overrightarrow{OP}$ or between $\overrightarrow{OA}$ and $\overrightarrow{OA}$. The correct subtraction of two components is enough to establish that subtraction is intended. The coordinates or position vector of $P$ must be given in terms of a parameter. Taking $P: (x, y, z)$ gains no marks although this can be recovered later. See **Additional Solutions**.

**A1:** (M1 on open) A correct expression for $\mathbf{AP}$. Again accept the reverse direction.

**dM1:** Depends on the previous M. Taking the scalar product of their expression for $\mathbf{AP}$ with $\mathbf{d}_1$ or a multiple of $\mathbf{d}_1$ and equating to 0 and obtaining an equation for $\lambda$. The equation must derive from an expression of the form $x_1x_2 + y_1y_2 + z_1z_2 = 0$. Differentiation can be used. See **Additional Solutions**.

**A1:** Solving to find $\lambda = \frac{1}{3}$.

**ddM1:** Depends on both previous Ms. Substitutes their value of the parameter into their expression for $\overrightarrow{OP}$. Substituting into $\mathbf{AP}$ is a common error which loses the mark.

**Note:** Needs 2 correct co-ordinates if $\lambda = \frac{1}{3}$ found and then $P$ stated without method to gain ddM1.
A1: $9 \frac{1}{3} \mathbf{i} + 14 \frac{1}{3} \mathbf{j} - 3 \frac{2}{3} \mathbf{k}$. Accept vector notation or coordinates. *Must be exact.*
7. (c) **Additional Solution 1:**

Taking \( \overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \), in itself, can gain no marks but this may be converted to a parameter at a later stage in the solution and, at that stage, any relevant marks can be awarded.

For example, \( \overrightarrow{AP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 4 \\ -16 \\ -3 \end{pmatrix} = \begin{pmatrix} x-4 \\ y-16 \\ z+3 \end{pmatrix} \)

leading to: \( \begin{pmatrix} x-4 \\ y-16 \\ z+3 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix} \)

\[ x - 4 + 4y - 64 - 2z - 6 = 0 \]

No marks gained at this stage.

Using, \( \overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 + \lambda \\ 13 + 4\lambda \\ -3 - 2\lambda \end{pmatrix} \) on \( x + 4y - 2z = 74 \)

which gives: \( 9 + \lambda + 4(13 + 4\lambda) - 2(-3 - 2\lambda) = 74 \)

\[ 21\lambda + 67 = 74 \Rightarrow \lambda = \frac{1}{3} \]

Position vector

\[ \overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \frac{1}{3} \\ 14 \frac{1}{3} \\ -3 \frac{2}{3} \end{pmatrix} \] or \[ \begin{pmatrix} 28 \\ 43 \\ 11 \end{pmatrix} \] ddM1 A1

**Additional Solution 2: Using Differentiation**

\[ \overrightarrow{AP} = \begin{pmatrix} 9 + \lambda \\ 13 + 4\lambda \\ -3 - 2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ -16 \\ -3 \end{pmatrix} = \begin{pmatrix} \lambda + 5 \\ 4\lambda - 3 \\ -2\lambda \end{pmatrix} \] \[ M1A1: \text{ As main scheme} \]

\[ AP^2 = (\lambda + 5)^2 + (4\lambda - 3)^2 + (-2\lambda)^2 = (21\lambda^2 - 14\lambda + 34) \]

\[ \frac{d}{d\lambda} (AP^2) = 42\lambda - 14 = 0 \] \[ M1 \]

leading to \( \lambda = \frac{1}{3} \) \[ A1: \text{ Solving to find } \lambda = \frac{1}{3}. \]

... then apply the main scheme.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. (a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| \[ \begin{align*} 
\frac{d\theta}{dt} &= \frac{(3-\theta)}{125} \\
\Rightarrow \int \frac{1}{3-\theta} \, d\theta &= \int \frac{1}{125} \, dt \\
\Rightarrow -\ln(\theta - 3) &= \frac{1}{125} \{+c\} \quad \text{or} \quad -\ln(3-\theta) = \frac{1}{125} \{+c\} 
\end{align*} \] | B1 | |
| & $\Rightarrow -\ln(\theta - 3) = \frac{1}{125} t + c$ \\
$\ln(\theta - 3) = -\frac{1}{125} t + c$ | M1 A1 | |
| $\theta - 3 = e^{-\frac{1}{125} t + c}$ or $e^{\frac{1}{125} t + c}$ | A1 | |
| $\theta = Ae^{-0.008t} + 3 \quad *$ | | |
| (b) $\begin{cases} t = 0, \theta = 16 \Rightarrow 16 = Ae^{-0.008(0)} + 3; \Rightarrow A = 13 \\
10 = 13e^{-0.008t} + 3 \\
e^{-0.008t} = \frac{7}{13} \Rightarrow -0.008t = \ln\left(\frac{7}{13}\right) \\
\left\{ \begin{array}{l}
\ln\left(\frac{7}{13}\right) \\
(-0.008)
\end{array} \right\} = 77.3799... = 77 \text{ (nearest minute)}
\end{cases}$ | See notes. | M1; A1 |

8. (a)

**B1: (M1 on open)** Separates variables as shown. $d\theta$ and $dt$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

**M1: Both** $\pm \lambda \ln(3-\theta)$ or $\pm \lambda \ln(\theta-3)$ and $\pm \mu t$ where $\lambda$ and $\mu$ are constants.

**A1:** For $-\ln(\theta - 3) = \frac{1}{125} t$ or $-\ln(3-\theta) = \frac{1}{125} t$ or $-125\ln(\theta - 3) = t$ or $-125\ln(3-\theta) = t$

\[ \begin{cases} 
\ln(-\frac{1}{125}) + c \text{ leading to } \theta - 3 = e^{\frac{1}{125} t + c} \quad \text{or} \quad \theta - 3 = e^{\frac{1}{125} t + c} + A, \text{ would be final} \\
\text{Note: } +c \text{ is not needed for this mark.}
\end{cases} \]

**A1:** Correct completion to $\theta = Ae^{-0.008t} + 3$. **Note:** $+c$ is needed for this mark.

\[ \begin{cases} 
\ln(\theta - 3) = -\frac{1}{125} t + c, \text{ leading to } \theta - 3 = e^{\frac{1}{125} t + c} \quad \text{or} \quad \theta - 3 = e^{\frac{1}{125} t + c} + A, \text{ would be final} \\
\text{Note: From } -\ln(\theta - 3) = \frac{1}{125} t + c, \text{ then } \ln(\theta - 3) = -\frac{1}{125} t + c \\
\Rightarrow \theta - 3 = e^{\frac{1}{125} t + c} \quad \text{or} \quad \theta - 3 = e^{\frac{1}{125} t + c} \Rightarrow \theta = Ae^{-0.008t} + 3 \text{ is required for A1.}
\end{cases} \]

**Note:** From $-\ln(3-\theta) = \frac{1}{125} t + c$, then $\ln(3-\theta) = -\frac{1}{125} t + c$

\[ \begin{cases} 
3 - \theta = e^{\frac{1}{125} t + c} \quad \text{or} \quad 3 - \theta = e^{\frac{1}{125} t + c} \Rightarrow \theta = Ae^{-0.008t} + 3 \text{ is sufficient for A1.}
\end{cases} \]

**Note:** The jump from $3 - \theta = Ae^{\frac{1}{125} t + c}$ to $\theta = Ae^{-0.008t} + 3$ is fine.
Note: \[ \ln(\theta - 3) = -\frac{1}{125} t + c \Rightarrow \theta - 3 = Ae^{-\frac{t}{125}}, \] where candidate writes \( A = e^c \) is also acceptable.
M1: (B1 on epen) Substitutes \( \theta = 16, t = 0 \), into either their equation containing an unknown constant or the printed

equation.  **Note:** You can imply this method mark.

A1: (M1 on epen)  \( A = 13 \).  **Note:**  \( \theta = 13e^{-0.008t} + 3 \) without any working implies the first two marks, M1A1.

M1: Substitutes \( \theta = 10 \) into an equation of the form \( \theta = Ae^{-0.008t} + 3 \), or equivalent.

where \( A \) is a positive or negative numerical value and \( A \) can be equal to 1 or -1.

M1: Uses correct algebra to rearrange their equation into the form \(-0.008t = \ln k\),

where \( k \) is a positive numerical value.

A1: awrt 77 or awrt 1 hour 17 minutes.

**Alternative Method 1 for part (b)**

\[
\int \frac{1}{3-\theta} \, d\theta = \int \frac{1}{125} \, dt \quad \Rightarrow \quad -\ln (\theta - 3) = \frac{1}{125}t + c
\]

\( \{t = 0, \theta = 16 \} \quad -\ln(16 - 3) = \frac{1}{125}(0) + c \)

\( \Rightarrow c = -\ln 13 \)

\(-\ln(\theta - 3) = \frac{1}{125}t - \ln 13 \) or \(-\ln(\theta - 3) = -\frac{1}{125}t + \ln 13 \)

\(-\ln(10 - 3) = \frac{1}{125}t - \ln 13 \)

\( \ln 13 - \ln 7 = \frac{1}{125}t \)

\( t = 77.3799... = 77 \) (nearest minute)

**Alternative Method 2 for part (b)**

\[
\int \frac{1}{3-\theta} \, d\theta = \int \frac{1}{125} \, dt \quad \Rightarrow \quad -\ln|3 - \theta| = \frac{1}{125}t + c
\]

\( \{t = 0, \theta = 16 \} \quad -\ln|3 - 16| = \frac{1}{125}(0) + c \)

\( \Rightarrow c = -\ln 13 \)

\(-\ln|3 - \theta| = \frac{1}{125}t - \ln 13 \) or \(-\ln|3 - \theta| = -\frac{1}{125}t + \ln 13 \)

\(-\ln(3 - 10) = \frac{1}{125}t - \ln 13 \)

\( \ln 13 - \ln 7 = \frac{1}{125}t \)

M1: Substitutes \( t = 0, \theta = 16 \),

into \(-\ln(\theta - 3) = \frac{1}{125}t + c \)

A1: \( c = -\ln 13 \)

M1: Substitutes \( \theta = 10 \) into an equation of the

form \( \pm \lambda \ln(\theta - 3) = \pm \frac{1}{125}t \pm \mu \)

where \( \lambda, \mu \) are numerical values.

M1: Uses correct algebra to rearrange their equation into the form \( \pm 0.008t = \ln C - \ln D \),

where \( C, D \) are positive numerical values.

A1: awrt 77.
where $C, D$ are positive numerical values.

A1: awrt 77.

8. (b) **Alternative Method 3 for part (b)**

\[ \int_{16}^{10} \frac{1}{3-\theta} \, d\theta = \int_{0}^{1} \frac{1}{125} \, dt \]

\[ = \left[ -\ln(3-\theta) \right]_{16}^{10} = \left[ \frac{1}{125} t \right]_{0}^{1} \]

\[-\ln 7 - -\ln 13 = \frac{1}{125} t \]

\[ t = 77.3799... \approx 77 \text{ (nearest minute)} \]

**M1A1:** $\ln 13$

**M1:** Substitutes limit of $\theta = 10$ correctly.

**M1:** Uses correct algebra to rearrange their own equation into the form

\[ \pm 0.008 t = \ln C - \ln D , \]

where $C, D$ are positive numerical values.

A1: awrt 77.

**Alternative Method 4 for part (b)**

\[ \{ \theta = 16 \Rightarrow \} \quad 16 = Ae^{-0.008t} + 3 \]

\[ \{ \theta = 10 \Rightarrow \} \quad 10 = Ae^{-0.008t} + 3 \]

\[-0.008t = \ln \left( \frac{13}{A} \right) \quad \text{or} \quad -0.008t = \ln \left( \frac{7}{A} \right) \]

\[ t_{(1)} = \frac{\ln \left( \frac{13}{A} \right)}{-0.008} \quad \text{and} \quad t_{(2)} = \frac{\ln \left( \frac{7}{A} \right)}{-0.008} \]

\[ t = t_{(1)} - t_{(2)} = \frac{\ln \left( \frac{13}{A} \right)}{-0.008} - \frac{\ln \left( \frac{7}{A} \right)}{-0.008} \]

\[ \left\{ \begin{array}{c}
\ln \left( \frac{7}{13} \right) \\
(-0.008)
\end{array} \right\} = 77.3799... \approx 77 \text{ (nearest minute)} \]

**M1:** Writes down a pair of equations in $A$ and $t$, for $\theta = 16$ and $\theta = 10$ with either $A$ unknown or $A$ being a positive or negative value.

**A1:** Two equations with an unknown $A$.

**M1:** Uses correct algebra to solve both of their equations leading to answers of the form

\[ -0.008t = \ln k , \]

where $k$ is a positive numerical value.

**M1:** Finds difference between the two times. (either way round).

**A1:** awrt 77. Correct solution only.
Mark Scheme (Results)

January 2013

GCE Decision Mathematics D1 6689/01
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

   In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\sqrt{\hphantom{0}}$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **oe** – or equivalent (and appropriate)
   - **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - *** The answer is printed on the paper
   - ** The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question/item is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
</tr>
<tr>
<td>aA</td>
<td></td>
</tr>
<tr>
<td>bM1</td>
<td></td>
</tr>
<tr>
<td>bA1</td>
<td></td>
</tr>
<tr>
<td>bB</td>
<td></td>
</tr>
<tr>
<td>bM2</td>
<td></td>
</tr>
<tr>
<td>bA2</td>
<td></td>
</tr>
</tbody>
</table>
# January 2013
## 6689 Decision Mathematics 1
### Mark Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>E</td>
</tr>
<tr>
<td>72</td>
<td>8</td>
<td>8.5</td>
</tr>
<tr>
<td>8.5</td>
<td>8.485 294 118</td>
<td>N</td>
</tr>
<tr>
<td>8.485 294 118</td>
<td>8.485 281 374</td>
<td>N</td>
</tr>
<tr>
<td>8.485 281 374</td>
<td>8.485 281 374</td>
<td>Y</td>
</tr>
</tbody>
</table>

Output is \( R = 8.485 \, 281 \, 4 \)

(b) We would get a negative output for \( R \). We would get the negative square root

(c) \( E \) cannot be zero

### Notes

- **a1M1**: At least two rows of cells in just \( E \) and \( R \) completed.
- **a1A1**: CAO first two rows correct giving exact values or awrt 7dp (the exact second value for \( R \) is \( \frac{577}{68} \)).
- **a2A1**: CAO third and fourth rows awrt 7dp
- **a3A1ft**: Output for \( R \) **must** follow through from their final value for \( R \) awrt 7dp – candidate **must** have answered ‘yes’ to score this mark. Output either on the answer line (or on the second page) or stated in the table but **must** be in the column for \( R \) below the row which contains ‘yes’.
- Condone \( N = 72 \) on each row and entries appearing on separate rows throughout for full marks. Allow e.g. ticks/crosses etc. for yes/no.
- **b1B1**: Mention of ‘negative’ scores B1 however do not accept incorrect statements but bod that ‘negative’ only is implicitly describing the effect on the output. Accept ‘other square root’.
- **c1B1**: CAO (nothing/null etc. scores B0). Condone \( E = 0 \).
2 (a)

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (a)</td>
<td>Pivot 1 $= \left\lceil \frac{14 + 26}{2} \right\rceil = 13.5 = 14$ letter N reject A – N</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Pivot 2 $= \left\lceil \frac{15 + 26}{2} \right\rceil = 20.5 = 21$ letter U reject U – Z</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Pivot 3 $= \left\lceil \frac{15 + 20}{2} \right\rceil = 17.5 = 18$ letter R reject R – T</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Pivot 4 $= \left\lceil \frac{15 + 17}{2} \right\rceil = 16$ letter P – located</td>
<td>A1 (4)</td>
</tr>
</tbody>
</table>

(b)

E.g. The maximum number of letters at the start of each iteration is 26, 13, 6, 3, 1

So a maximum of 5 iterations is necessary

Notes

a1M1: Choosing middle right pivot (choosing middle left is M0) + discarding/retaining half the list. M1 only for an ‘incorrect’ list - allow 1 error (e.g. two letters interchanged) or one omission or 1 extra letter.

a1A1: First pass correct i.e. N found as pivot for a correct list and either using O to Z in 2nd pass or discarding A to N (so therefore no ‘sticky’ pivots – sticky is when the letter being considered is retained in the next pass)

a2A1: Second and third passes correct i.e. U and R (no sticky pivots).

Special case: Allow recovery for this mark if a sticky pivot is used in first pass but sticky pivots are not used in the 2nd and 3rd passes. So after retaining N incorrectly the 2nd pass would give T and the 3rd pass would give Q leaving a list with N O P.

a3A1: CSO (correct solution only – all three previous marks must have been awarded to score this mark) search complete + ‘found’ (accept ‘found’, ‘located’, ‘stop’, etc. but not just the letter; must be convinced that P has been located).

If no alphabetical list seen then withhold the final A mark in part (a). If the alphabetical list is not given then bod that candidate is using the correct ordered list (which is implied by the correct passes). Listing the alphabet and then numbering the alphabet and referring to the corresponding numbers is fine for full marks. Candidates may renumber their list for each pass to calculate pivots. However, use of numbers and comparing to 16 without any reference to the alphabet is M0.

b1M1: Numerical argument; listing size of list, using logs, etc.

b1A1: Correct complete argument.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| **3 (a)** | (i) \(C - 4 = N - 6 = J - 3 = R - 2\) or (ii) \(O - 6 = J - 3 = R - 2\)  
Change status to give  
(i) \(C = 4 - N = 6 - J = 3 - R = 2\) or (ii) \(O = 6 - J = 3 - R = 2\)  
Improved matching is:  
| | | | |
| | | C | G | J | N | O | R |
| (i) | 4 | 5 | 3 | 6 | 2 |
| (ii) | 5 | 3 | 4 | 6 | 2 | A1 | (3) |
| E.g. Tasks 1 and 5 can only be done by George  
E.g. Charlie can only do task 4 and Olivia can only do task 6 which means that Nurry can’t be allocated a task as Nurry can only do tasks 4 and 6 | | | B2, 1, 0 | (2) |
| **3 (b)** | \(O - 6 = N - 4 = C - 5 = G - 1\) or \(C - 5 = G - 1\)  
Change status to give \(O = 6 - N = 4 - C = 5 - G = 1\) or \(C = 5 - G = 1\)  
Maximum matching is: \(C = 5, G = 1, J = 3, N = 4, O = 6, R = 2\) | | | | A1 | (3) |
| **Total 8 marks** | | | | | | | |

**Notes**

a1M1: An alternating path (e.g. letter - number - letter - …) from C or O to 2 or vice versa.

a1A1: CAO – a correct path including change status either stated (only accept ‘change (of) status’ or ‘c.s.’) or shown (all symbols e.g. (... = ... = ...) interchanged (... = ... = ...)). Chosen path clear.

a2A1: CAO must follow from the correct stated path. Accept on a clear diagram (with five arcs only).

b1B1: Correct idea, may be imprecise or muddled (bod gets B1) all relevant nodes must be referred to and must be correct.

b2B1: Good, clear, complete, correct answer (this needs to be checked carefully e.g. G can only do tasks 1 and 5 is B1 only).

c1M1: A second alternating path from O or C to 1 (whichever letter (of O or C) that they didn’t use before) or vice versa.

C1A1: CAO including change status (stated or shown), chosen path clear.

C2A1: CAO must follow from two correct stated paths (so both previous M marks must have been awarded). Accept on a clear diagram (with six arcs only).
A path is a (i) finite sequence of edges, such that (ii) the end vertex of one edge in the sequence is the start vertex of the next, and in which (iii) no vertex appears more than once.

Shortest path: SBADET
Length: 40 (miles)

Shortest distance S to F = 29 (miles)

SADET or SCDET; of length 41 (miles)

Notes
a1B1: One of the three points made clearly or two suggested. Arcs (edges)/vertices (nodes) must be referred to correctly. Do not condone incorrect technical language e.g. point for vertex.

a2B1: All three points made clearly.

b1M1: A larger value replaced by a smaller value at least once at A or D or E or F or T.

b1A1: All values in S, A, B and C correct. The working values at A must be in the correct order. Condone lack of 0 in S’s working value.

b2A1ft: All values in D and F ft correctly and working values in the correct order. F must be labelled before E but penalise order of labelling only once per question.

b3A1: All values in E and T correct and working values in the correct order. Penalise order of labelling only once per question.

b1B1: Route CAO

b2B1ft: Their final value ft (if answer is not 40 ft their final value at T)

c1B1ft: Their final value ft (if answer is not 29 ft their final value at F)

B1 B1  (2)

Total 11 marks
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 (a)</td>
<td>AC (32) CF (14) DF (12) EF (17); BE (15) FI(18); IJ (10) GJ (9) DH (19)</td>
<td>M1 A1; A1 (3)</td>
</tr>
<tr>
<td>(b)</td>
<td>$146 \times 80 = (£) 11,680$</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(c)</td>
<td>$BF + GH = 32 + 40 = 72$&lt;br&gt;$BG + FH = 39 + 25 = 64^*$&lt;br&gt;$BH + FG = 57 + 37 = 94$&lt;br&gt;Roads BE, EG and FH need repeating</td>
<td>M1 A3,2,1,0</td>
</tr>
<tr>
<td></td>
<td>$379 + 64 = 443$ (km)</td>
<td>B1ft (1)</td>
</tr>
<tr>
<td>(e)</td>
<td>Ben should choose to repeat FH (25) since this is the shortest.&lt;br&gt;He should choose B and G as his start and finish vertices&lt;br&gt;Route length is $379 + 25 = 404$ (km)</td>
<td>M1 A1 A1 (3)</td>
</tr>
</tbody>
</table>

**Notes**
Accept the weight of each arc to represent the arcs (as each value is unique).

a1M1: First four arcs correctly chosen or first five nodes correctly chosen (A, C, F, D, E, …). Any rejections seen during selection scores M0. Order of nodes may be seen at the top of a matrix.<br>a1A1: First six arcs correctly chosen or all nodes correctly chosen (A, C, F, D, E, B, I, J, G, H). Order of nodes may be seen at the top of a matrix.<br>a2A1: CSO (must be considering arcs for this final mark).<br>b1M1: $80 \times$ their MST weight. Accept a value in the interval $[114,178] \times 80$ for this mark. If no working is seen then M0 unless answer is correct.<br>b1A1: CAO (11680 with no working scores both marks).<br>c1M1: Three distinct pairings of their four odd nodes.<br>c1A1: Any one row correct including pairing and total.<br>c2A1: Any two rows correct including pairing and total.<br>c3A1: All three rows correct including pairing and total.<br>c4A1ft: Their smallest arcs repeated (e.g. accept BEG or BG via E but not just BG). BEG (or e.g. BG via E) could appear in their working.<br>c5A1: CAO BE, EG and FH. Accept BEG or BG via E (could appear in working) but not just BG.<br>d1B1ft: correct answer of 443 or 379 + their least out of a choice of at least two totals given in part (c).<br>e1M1: FH (or 25) specifically identified as least.<br>e1A1: B and G identified as the start and finish nodes.<br>e2A1: 404 CAO (condone lack of (or incorrect) units throughout).
Misread in (a): Starting at a node other than A scores **M1 only – must** have the first four arcs (or five nodes or numbers) correct.

<table>
<thead>
<tr>
<th>Starting at</th>
<th>Minimum Arcs required for M1 only</th>
<th>Nodes</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>BE,EF,DF,CF</td>
<td>B, E, F, D, C</td>
<td>(10)15423(8967)</td>
</tr>
<tr>
<td>C</td>
<td>CF,DF,EF,BE</td>
<td>C, F, D, E, B</td>
<td>(10)51342(8967)</td>
</tr>
<tr>
<td>D</td>
<td>DF,CF,EF,BE</td>
<td>D, F, C, E, B</td>
<td>(10)53142(8967)</td>
</tr>
<tr>
<td>E</td>
<td>BE,EF,DF,CF</td>
<td>E, B, F, D, C</td>
<td>(10)25413(8967)</td>
</tr>
<tr>
<td>F</td>
<td>DF,CF,EF,BE</td>
<td>F, D, C, E, B</td>
<td>(10)53241(8967)</td>
</tr>
<tr>
<td>G</td>
<td>GJ,IJ,FI,DF</td>
<td>G, J, I, F, D</td>
<td>(10)(86)5(7)41(9)32</td>
</tr>
<tr>
<td>H</td>
<td>DH,DF,CF,EF</td>
<td>H, D, F, C, E</td>
<td>(10)(6)4253(9)1(78)</td>
</tr>
<tr>
<td>I</td>
<td>IJ,GJ,FI,DF</td>
<td>I, J, G, F, D</td>
<td>(10)(86)5(7)43(9)12</td>
</tr>
<tr>
<td>J</td>
<td>GJ,IJ,FI,DF</td>
<td>J, G, I, F, D</td>
<td>(10)(86)5(7)42(9)31</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[2x + y = 70\]

\[5y = x\]

\[4x + 3y = 200\]
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td>5y ≥ x</td>
<td>B1 B1 (2)</td>
</tr>
<tr>
<td>(b)</td>
<td>2x + y ≥ 70 and 4x + 5y ≥ 200</td>
<td>B3,2,1 (3)</td>
</tr>
<tr>
<td>(c)</td>
<td>Two lines correctly added</td>
<td>B1 B1 (2)</td>
</tr>
<tr>
<td>(d)</td>
<td>R correctly labelled</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(e)</td>
<td>(T =) 10x + 4y</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(f)</td>
<td>Vertex</td>
<td>Time (mins)</td>
</tr>
<tr>
<td></td>
<td>(20,30)</td>
<td>320</td>
</tr>
<tr>
<td></td>
<td>(25, 20)</td>
<td>330</td>
</tr>
<tr>
<td></td>
<td>(40, 8)</td>
<td>432</td>
</tr>
<tr>
<td></td>
<td>(60,12)</td>
<td>648</td>
</tr>
<tr>
<td></td>
<td>(60,30)</td>
<td>720</td>
</tr>
</tbody>
</table>

So produce 20 celebration arrangements, 30 party arrangements taking 320 minutes (minutes)

**Notes**

a1B1: Ratio of coefficients correct (i.e. equation of line correct)

a2B1: Inequality correct way round (ay ≥ bx o.e.) do not accept a strict inequality

b1B1: One equation correct

b2B1: One constraint correct, including inequality (but accept strict inequality here)

b3B1: Both constraints correct, including correct inequalities

c1B1: One line drawn correctly. Must pass within one small square of (25, 20) and if line extended must go from axis to axis through the points of intersection with the axes within one small square. Line must be long enough to form the feasible region. Check using length measurement tool if required. Ignore shading.

c2B1: Both lines drawn correctly. See above for accuracy. Ignore shading.

d1B1: R labelled (not just implied by shading) – must have scored both marks in (c).

e1B1: CAO (isw if (T =)10x + 4y ‘simplified’ to k(10x + 4y) but if (T =)10x + 4y not stated then B0)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1M1: At least three of their (or the correct) R vertices found (by either reading off their graph or using simultaneous equations) and tested using their T (or the correct T). Objective line method (only) is M0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f1A1: Three vertices found and tested correctly CAO (must be using three of the correct vertices (see table above) and the values for T must be correct).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f2A1: All five vertices found and tested correctly CAO (all values of T must be correct).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f3A1: CAO number of each and time, both correct and it must be clear that $x = 20$ and $y = 30$ (accept as coordinates). If values appear in e.g. a table it must be clear that (20, 30) and 320 has been selected (condone lack of/incorrect units on the time).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Activity K depends on activities E, F and B, but activity I depends on F and B only.

Critical activities are: A, F, I, L

Total float on G = 15 – 6 – 6 = 3
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f)</td>
<td>Activities A, C and D must be happening at time 5.5</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(g)</td>
<td>E.g. Activities F, B, C and G together with 9 &lt; time &lt; 10 stated. So 4 workers are needed</td>
<td>M1 A1 (2)</td>
</tr>
</tbody>
</table>

**Notes**

- a1B1: K, I, E and at least one of B or F referred to. Correct statement but may be incomplete give bod here.
- a2B1: Clear correct statement no bod (at least one of only B or F referred to can score this mark).
- b1M1: All top boxes complete, values generally increasing left to right, condone one 'rogue' (if values do not increase from left to right then if one value is ignored and then the values do increase from left to right then this is considered to be only one rogue value).
- b2A1: CAO
- b2M1: All bottom boxes complete, values generally decreasing right to left, condone one rogue. Condone missing 0 or 21 for the M only.
- b2A1: CAO
- c1B1: CAO
- d1M1: Correct calculation seen, all three numbers correct (ft), float ≥ 0
- d1A1: CAO (no ft on this mark)
- e1M1: At least 9 activities including at least 5 floats. Scheduling diagram scores M0.
Mark Scheme (Results)

January 2013

GCE Further Pure Mathematics FP1 (6667/01)
Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

   In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
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   - ¿ The answer is printed on the paper
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<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>aA</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bM1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA1</td>
<td></td>
<td>●</td>
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<tr>
<td>bB</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bM2</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA2</td>
<td></td>
<td>●</td>
</tr>
</tbody>
</table>
### Jan 2013
Further Pure Mathematics FP1 6667
Mark Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1.              | \[ \sum_{r=1}^{n} 3(4r^2 - 4r + 1) = 12 \sum_{r=1}^{n} r^2 - 12 \sum_{r=1}^{n} r + \sum_{r=1}^{n} 3 \]  
\[ = \frac{12}{6} n(n+1)(2n+1) - \frac{12}{2} n(n+1), \quad +3n \]  
\[ = n\left[2(n+1)(2n+1) - 6(n+1) + 3\right] \]  
\[ = n\left[4n^2 - 1\right] = n(2n+1)(2n-1) \] | M1  
A1, B1  
M1  
A1 cso |

**Notes:**  
Induction is not acceptable here  
First M for expanding given expression to give a 3 term quadratic and attempt to substitute.  
First A for first two terms correct or equivalent.  
B for +3n appearing  
Second M for factorising by n  
Final A for completely correct solution
### Question 2

<table>
<thead>
<tr>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( \frac{50}{3+4i} = \frac{50(3-4i)}{(3+4i)(3-4i)} = \frac{50(3-4i)}{25} = 6-8i )</td>
</tr>
<tr>
<td>(b) ( z^2 = (6-8i)^2 = 36-64-96i = -28-96i )</td>
</tr>
<tr>
<td>(c) (</td>
</tr>
<tr>
<td>(d) ( \tan \alpha = \frac{-96}{-28} )</td>
</tr>
</tbody>
</table>

**Alternatives**

<table>
<thead>
<tr>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c) (</td>
</tr>
<tr>
<td>(d) ( \arg(3+4i) = 53.13\ldots ) so ( \arg\left(\frac{50}{3+4i}\right)^2 = -2 \times 53.13\ldots = -106.3 )</td>
</tr>
</tbody>
</table>

**Notes:**

(a) M for \( \times \frac{3-4i}{3-4i} \) (accept use of \(-3+4i\)) and attempt to expand using \(i^2=-1\), A for 6-8i only

(b) M for attempting to expand their \( z^2 \) using \(i^2=-1\), A for -28-96i only. If using original z then must attempt to multiply top and bottom by conjugate and use \(i^2=-1\).

(c) M for \( \sqrt{a^2+b^2} \), A for ‘their 10’

(d) M for use of tan or tan\(^{-1}\) and values from their \( z^2 \) either way up ignoring signs. Radians score A0.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>(a) $f'(x) = x^{-\frac{1}{3}} - \frac{1}{2}x^{-\frac{3}{2}}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(b) $f(5) = -0.0807$ $f'(5) = 0.4025$</td>
<td>B1 M1 M1 A1</td>
</tr>
<tr>
<td></td>
<td>$x_i = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{-0.0807}{0.4025}$ $= 5.2(0)$</td>
<td>(4)</td>
</tr>
</tbody>
</table>

**Notes**

The B and M marks are implied by a correct answer only with no working or by $\frac{3}{5}(10\sqrt{5} - 13)$

(a) M for at least one of $\pm ax^{\frac{1}{2}}$ or $\pm bx^{\frac{3}{2}}$, A for correct (equivalent) answer only

(b) B for awrt -0.0807, first M for attempting their $f'(5)$, M for correct formula and attempt to substitute, A for awrt 5.20, but accept 5.2
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| (a) \[
\begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}
\] | B1     | (1)   |
| (b) \[
\begin{pmatrix}
0 & -1 \\
-1 & 0
\end{pmatrix}
\] | B1     | (1)   |
| (c) \( R = QP \) | B1     | (1)   |
| (d) \[\begin{pmatrix}
0 & -1 \\
-1 & 0
\end{pmatrix} \begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix} = \begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}\] | M1 A1 cao | (2) |
| (e) Reflection in the \( y \) axis | B1     | (2)   |

**Notes**

(a) and (b) Signs must be clear for B marks.

(c) Accept \( QP \) or their 2x2 matrices in the correct order only for B1.

(d) M for their \( QP \) where answer involves \( \pm 1 \) and 0 in a 2x2 matrix, A for correct answer only.

(e) First B for Reflection, Second B for ‘y axis’ or ‘\( x=0 \)’. Must be single transformation. Ignore any superfluous information.
5. (a) \(4x^2 + 9 = 0 \Rightarrow x = ki, \quad x = \pm \frac{3}{2}i\) or equivalent

Solving 3-term quadratic by formula or completion of the square

\[ x = \frac{6 \pm \sqrt{36 - 136}}{2} \text{ or } (x - 3)^2 - 9 + 34 = 0 \]

\[ = 3 + 5i \text{ and } 3 - 5i \]

(b) Two roots on imaginary axis

Two roots – one the conjugate of the other

Accept points or vectors

Notes

(a) Final A follow through conjugate of their first root.
(b) First B award only for first pair imaginary,
Second B award only if second pair complex.
Complex numbers labelled, scales or coordinates or vectors required for B marks.
### Question 6.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Determinant: $2 - 3a = 0$ and solve for $a = \frac{2}{3}$ or equivalent</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(b) Determinant: $(1\times2) - (3\times1) = 5 \quad (\Delta)$</td>
<td>M1A1 (2)</td>
</tr>
<tr>
<td>$Y^{-1} = \frac{1}{5} \begin{pmatrix} 2 &amp; 1 \ -3 &amp; 1 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$= \begin{pmatrix} 0.4 &amp; 0.2 \ -0.6 &amp; 0.2 \end{pmatrix} = \begin{pmatrix} 0.4 &amp; 0.2 \ -0.6 &amp; 0.2 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\lambda - \lambda - \lambda - \lambda = \frac{1}{5} \begin{pmatrix} 2 - 2\lambda + 7\lambda - 2 \ 3 + 3\lambda + 7\lambda - 2 \end{pmatrix} = \begin{pmatrix} \lambda \ 2\lambda - 1 \end{pmatrix}$</td>
<td>M1depM1A1 A1 (4) [8]</td>
</tr>
<tr>
<td>(c) Alternative method for (c)</td>
<td></td>
</tr>
<tr>
<td>$\begin{pmatrix} 1 &amp; -1 \ 3 &amp; 2 \end{pmatrix} \begin{pmatrix} x \ y \end{pmatrix} = \begin{pmatrix} 1 - \lambda \ 7\lambda - 2 \end{pmatrix}$ so $x - y = 1 - \lambda$ and $3x + 2y = 7\lambda - 2$</td>
<td>M1M1</td>
</tr>
<tr>
<td>Solve to give $x = \lambda$ and $y = 2\lambda - 1$</td>
<td>A1A1</td>
</tr>
<tr>
<td>Notes</td>
<td></td>
</tr>
<tr>
<td>(b) M for $\frac{1}{\text{their det}} \begin{pmatrix} 2 &amp; 1 \ -3 &amp; 1 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>(c) First M for their $Y^{-1}B$ in correct order with $B$ written as a 2x1 matrix, second M dependent on first for attempt at multiplying their matrices resulting in a 2x1 matrix, first A for $\lambda$, second A for $2\lambda - 1$</td>
<td></td>
</tr>
<tr>
<td>Alternative for (c)</td>
<td></td>
</tr>
<tr>
<td>First M to obtain two linear equations in $x, y, \lambda$</td>
<td></td>
</tr>
<tr>
<td>Second M for attempting to solve for $x$ or $y$ in terms of $\lambda$</td>
<td></td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
</tr>
<tr>
<td>7.</td>
<td></td>
</tr>
<tr>
<td>(a) $y = \frac{25}{x}$ so $\frac{dy}{dx} = -25x^{-2}$</td>
<td>$M1$</td>
</tr>
<tr>
<td>$\frac{dy}{dx} = -25 = -\frac{1}{(5p)^2} = -\frac{1}{p^2}$</td>
<td>$A1$</td>
</tr>
<tr>
<td>$y - \frac{5}{p} = -\frac{1}{p^2}(x - 5p) \Rightarrow p^2y + x = 10p \quad (*)$</td>
<td>$M1 \quad A1$</td>
</tr>
<tr>
<td>(b) $q^2y + x = 10q$ only</td>
<td>$B1$</td>
</tr>
<tr>
<td>(c) $(p^2 - q^2)y = 10(p - q)$ so $y = \frac{10(p - q)}{(p^2 - q^2)} = \frac{10}{p + q}$</td>
<td>$M1 \quad A1 \quad cso$</td>
</tr>
<tr>
<td>$x = 10p - p^2 \frac{10}{p + q} = \frac{10pq}{p + q}$</td>
<td>$M1 \quad A1 \quad cso$</td>
</tr>
<tr>
<td>(d) Line $PQ$ has gradient $\frac{5}{p - 5q} = -\frac{1}{pq}$</td>
<td>$M1 \quad A1$</td>
</tr>
<tr>
<td>$ON$ has gradient $\frac{p + q}{10pq} = \frac{1}{pq}$ or $\frac{-1}{pq} = \frac{pq}{pq}$ could be as unsimplified equivalents seen anywhere</td>
<td>$B1$</td>
</tr>
<tr>
<td>As these lines are perpendicular $\frac{1}{pq}x - \frac{1}{pq} = -1$ so $p^2q^2 = 1$</td>
<td></td>
</tr>
<tr>
<td><strong>OR for $ON$</strong></td>
<td></td>
</tr>
<tr>
<td>$y - y_i = m(x - x_i)$ with gradient (equivalent to) $pq$ and sub in points $O$</td>
<td></td>
</tr>
<tr>
<td>AND $N$ to give $p^2q^2 = 1$</td>
<td></td>
</tr>
<tr>
<td><strong>OR for $PQ$</strong></td>
<td></td>
</tr>
<tr>
<td>$y - y_i = m(x - x_i)$ with gradient (equivalent to) $-pq$ and sub in points $P$</td>
<td></td>
</tr>
<tr>
<td>AND $Q$ to give $p^2q^2 = 1$. NB $-pq$ used as gradient of $PQ$ implies first $M1A1$</td>
<td></td>
</tr>
</tbody>
</table>
### Alternatives for first M1 A1 in part (a)

\[
x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}
\]

So at \( P \) gradient = \(-\frac{5}{p} \) = \(-\frac{1}{p^2}\)  

Or \( x = 5t, \ y = \frac{5}{t} \Rightarrow \frac{dx}{dt} = 5, \ \frac{dy}{dt} = -\frac{5}{t^2} \) so \( \frac{dy}{dx} = \)

\[-\frac{\frac{5}{t}}{5} = -\frac{1}{t^2}\]  so at \( P \) gradient = \(-\frac{1}{p^2}\)

### Notes

(a) First M for attempt at explicit, implicit or parametric differentiation not using \( p \) or \( q \) as an initial parameter, first A for \(-\frac{1}{p^2}\) or equivalent. Quoting gradient award first M0A0. Second M for using \( y - y_1 = m(x - x_1) \) and attempt to substitute or \( y = mx + c \) and attempt to find \( c \); gradient in terms of \( p \) only and using \( \left(5p, \frac{5}{p}\right) \), second A for correct solution only.

(c) First M for eliminating \( x \) and reaching \( y = f(p, q) \), second M for eliminating \( y \) and reaching \( x = f(p, q) \), both As for given answers. Minimum amount of working given in the main scheme above for 4/4, but do not award accuracy if any errors are made.

(d) First M for use of \( \frac{y_2 - y_1}{x_2 - x_1} \) and substituting, first A for \(-\frac{1}{pq}\) or unsimplified equivalent .

Second M for their product of gradients = \(-1\) (or equating equivalent gradients of \( ON \) or equating equivalent gradients of \( PQ \)), second A for correct answer only.
(a) If \( n = 1 \), \( \sum_{r=1}^{n} r(r + 3) = 1 \times 4 = 4 \) and \( \frac{1}{3} n(n+1)(n+5) = \frac{1}{3} \times 1 \times 2 \times 6 = 4 \), so true for \( n = 1 \). Assume true for \( n = k \)

So \( \sum_{r=1}^{k+1} r(r + 3) = \frac{1}{3} k(k + 1)(k + 5) + (k + 1)(k + 4) \)

\[ = \frac{1}{3} (k + 1) [k(k + 5) + 3(k + 4)] = \frac{1}{3} (k + 1) [k^2 + 8k + 12] \]

\[ = \frac{1}{3} (k + 1)(k + 2)(k + 6) \] which implies is true for \( n = k + 1 \)

As result is true for \( n = 1 \) this implies true for all positive integers and so result is true by induction

(b) \( u_1 = 1^2(1 - 1) + 1 = 1 \)

(so true for \( n = 1 \). Assume true for \( n = k \))

\( u_{k+1} = k^2(k - 1) + 1 + k(3k + 1) \)

\[ = k(k^2 - k + 3k + 1) + 1 = k(k + 1)^2 + 1 \] which implies is true for \( n = k + 1 \)

As result is true for \( n = 1 \) this implies true for all positive integers and so result is true by induction

**Notes**

(a) First B for LHS=4 and RHS =4

First M for attempt to use \( \sum_{r=1}^{k} r(r + 3) + u_{k+1} \)

First A for \( \frac{1}{3} (k + 1) \), \( \frac{1}{3} (k + 2) \) or \( \frac{1}{3} (k + 6) \) as a factor before the final line

Second A dependent on first for \( \frac{1}{3} (k + 1)(k + 2)(k + 6) \) with no errors seen

Second M dependent on first M and for any 3 of ‘true for \( n=1 \’) ‘assume true for \( n=k \’) ‘true for \( n=k+1 \’\), ‘true for all \( n \’) (or ‘true for all positive integers’) seen anywhere

Third A for correct solution only with all statements and no errors
(b) First B for both some working and 1.

First M for \( u_{k+1} = u_k + k(3k + 1) \) and attempt to substitute for \( u_k \)

First A for \( k(k + 1)^2 + 1 \) with some correct intermediate working and no errors seen

Second M dependent on first M and for any 3 of ‘true for \( n=1 \)’ ‘assume true for \( n=k \)’ ‘true for \( n=k+1 \)’, ‘true for all \( n \)’ (or ‘true for all positive integers’) seen anywhere

Second A for correct solution only with all statements and no errors
### Question 9.

(a) \( y = 6x^2 \) so \( \frac{dy}{dx} = 12x \)

Gradient when \( x = 4 \) is \( \frac{1}{2} \) and gradient of normal is \( -\frac{3}{2} \)

So equation of normal is \( (y - 12) = -\frac{3}{2}(x - 4) \) (or \( 3y + 2x = 44 \)) (M1 A1)

(b) \( S \) is at point \( (9,0) \)
\( N \) is at \( (22,0) \), found by substituting \( y = 0 \) into their part (a)
Both B marks can be implied or on diagram.
So area is \( \frac{1}{2} \times 12 \times (22 - 9) = 78 \) (M1 A1 cao)

Alternatives:
First M1 for \( ky \frac{dy}{dx} = 36 \) or for
\[ x = 9t^2, y = 18t \rightarrow \frac{dx}{dt} = 18t, \frac{dy}{dt} = 18 \rightarrow \frac{dy}{dx} = \frac{1}{t} \]

### Notes

(a) First M for \( \frac{dy}{dx} = ax^{\frac{1}{2}} \),

Second M for substituting \( x = 4 \) (or \( y = 12 \) or \( t = 2/3 \) if alternative used) into their gradient and applying negative reciprocal.
First A for \( -\frac{2}{3} \)

Third M for \( y - y_i = m(x - x_i) \) or \( y = mx + c \) and attempt to substitute a changed gradient AND (4,12)
Second A for \( 3y + 2x = 44 \) or any equivalent equation

(b) M for Area = \( \frac{1}{2} \) base x height and attempt to substitute including their numerical ‘\( (22-9) \)’ or equivalent complete method to find area of triangle \( PSN \).
Mark Scheme (Results)

January 2013

GCE Mechanics M1 (6677/01)
Edexcel and BTEC Qualifications

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January 2013
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

   In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>aA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bM1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bM2</td>
<td></td>
<td>●</td>
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<tr>
<td>bA2</td>
<td></td>
<td>●</td>
</tr>
</tbody>
</table>
# Jan 2013
## 6677 Mechanics M1
### Mark Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a)</td>
<td>$4m.2u - m.5u = -4m.\frac{1}{2}u + mv$  $3mu = -2mu + mv$ $v = 5u$, opposite direction</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$I = 4m(\frac{1}{2}u - 2u)$ OR $I = m(5u - 5u)$  $= 10mu$  $= 10mu$</td>
<td>A1, A1 cso (4)</td>
</tr>
<tr>
<td>2. (a)</td>
<td>$M(D)$,  $8R = (80g \times 6) + (200g \times 4)$  $R = 160g, 1600, 1570$</td>
<td>M1 A1 A1 (3)</td>
</tr>
<tr>
<td></td>
<td>($\uparrow$),  $2S = 80g + 200g$  $S = 140g, 1400, 1370$</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td></td>
<td>$M(B)$,  $Sx + (S \times 10) = (80g \times 8) + (200g \times 6)$  $140x + 1400 = 640 + 1200$  $140x = 440$  $x = \frac{440}{140} = \frac{2}{7}$</td>
<td>M1 A2</td>
</tr>
<tr>
<td></td>
<td>($\uparrow$),  $T \cos 30 + F \cos 60 = 2g$  ($\rightarrow$),  $T \cos 60 - F \cos 30 = 0$  $F = g = 9.8$  $T = \sqrt{3}g = 17$ or 17.0</td>
<td>M1 A1 M1 A1</td>
</tr>
<tr>
<td></td>
<td>OR:  $F = 2g \cos 60$  $T = 2g \cos 30$  $F = g = 9.8$  $T = \sqrt{3}g = 17$ or 17.0</td>
<td>M1 A1 M1 A1</td>
</tr>
</tbody>
</table>
4. 
\[
12.6^2 = 2a.50 \quad (\Rightarrow a = 1.5876)
\]
\[
800gsin15 - F = 800a
\]
\[
R = 800gcos15
\]
\[
F = \mu R
\]
\[
800gsin15 - \mu 800gcos15 = 800 \times 1.5876
\]
\[
\mu = 0.1, 0.10, 0.100
\]

5. (a) 
\[
30^2 = 2a.300
\]
\[
a = 1.5
\]

(b) 
\[
0^2 = 30^2 - 2 \times 1.25s \quad OR \quad 0 = 30 - 1.25t_2
\]
\[
s = 360\quad t_2 = 24
\]
\[
300 + 30T + 360 = 1500 \quad \frac{(20 + T + 24 + T)}{2} \times 30 = 1500
\]
\[
T = 28
\]

(c) 
triangle, drawn on the diagram, with base coinciding with base of trapezium, top vertex above line \( v = 30 \) and meeting trapezium at least once

\( V \) marked correctly

(d) 
\[
30 = 1.5t_1 \Rightarrow t_1 = 20
\]
\[
30 = 1.25t_2 \Rightarrow t_2 = 24
\]
\[
\frac{1}{2}(20 + 28 + 24)V = 1500
\]
\[
V = \frac{750}{18} = 41.67
\]
\[
= \frac{125}{3} \quad (oe) \text{ or } 42 \text{ (or better)}
\]
6.(a) \[
\frac{(i - 4j) - (4i - 8j)}{0.5}; (\pm 6i \pm 8j)
\]
\[
\sqrt{(\pm 6)^2 + (\pm 8)^2} = 10
\]
\[
\mathbf{r} = (4i - 8j) + t(-6i + 8j)
\]
\[
= (4i - 8j) - 6t\mathbf{i} + 8t\mathbf{j}
\]
\[
= (4 - 6t)i + (8t - 8)j
\]

(b) \[
\mathbf{r} = (4i - 8j) + t(-6i + 8j)
\]
\[
= (4i - 8j) - 6t\mathbf{i} + 8t\mathbf{j}
\]

At 10 am, \( \mathbf{r} = -2\mathbf{i} \)

At 10.30 am, \( \mathbf{r} = -5\mathbf{i} + 4\mathbf{j} \)

\( l = ki, \ k < -2 \)

\( k = -5 - 4 = -9 \)

\( l = -9\mathbf{i} \)

(c) Inextensible string

\[
4mg - T = 4ma
\]
\[
T - 2mg \sin \alpha - F = 2ma
\]

\( F = 0.25R \)

\( R = 2mg \cos \alpha \)

\( \cos \alpha = 0.8 \) or \( \sin \alpha = 0.6 \)

Eliminating \( R, F \) and \( T \)

\( a = 0.4g = 3.92 \)

(d) \[
\mathbf{v}^2 = 2 \times 0.4gh
\]

\( -2mg \sin \alpha - F = 2ma' \)

\( a' = -0.8g \)

\( 0^2 = 0.8gh - 2 \times 0.8g \times s \)

\( s = 0.5h \)

\( XY = 0.5h + h = 1.5h \)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

   In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \sqrt{ } \) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
<td>⬤</td>
</tr>
<tr>
<td>aA</td>
<td></td>
<td>⬤</td>
</tr>
<tr>
<td>bM1</td>
<td></td>
<td>⬤</td>
</tr>
<tr>
<td>bA1</td>
<td></td>
<td>⬤</td>
</tr>
<tr>
<td>bB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bM2</td>
<td>⬤</td>
<td></td>
</tr>
<tr>
<td>bA2</td>
<td>⬤</td>
<td></td>
</tr>
<tr>
<td>Q.</td>
<td>Scheme</td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>1. (a)</td>
<td><img src="" alt="Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>

Moments equation with lengths $\frac{1}{4}$, 1 and (ratio of) masses 2, 3. Allow moments about a parallel axis. Use of length for mass is M0.

For distance from BC

(b)  

$\tan \theta = \frac{0.6}{0.5 - 0.1}$

$\theta = \tan^{-1} \left( \frac{6}{4} \right) = 56.3^\circ = 56^\circ$
<table>
<thead>
<tr>
<th>Q.</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (a)</td>
<td><img src="image" alt="Diagram" /></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$R \leftarrow 1800 \text{ kg} \rightarrow T$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$T = \frac{30000}{20} \quad (= 1500)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$T - R = 1800a$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$T - R = 1800 \times 0.4$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$R = 1500 - 1800 \times 0.4$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= 780$</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td><img src="image" alt="Diagram" /></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\begin{align*} T - 1800g \sin \alpha - R &amp;= 0 \ T &amp;= 1800 \times \frac{1}{12}g + 780 \ \text{Power} &amp;= \left(1800 \times \frac{1}{12}g + 780\right) \times 20 \end{align*}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= 45000 \text{ W or } 45 \text{ kW}$</td>
<td>A1</td>
</tr>
</tbody>
</table>

- Use of $P = Fv$
- Equation of motion. Need all 3 terms.
- Condone sign errors
- Equation correct (their $T$)
- Only
- Equation of motion. Need all 3 terms. Weight must be resolved.
- Condone cos for sin.
- Condone sign errors
- Correct equation. Allow with $R$ not substituted or with their $R$. 
- Use of $P = Tv$
- Correctly substituted equation (for their $R$)
- cao
\[ F = \mu N \]

\[ R \uparrow 18g + 60g = N \]
\[ = 78g \]
\[ R \rightarrow R = F = \mu N \]

\[ P \]
\[ 2.5 \times 18g \cos \alpha + 3 \times 60g \cos \alpha = 5F \sin \alpha \]

\[ A \]
\[ 18g \times 2.5 \cos \alpha + 60g \times 3 \cos \alpha = R \times 5 \sin \alpha \]

\[ C \]
\[ \frac{1}{2} \cos \alpha \times 18g + 3 \sin \alpha F + 2 \sin \alpha R = 3 \cos \alpha N \]

\[ B \]
\[ 5 \cos \alpha N = 5 \sin \alpha F + 2.5 \cos \alpha \times 18g + 2 \cos \alpha \times 60 \]
\[ \frac{1}{2} \sin \alpha = 2.5R \sin \alpha + 2.6F \sin \alpha \]

\[ 45 \times \frac{3}{5}g + 180 \times \frac{3}{5}g = 4R \]

\[ R = \frac{135}{4} g \]

\[ 78g \mu = \frac{135}{4} g \]

\[ \mu = \frac{135}{4 \times 78} = \frac{135}{312} = 0.432\ldots = 0.43 \]

NB If use just two moments equations, M1A2 for the better attempt, M1A1 for the other. Remaining marks as above.

\[ 4 \]

\[ B1 \]
<table>
<thead>
<tr>
<th>Q</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$t = \frac{5}{4}$</td>
<td>M1 1.25</td>
</tr>
<tr>
<td>(b)</td>
<td>$\mathbf{r} = (2t^2 - 5t)\mathbf{i} + 3t\mathbf{j}(+c)$</td>
<td>A1 Integrate the velocity vector</td>
</tr>
<tr>
<td></td>
<td>$t = 0 \quad 2\mathbf{i} + 5\mathbf{j} = \mathbf{e}$</td>
<td>DM1 NB Also correct to use suvat with $a = 4\mathbf{i}$ and $\mathbf{u} = -5\mathbf{i} + 3\mathbf{j}$</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{r} = (2t^2 - 5t)\mathbf{i} + (3t + 5)\mathbf{j}$</td>
<td>A1 Correct</td>
</tr>
<tr>
<td></td>
<td>$(2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$</td>
<td>Use $\mathbf{r}_0$ to find $C$</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{r}_0 = 1\mathbf{i} + 2\mathbf{j} - 2\mathbf{t} + \mathbf{c}\mathbf{t}\mathbf{j}$</td>
<td>B1 oe</td>
</tr>
<tr>
<td>(c)</td>
<td>$(11 - 2t)\mathbf{i} + (2 + ct)\mathbf{j}$</td>
<td>Correct $\mathbf{j}$ component of $\mathbf{r}_Q$</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{r}_p = (2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$</td>
<td>Do not actually require the whole thing - can answer the Q by considering only the $\mathbf{j}$ component.</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{r}_Q = \mathbf{r}_p = d\mathbf{i} + 14\mathbf{j}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3t + 5 = 14 \quad \frac{2t^2 - 3t - 9}{(2t + 3)(t - 3)} = 0$</td>
<td>M1 Form an equation in $t$ only</td>
</tr>
<tr>
<td></td>
<td>$t = 3$</td>
<td>A1 ft Their $t$</td>
</tr>
<tr>
<td></td>
<td>$2 + ct = 14 \Rightarrow c = 4$</td>
<td>A1 ft Their $t$</td>
</tr>
<tr>
<td></td>
<td>$d = 11 - 2 \times 3 = 5 \quad$ or</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d = 2 \times 3^2 - 5 \times 3 + 2 \Rightarrow d = 5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Alt: $2t^2 - 5t + 2 = 11 - 2t = d \Rightarrow t = \frac{11-d}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2\left(\frac{11-d}{2}\right)^2 - 5\left(\frac{11-d}{2}\right) + 2 = d$,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d^2 - 19d + 70 = 0 = (d - 5)(d - 14)$</td>
<td></td>
</tr>
<tr>
<td>Q.</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>5</td>
<td><img src="image" alt="Diagram" /></td>
<td>Resolve perpendicular to plane. Condone trig confusion.</td>
</tr>
<tr>
<td>(a)</td>
<td>( N = 2g \cos \theta = \frac{14}{25}g )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( F = \mu N = \frac{5}{12} \times \frac{14}{25}g = \frac{7g}{30} )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Work done = ( \frac{7}{30}g \times 1.5 = 3.43 \ldots = 3.4 \text{ J} )</td>
<td>DM1</td>
</tr>
<tr>
<td>(b)</td>
<td>( 3.43 + 2g \sin \theta \times 1.5 = \frac{1}{2} \times 2U^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( U = 5.626 \ldots = 5.6 )</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td><img src="image" alt="Diagram" /></td>
<td>Energy equation - needs all three terms. Condone sign errors &amp; trig confusion. Must have an expression for the vertical height.</td>
</tr>
<tr>
<td></td>
<td>( 2g \sin \theta \times 1.5 = 3.43 + \frac{1}{2} \times 2v^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>OR: ( \frac{1}{2} \times 2U^2 = \frac{3}{2} \times 3.43 + \frac{1}{2} \times 2v^2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v^2 = 3g \sin \theta - 3.43 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v = 4.979 \ldots )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Speed = 5.0 m s(^{-1})</td>
<td></td>
</tr>
<tr>
<td>Alt</td>
<td><img src="image" alt="Diagram" /></td>
<td>Equation of motion - needs all three terms. Condone sign errors &amp; trig confusion. Together with \textit{suvat}</td>
</tr>
<tr>
<td>(c)</td>
<td>( mg \sin \theta - F = ma )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>and ( v^2 = (u^2) + 2as )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 2g \sin \theta = \frac{7g}{25} - \frac{7g}{30} = 2a )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( a = \frac{58g}{300} = 6.26 \ldots )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v^2 = 24.754 ) ( v = 5.0 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Accept 4.98</td>
<td></td>
</tr>
<tr>
<td>Q.</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td><strong>6 (a)</strong></td>
<td>[2 = -2u \sin \theta + \frac{1}{2} g \times 4 ] [\left(2 = u \cos \theta - \frac{1}{2} gt^2\right)] [u \sin \theta = g - 1] [2u \cos \theta = 8 \quad \left(u \cos \theta = 4\right)] [\tan \theta = \frac{g-1}{4} = 2.2 \quad \ast]</td>
<td>M1 Vertical distance. Condone sign errors. Must have used (t = 2), but could be using (u_x = 4) o.e. A1 All correct</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>[u \cos \theta = 4] [u = \frac{4}{\cos \theta} = 9.66... = 9.7]</td>
<td>M1 Horizontal distance. Accept (u_x = 4) o.e. A1 Divide to obtain expression for (\tan \theta)</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>[g = (1 - g)T + \frac{1}{2} \times 9.8T^2] [4.9T^2 - 8.8T - 6 = 0] [T = \frac{8.8 \pm \sqrt{(-8.8)^2 + 4 \times 9.8}}{9.8}] [T = 2.323... = 2.32 \quad \text{or} \quad 2.3]</td>
<td>M1 Given answer Equation for vertical distance = (\pm g) to give a quadratic in (T). Allow their (u_y) B1 Horizontal distance. Accept (u_x = 4) o.e. A1 Correct method for (\alpha) 74.01... = 74° Allow 106</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>[v^2 = 8.8^2 + 2g \times 6 \quad \text{or} \quad v = -8.8 + gT] [v = 13.96...] [\text{Horiz speed} = 4] [\tan \alpha = \frac{v}{4}] [\alpha = 74.01... = 74°]</td>
<td>M1 Use \textit{suvat} to find vertical speed A1 Correct equation their (u_y), (T) DM1 Solve a 3 term quadratic A1 2.3 or 2.32 only</td>
</tr>
<tr>
<td>Alternative:</td>
<td>[\frac{1}{2}u_x(9.66^2) + 6ug = \frac{1}{2}mv^2] [v = 16.52719...] [\cos \alpha = -\frac{4}{14.5}] [\alpha = 74.01... = 74°]</td>
<td>M1 Conservation of energy to find speed A1 Correct method for (\alpha) A1 Allow 106</td>
</tr>
<tr>
<td>Q</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7(a)</td>
<td><img src="image" alt="Diagram" /></td>
<td>If the signs on their diagram and in their working are inconsistent, ignore the diagram. Penalise inconsistency between the two equations in the second accuracy mark. CLM. Allow for $v$ in either direction. Needs all 3 terms. Condone sign errors.</td>
</tr>
<tr>
<td></td>
<td>$mu = -mv + 3mw$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$u = -v + 3w$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$eu = w + v$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$w = \frac{u}{4}(1+e)$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$v = -w + eu = \frac{u}{4}(3e-1)$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
<td>Solve for $v$ or $w$.</td>
</tr>
<tr>
<td></td>
<td>$3m\mu - m\nu + 3m\sigma$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\nu = -w + eu = \frac{u}{4}(3e-1)$</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td><img src="image" alt="Diagram" /></td>
<td>If the signs on their diagram and in their working are inconsistent, ignore the diagram. Penalise inconsistency between the two equations in the B mark.</td>
</tr>
<tr>
<td></td>
<td>$3mw = 4m\nu - 3m\sigma$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$2\nu w = \frac{\nu}{2}(1+e) - \frac{3\nu}{4}(1+e) = 7\sigma$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\nu &gt; 0$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\nu &lt; \frac{3}{8}$</td>
<td>NB No longer ft. Condon $\leq$.</td>
</tr>
<tr>
<td>(c)</td>
<td><img src="image" alt="Diagram" /></td>
<td>For a second collision their $Y &gt;$ their $\nu$</td>
</tr>
<tr>
<td></td>
<td>$\frac{u}{28}(1+e)(8e-3) &gt; \frac{u}{4}(3e-1)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$2e^2 - 4e + 1 &gt; 0$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$e = \frac{4 \pm \sqrt{16 - 8}}{4} = 1.707, 0.293$</td>
<td>Obtain the critical values</td>
</tr>
<tr>
<td></td>
<td>$2e^2 - 4e + 1 &lt; 0$ for $\frac{3}{8} &lt; e \leq \frac{1}{2}$ so no second collision.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{8} &lt; e &lt; \frac{1}{2}$</td>
<td>Compare 0.293 (o.e.) with $\frac{3}{8}$ to reach correct conclusion for correct reason.</td>
</tr>
</tbody>
</table>
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January 2013
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EDEXCEL GCE MATHEMATICS

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These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
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- the symbol \( \sqrt{ } \) will be used for correct ft
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- cso - correct solution only. There must be no errors in this part of the question to obtain this mark.
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- \* or AG: The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

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6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>aA</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bM1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bB</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bM2</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA2</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>1.</td>
<td>$v \frac{dv}{dx} = 9x$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}v^2 = 9x + c$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$v^2 = 9x^2 + c$</td>
<td>M1dep</td>
</tr>
<tr>
<td></td>
<td>$x = 2, v = 6 \Rightarrow 36 = 9 \times 4 + c \Rightarrow c = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v^2 = 9x^2$</td>
<td>A1</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>2 (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass:</td>
<td>( \frac{2}{3} \pi r^3 ) ( \frac{1}{3} \pi r^3 )</td>
<td></td>
</tr>
<tr>
<td>( 2 ) ( k ) ( 2+k )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>Dist from ( O ):</td>
<td>( -\frac{3}{8} r ) ( \frac{1}{4} kr )</td>
<td>( \bar{x} )</td>
</tr>
<tr>
<td>( -\frac{3}{4} r + \frac{k^2 r}{4} = \bar{x}(2+k) )</td>
<td></td>
<td>M1A1fi</td>
</tr>
<tr>
<td>( \bar{x} = \frac{(k^2 - 3)r}{4(k + 2)} ) *</td>
<td></td>
<td>A1</td>
</tr>
</tbody>
</table>

<p>| (b)             |        |       |
| Tan ( \theta ) | ( \frac{(k^2 - 3)r}{4(k + 2)} + r ) | M1A1 |
| ( \frac{(k^2 - 3)}{4(k + 2)} = \frac{11}{14} ) |        |       |
| ( 14k^2 - 42 = 44k + 88 ) |        |       |
| ( 7k^2 - 22k - 65 = 0 ) |        |       |
| ( (7k + 13)(k - 5) = 0 ) |        |       |
| ( k = 5 ) |        | M1depA1 |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3</strong> <strong>(a)</strong></td>
<td>$0.6a = -\frac{12}{(t+2)^2}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$0.6\int dv = -\int \frac{12}{(t+2)^2} , dt$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.6v = \frac{12}{t+2} \ (c+c)$</td>
<td>M1depA1</td>
</tr>
<tr>
<td></td>
<td>$t=0 \quad v=15 \quad 0.6 \times 15 = 6 + c \Rightarrow c = 3$</td>
<td>M1dep</td>
</tr>
<tr>
<td></td>
<td>$0.6v = \frac{12}{(t+2)} + 3 \quad v = \frac{20}{t+2} + 5 = 5 \left( \frac{4}{t+2} + 1 \right)$ *</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$\frac{dx}{dt} = 5 \left( \frac{4}{t+2} + 1 \right)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x = \int 5 \left( \frac{4}{t+2} + 1 \right) , dt$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 5 \left( 4 \ln (t+2) + t \right) \ (+c'')$</td>
<td>M1depA1</td>
</tr>
<tr>
<td></td>
<td>$t=0, \ x=0 \quad c'' = -20 \ln 2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t=5 \quad x = 5 \left( 4 \ln 7 + 5 \right) - 20 \ln 2$</td>
<td>M1dep</td>
</tr>
<tr>
<td></td>
<td>$\quad = 50.05... \quad = 50.1 \text{ or better}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\quad \text{or} \quad 20 \ln \left( \frac{7}{2} \right) + 25$</td>
<td>A1</td>
</tr>
</tbody>
</table>
### Question 4

<table>
<thead>
<tr>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ R(\uparrow) \quad T \cos \theta = mg ]</td>
</tr>
<tr>
<td>[ T \times \frac{2a}{(2a+x)} = mg ]</td>
</tr>
<tr>
<td>Hooke's Law: [ T = \frac{6mgx}{2a} = \frac{3mgx}{a} ]</td>
</tr>
<tr>
<td>[ \frac{3mgx}{a} \times \frac{2a}{(2a+x)} = mg ]</td>
</tr>
<tr>
<td>[ 6x = 2a + x ]</td>
</tr>
<tr>
<td>[ x = \frac{2}{5}a \quad \text{(*)} ]</td>
</tr>
<tr>
<td>[ T \sin \theta = \frac{mv^2}{r} ]</td>
</tr>
<tr>
<td>[ 3mg \times \frac{2}{5} \sin \theta = \frac{mv^2}{\left(\frac{12a}{5}\right) \sin \theta} ]</td>
</tr>
<tr>
<td>[ v^2 = \frac{6}{5} g \times \frac{12a}{5} \sin^2 \theta ]</td>
</tr>
<tr>
<td>[ \sin^2 \theta = 1 - \left(\frac{4a^2}{\left(\frac{12a}{5}\right)^2}\right) = \frac{11}{36} ]</td>
</tr>
<tr>
<td>[ v^2 = \frac{72ag}{25} \times \frac{11}{36} = \frac{22ag}{25} ]</td>
</tr>
<tr>
<td>Question Number</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>(a)</td>
</tr>
<tr>
<td>( x = a \sin \omega t )</td>
</tr>
<tr>
<td>0.125 = 0.25 \sin 0.1\omega</td>
</tr>
<tr>
<td>( \sin 0.1\omega = \frac{1}{2} )</td>
</tr>
<tr>
<td>0.1\omega = \frac{\pi}{6}</td>
</tr>
<tr>
<td>( \omega = \frac{\pi}{0.6} = \frac{10\pi}{6} )</td>
</tr>
<tr>
<td>Period ( \frac{2\pi}{\omega} = \frac{6}{5} ) (( = 1.2 ))</td>
</tr>
<tr>
<td>(b)</td>
</tr>
<tr>
<td>( x = 0.25 \sin \frac{5}{3} \pi t )</td>
</tr>
<tr>
<td>( t = 2 \quad x = 0.25 \sin \left( 2 \times \frac{5}{3} \pi \right) )</td>
</tr>
<tr>
<td>( x = -0.2165... )</td>
</tr>
<tr>
<td>Dist from ( B ) = 0.25 + x = 0.033 m</td>
</tr>
<tr>
<td>(c)</td>
</tr>
<tr>
<td>Max accel ( = a\omega^2 = 0.25 \times \left( \frac{5\pi}{3} \right)^2 = 6.853... = 6.85 )</td>
</tr>
<tr>
<td>(d)</td>
</tr>
<tr>
<td>Max speed ( a\omega = 0.25 \times \left( \frac{5\pi}{3} \right) = 1.308... = 1.31 )</td>
</tr>
<tr>
<td>Question Number</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>(a) At B $mg \cos 60^\circ + R = \frac{mv^2}{a}$</td>
</tr>
<tr>
<td>$\frac{1}{2} g = \frac{v^2}{a}$</td>
</tr>
<tr>
<td>$v = \sqrt{\frac{ag}{2}}$</td>
</tr>
<tr>
<td>(b) Energy A to B: $\frac{1}{2} mu^2 - \frac{1}{2} m\left(\frac{ag}{2}\right) = mga \sin 30^\circ$</td>
</tr>
<tr>
<td>$u^2 = \frac{ag}{2} + 2ag \times \frac{1}{2}$</td>
</tr>
<tr>
<td>$u = \sqrt{\frac{3ag}{2}}$</td>
</tr>
<tr>
<td>(c) Horiz speed $= \sqrt{\frac{ag}{2}} \cos 60^\circ \left(= \frac{1}{2} \sqrt{\frac{ag}{2}}\right)$</td>
</tr>
<tr>
<td>Initial vert speed $= (-) \sqrt{\frac{ag}{2}} \sin 60^\circ \left(= (-)\frac{1}{2} \sqrt{\frac{3ag}{2}}\right)$</td>
</tr>
<tr>
<td>$v^2 = \frac{1}{4} \times \frac{3ag}{2} + 2g \times \frac{a}{2}$</td>
</tr>
<tr>
<td>$v^2 = \frac{11ag}{8}$</td>
</tr>
<tr>
<td>$\tan \theta = \frac{\text{vert}}{\text{horiz}} = \frac{\sqrt{\frac{11ag}{8}} \times \frac{8}{ag}}{\sqrt{11}}$</td>
</tr>
<tr>
<td>$\theta = 73.22... \approx 73^\circ$</td>
</tr>
<tr>
<td>Question Number</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td><strong>7 (a)</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
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- sf significant figures
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   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
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7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question/item is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

```
   | 0 | 1 |
---|---|---|
aM |   | ● |
aA |   | ● |
bM1|   | ● |
bA1|   | ● |
bB |   | ● |
bM2|   | ● |
bA2|   | ● |
```
# January 2013

## 6683 Statistics S1

### Mark Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>( (S_u) = 8702 - \frac{258^2}{10} ) or ( S_{gr} = 1550.2 - \frac{258 \times 63.6}{10} )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>((S_u =) 2045.6), ((S_{gr} =) -90.68) awrt (2046), awrt -90.7</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>( r = \frac{-90.68}{\sqrt{2045.6 \times 7.864}} ) awrt -0.715</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(c)</td>
<td>Positive e.g. high ( v ) corresponds to low ( t ) and low ( t ) corresponds to high ( g ) so expect high ( v ) to correspond to high ( g ) or expect more revision to result in a better grade</td>
<td>B1</td>
</tr>
</tbody>
</table>

### Notes

(a) M1 for at least one correct expression
   1\(^{st}\) A1 for \( S_u = \) awrt 2046 (Condone \( S_{xx} = \ldots \) or even \( S_{yy} = \ldots \))
   2\(^{nd}\) A1 for \( S_{gr} = \) awrt -90.7 (Condone \( S_{xy} = \ldots \))

(b) M1 for attempt at correct formula.
   Must have their \( S_u \), \( S_{gr} \) and given \( S_{gg} \) in the correct places. Condone missing “−”
   Award M1A0 for awrt -0.71 with no expression seen
   M0 for \( \frac{1550.2}{\sqrt{8702 \times 7.864}} \)
   Correct answer only is 2/2

(c) 1\(^{st}\) B1 for saying "positive". Ignore mention of skew.
   2\(^{nd}\) B1 for suitable reason that mentions at least \( v \) and \( g \) and supports positive correlation.
   e.g. “the less revision done the lower the grade” is B1
   “should do better with more revision” is B0 since does not mention grades
   “both coefficients are similar” or two sketches of negative correlation with labelled axes is B1 since \( v \), \( t \) and \( g \) are implied
   Allow use of letters \( v \) and \( g \)
   Allow equivalent terms e.g. “study” instead of “revision” or “score” instead of “grade”
### Question 2.1

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| **(a)** | F(3) = 1 gives \( \frac{3^3 + k}{40} = 1 \)  
So \( k = 13 \) | M1 |
| **(b)** | \( P(X = 1) = \frac{14}{40} \) or 0.35 (o.e.)  
Use of \( P(X = 2) = F(2) - F(1) \) or \( P(X = 3) = F(3) - F(2) \) | B1 |
| | \( P(X = 2) = \frac{7}{40} \) or 0.175, \( P(X = 3) = \frac{19}{40} \) or 0.475 | M1 |
| **(c)** | \( \text{Var}(4X - 5) = 4^2 \text{Var}(X) \) 
So \( \text{Var}(4X - 5) = \frac{259}{20} \) or 12.95 | M1 |

### Notes

| **(a)** | M1 for use of \( F(3) = 1 \) Attempt at \( \frac{3^3 + k}{40} = 1 \) must be seen  
27 + \( k \) = 40 without reference to \( F(3) = 1 \) is M0  
A1cso for no incorrect working seen and M1 scored.  
Verify Allow M1 for \( \frac{3^3 + 13}{40} = 1 \) but the A1 requires an explicit comment such as “so \( k = 13 \)” |

If a table such as this is seen then award B1M1A1A1. Ignore labels on 2nd row

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{7}{40} ) or 0.35</td>
<td>( \frac{7}{40} ) or 0.175</td>
<td>( \frac{19}{40} ) or 0.475</td>
</tr>
</tbody>
</table>

Otherwise apply the following:

| **(b)** | B1 for \( \frac{14}{40} \) or 0.35 or any exact equivalent. Can be labelled \( F(1), P(X = 1) \) or \( p(x) \) and associated with \( x = 1 \) or given in a table but must have a label  
M1 for clear method showing how to obtain \( P(X = \ldots) \) from \( F(x) \)  
M1 can be implied if either \( P(X = 2) \) or \( P(X = 3) \) is correct  
1st A1 for \( P(X = 2) = \frac{7}{40} \) or 0.175 or exact equivalent  
2nd A1 for \( P(X = 3) = \frac{19}{40} \) or 0.475 or exact equivalent |

| **(c)** | M1 for correct use of the variance formula (\( 4^2 \text{Var}(X) \) alone secures M1)  
A value for \( \text{Var}(X) \) is not required for this M1  
A1 for any exact equivalent to 12.95 Correct answer only is 2/2 |
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| (a)             | \[ \sum t = 140 \text{ (or } \bar{t} = 17.5) \text{ and } \sum m = 32 \text{ (or } \bar{m} = 4) \]  
|                 | \[ (S_{tm}) = 469.5 - \frac{"140\times"32}{8} \]  
|                 | \[ (S_{tm}) = -90.5 \]  
|                 | B1 B1 |       |
| (b)             | \[ b = \frac{S_{tm}}{S_m} = \frac{-90.5}{354} \]  
|                 | \[ b = -0.255649... \text{ (allow } \frac{181}{708}) \]  
|                 | \[ a = \frac{32}{8} - b \times \frac{140}{8} \]  
|                 | \[ \text{So equation of the line is } m = 8.47 - 0.256t \]  
|                 | A1    |       |
|                 | \[ (8.47 - 0.256\times10 = )5.9... \text{ awrt 5.9} \]  
|                 | B1    | (1)   |
| (d)             | Should be reliable since 10 is in the range (of the data) \]  
|                 | B1    |       |

Notes

(a) 1st B1 for 140 seen in correct context or correctly labelled  
2nd B1 for 32 seen in correct context or correctly labelled.  
(allow a fully correct expression – not ‘...+...+...’) 4480 used correctly is B1B1  
M1 for attempting a correct expression. Follow through their 140 and their 32  
A1cso requires a correct expression seen and no incorrect working leading to -90.5  
You may see attempt at \[ \sum (t - \bar{t})(m - \bar{m}) \]. This must have all the products seen.  

(b) 1st M1 for a correct expression for \( b \). Follow through their \( S_{tm} \). Condone missing ‘-’  
1st A1 for awrt -0.26 or condone -0.25  
2nd M1 for a correct method for \( a \). Follow through their sums from part (a) and their value of \( b \)  
2nd A1 for a correct equation for \( m \) and \( t \) with \( a = \text{awrt } 8.47 \) and \( b = \text{awrt } -0.256 \)  
Must be an equation in \( m \) and \( t \), use of \( x \) or \( y \) scores A0 here.  

(c) B1 for awrt 5.9 Accept 6 if the correct expression \( (\text{awrt } 8.47 - 10 \times \text{awrt } 0.256) \) is seen  

(d) B1 for suggesting it is reliable and mentioning 10 within the range (of the data)  
or suggesting it is reliable since interpolating or not extrapolating  

NB “it is reliable since it is in the range” is B0 since “it” is not explicit enough  
Condone extra non-relevant comments but penalise contradictory comments.  
e.g. “near the extreme so not reliable but not extrapolated so reliable” is B0 since contradicts  
“reliable since 10 is within the range (of temps) and 5.9 within range of times” is B1 since irrelevant
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>127 (-100) (15)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>So (P(L &gt; 127) = P(Z &gt; 1.8)) or (1 - P(Z &lt; 1.8)) o.e.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(= 1 - 0.9641 = 0.0359) (awrt 0.0359)</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>(d - 100 \div 15 = -1.2816) (Calculator gives (-1.2815515\ldots))</td>
<td>M1, B1</td>
</tr>
<tr>
<td></td>
<td>(d = 80.776) (awrt 80.8)</td>
<td>A1</td>
</tr>
<tr>
<td>(c)</td>
<td>Require (P(L &gt; 133 \mid L &gt; 127))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ \frac{P(L &gt; 133)}{P(L &gt; 127)} = \frac{P(Z &gt; 2.2)}{P(L &gt; 127)} ]</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>[ = \frac{1 - 0.9861}{1 - 0.9641} = \frac{0.0139}{0.0359} ]</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(= 0.3871\ldots) = awrt 0.39</td>
<td>A1</td>
</tr>
<tr>
<td>S.C.</td>
<td>An attempt at (P(L &lt; 133 \mid L &gt; 127)) that leads to awrt 0.61 (M0M1A0A0)</td>
<td>10</td>
</tr>
</tbody>
</table>

**Notes**

(a) M1 for attempting to standardise with 127, 100 and 15. Allow ± 1st A1 for \(Z > 1.8\). Allow a diagram but must have 1.8 and correct area indicated. Must have the \(Z\) so \(P(L > 127)\) with or without a diagram is insufficient. May be implied by 0.0359 2nd A1 for awrt 0.0359 (calc. gives 0.035930266\ldots). Correct ans only 3/3. M1A0A1 not poss.

(b) M1 for an attempt to standardise with 100 and 15 and set \(= \pm\) any \(z\) value (\(|z| > 1\)) B1 for \(z = \pm 1.2816\) (or better) seen anywhere [May be implied by 80.776(72\ldots) or better seen] A1 for awrt 80.8 (can be scored for using 1.28 but then they get M1B0A1) The 80.8 must follow from correct working.

Calc If answer is awrt 80.8 and awrt 80.777 or 80.776… or better seen then award M1B1A1
If answer is awrt 80.8 or 80.77 then award M1B0A1 (unless of course \(z = 1.2816\) is seen)

(c) 1st M1 for clear indication of correct conditional probability or attempt at correct ratio So clear attempt at \(\frac{P(L > 133)}{P(L > 127)}\) is sufficient for the 1st M1

2nd dM1 dependent on 1st M1 for \(P(L > 133)\) leading to \(P(Z > 2.2)\). 1st A1 for 0.0139 or better seen coming from \(P(Z > 2.20)\). Dependent on both Ms 2nd A1 for awrt 0.39. Both Ms required

ALT If they assume Alice did not check that the phone was working you may see:
\[P(L < 127).0) + P(L > 127).P(L > 133 \mid L > 127)\) Provided the conditional probability is seen as part of this calculation the 1st M1 can be scored and their final answer will be 0.0139(4/4) An answer of 0.0139 without sight of the conditional probability is 0/4.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 5. (a) | Width = 4 (cm)  
Area of 14 cm$^2$ represents frequency 28 and area of $4h$ represents 18  
Or $\frac{4h}{18} = \frac{14}{28}$ (o.e.)  
$h = 2.25$ (cm) | B1  
M1  
A1 |
| (b) | $m = (240) + \frac{10}{22} \times 80$ (o.e.)  
$= 276.36...$ (awrt $276.5$)  
$(\text{£}276.0 \leq m < \text{£}276.5)$ | M1  
A1 |
| (c) | $\sum f\bar{y} = 31600$ leading to $\bar{y} = 316$  
$\sigma_y = \sqrt{\frac{12452800}{100} - (\bar{y})^2} = 157.07...$ (awrt 157)  
Allow $s = 157.86...$ | M1A1  
M1A1 |
| (d) | Skewness = 0.764...  
(awrt 0.76 or 0.75)  
[If $n+1$ used in (b) and $m = \£278$ accept awrt 0.73 or 0.72]  
Positive skew | B1  
B1ft |
| (e) | $z = \pm \frac{80}{150}$  
P(240 < $X$ < 400) = 0.40 ~ 0.41 | M1  
A1 |
| (f) | (e) suggests a reasonable fit for this range  
BUT (d) since skew it will not be a good fit overall | B2/1/0  
(2) |

Notes

(a) B1 for width (ignore units)  
M1 for clear method using area and frequency or their width $\times$ their height = 9  
e.g. seeing both fd of 0.7 and 0.225 (may see fd in the table) [Must use correct interval]
(b) M1 for $\frac{10}{22} \times 80$ or $\frac{10.5}{22} \times 80$ (o.e.). Allow use of $(n + 1)$ leading to £278.18... or [278, 278.5)  
A1 Do not award if incorrect end-point seen but answer only is 2/2
(c) 1st M1 attempt at $\sum f\bar{y}$ with at least 3 correct products or ans. that rounds to 30 000 (to 1 sf) &/100  
2nd M1 for correct expression including $\sqrt{\bar{y}}$. Follow through $\bar{y}$. Need $\sum f\bar{y}^2$ correct but condone a minor transcription error e.g. 12458200.
(d) 1st B1 for awrt 0.76/0.75 for $m = \£276$ or awrt 0.73/0.72 for $m = \£278$  
2nd B1ft for a correct description of their skew based on their measure or if no measure given based on their values of mean and median. (correlation is B0)
(e) M1 for an attempt to standardise using the 320 and 150 and either 240 or 400 (implied by 0.53)  
A1 for answer in range [0.40, 0.41] (tables gives 0.4038, calculator 0.40619...) Ans only 2/2
(f) For B2 we need 2 comments that make reference to each of part (e) and part (d)  
One comment should suggest it is not good since skew. The other it is since matches range in (e)  
1st B1 for one relevant comment  
2nd B1 for both comments  
NB Do not use B0B1
Question Number | Scheme | Marks
---|---|---
6. (a) | \[
\begin{array}{|c|c|c|}
\hline
b & 1 & 3 \\
\hline
P(B = b) & \frac{1}{3} & \frac{1}{3} \\
\hline
\end{array}
\]
Also allow \(b\) values 1, 1, 3, 3, 5, 5 and probabilities all \(\frac{1}{6}\)
| B1 B1 |
(b) Discrete Uniform \{distribution\} | B1 |
(c) \([E(B) = ] 3\) (by symmetry) | B1 |
(d) \([E(R) = ] 2 \times \frac{2}{3} + 4 \times \frac{1}{6} + 6 \times \frac{1}{6} = 3\) | M1 A1 |
(e) \([E(R^2) = ] 2^2 \times \frac{2}{3} + 4^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} = \frac{34}{3}\) | M1 |

\([\text{Var}(R) = ] \frac{34}{3} - 3^2 = \frac{7}{3}\) (or any exact equivalent. NB 2.33 is A0) | dM1, A1 |
(f) Coin lands on 2, choose blue die; coin lands on 5 choose red die
\[
P(\text{Avisha wins}) = \frac{1}{2} \times \left( \frac{1}{3} + \frac{1}{3} \right) + \frac{1}{2} \times \frac{1}{6}
\]
\[
= \frac{5}{12} \quad (\text{allow awrt 0.417})
\]
| B2/1/0 |

Notes
(a) 1\(^{st}\) B1 for correctly identifying values of \(b\) as 1, 3, 5 or 1, 1, 3, 3, 5, 5
2\(^{nd}\) B1 for probabilities all = \(\frac{1}{6}\) or exact equivalent (or of course 6 cases of \(\frac{1}{6}\))
Any correct probability distribution or probability function is 2/2. Must be in part (a)
(b) B1 for "Discrete Uniform". Both words required.
(c) B1 for answer of 3 o.e. Accept \(E(X) = 3\)
(d) M1 for an attempt at correct formula. At least 2 correct products seen. If later divide by \(n( \neq 1)\) M0 A1 for an answer of 3. Correct answer only scores both marks.
(e) 1\(^{st}\) M1 for a correct attempt at \(E(R^2)\). At least 2 correct products seen. Condone \(\text{Var}(R) = \) etc
May be implied by sight of \(\frac{34}{3}\) or 11.3 or better.
2\(^{nd}\) dM1 Dep. on 1\(^{st}\) M1 for clear attempt at \(E(R^2) - [E(R)]^2\) Must see their values used.
NB \(\text{Var}(R) = E(R^2) - [E(R)]^2 = 3.44 - 3 = 0.44\) is M1M0A0 since do not use \([E(R)]^2\)
(f) B2/1/0 Both correct B1B1, one correct B1B0. Do not use B0B1[e.g. always red or RR is B1B0]
NB Allow other descriptions of the die e.g. 1\(^{st}\) or fair for blue, 2\(^{nd}\) for red if they are clear.
M1 for evaluating correct probabilities i.e. only \(\frac{1}{3}, \frac{1}{6}\) seen or if incorrect choice made:
M1 for an answer of: if choose RR (\(\frac{1}{4}\)), if choose BB (\(\frac{1}{3}\)), if choose RB (\(\frac{5}{6}\))
NB \(\frac{5}{12}\) as answer scores M1A1. Need to see choices of die stated for B marks.
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) P(A ∪ B) = 0.35 + 0.45 − 0.13 or 0.22 + 0.13 + 0.32 = 0.67</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td>(b) P(A’</td>
<td>B’) = P(A’ ∩ B’) / P(B’) or 0.33 0.55</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td>(c) P(B ∩ C) = 0.45 × 0.2 = 0.09</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td>(d)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Venn Diagram" /></td>
<td></td>
</tr>
<tr>
<td>(e) P(B ∪ C)’ = 0.22 + 0.22 or 1−[0.56] or 1−[0.13 + 0.23 + 0.09 + 0.11] o.e. = 0.44</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
</tbody>
</table>

**Notes**

NB May see Venn diagram for A and B only used for (a) and (b) but M marks are awarded for correct expressions only. No ft from an incorrect diagram for M marks.

(a) M1 for attempt to use the addition rule. Correct substitution i.e. correct expression seen A1 for 0.67 only. Correct answer only scores 2/2

(b) M1 for a correct ratio of probabilities or a correct formula and at least one correct prob.
   For a correct formula allow “1 − their (a)” instead of 0.33 but not for correct ratio case.
   Do not award for assuming independence i.e. \( \frac{P(A \cap C)}{P(B')} = \frac{0.66+0.55}{0.55} \) is M0. M0 if num>denom A1 for 3/5 or any exact equivalent.

(c) M1 for correct expression. Need correct values for P(B) and P(C) seen. A1 for 0.09 or any exact equivalent. Correct answer only is 2/2

(d) No labels A, B, C in (d) loses 1st B1 but can score the other 3 by implication
   B1 for box with B intersecting A and C but C not intersecting A. No box is B0
   B1ft for 0.13 and their 0.09 in correct places. [ft P(B ∩ C) from (c)]
   B1 for any 2 of 0.22, 0.22, 0.11 and 0.23 correct
   B1 for all 4 values correct

(e) M1 for a correct expression or follow through from their Venn diagram
   NB P(B’) × P(C’) = 0.55 × 0.8 is OK. Do not ft “blank” for zero and M0 for negative probs.
   A1 for 0.44 only. Correct answer only is 2/2
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.

- bod – benefit of doubt
- ft – follow through
- the symbol ✓ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
8. The maximum mark allocation for each question/part question/item is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>aM</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>aA</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bM1</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA1</td>
<td></td>
<td>●</td>
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<tr>
<td>bB</td>
<td></td>
<td>●</td>
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<tr>
<td>bM2</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>bA2</td>
<td></td>
<td>●</td>
</tr>
</tbody>
</table>
### Question

<table>
<thead>
<tr>
<th>Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>$n$ large</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$p$ small</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>Let $X$ be the random variable the number of letters delivered to the wrong house</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>$X \sim B(1000,0.01)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Po(10)$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$P(X \geq 4) = 1 - P(X \leq 3)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 1 - 0.0103$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 0.9897$</td>
<td>A1</td>
</tr>
</tbody>
</table>

#### Notes

(a) B1 Accept $n$ (the number of trials) large / high / big / $n > 50$ (accept any number larger than 50)

(b) B1 Accept $p$ (the probability) small / close to 0 / $p < 0.2$ (accept any number less than 0.2). Do not accept low.

These must appear in part (a).

(b) B1 writing or using $Po(10)$

M1 using a Poisson ($\lambda$ need not equal 10) and for writing or using $1 - P(X \leq 3)$. (Do not accept writing $1 - P(X < 4)$ unless they have used $1 - P(X \leq 3)$.

A1 0.9897 cao must be 4 dp

**NB**

An awrt 0.990 on its own gains B0M0A0 unless there is evidence that $Po(10)$ is used. In which case it gets B1M1A0

Using $B(1000,0.01)$ gives 0.989927…. and gains B0M0A0
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (a)</td>
<td>Let ( X ) be the random variable the number power cuts. ( X \sim \text{Po}(3) )</td>
<td>B1</td>
</tr>
<tr>
<td>(i)</td>
<td>( P(X = 7) = P(X \leq 7) - P(X \leq 6) \quad \text{or} \quad \frac{e^{-3}3^7}{7!} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = 0.9881 - 0.9665 ) ( = 0.0216 ) awrt 0.0216 A1</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>( P(X \geq 4) = 1 - P(X \leq 3) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = 1 - 0.6472 ) ( = 0.3528 ) awrt 0.353 A1</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>( X \sim \text{Po}(30) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( N(30,30) )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( P(X &lt; 20) = P\left( Z &lt; \frac{19.5 - 30}{\sqrt{30}} \right) ) ( = P( Z &lt; -1.92) )</td>
<td>M1M1 A1</td>
</tr>
<tr>
<td></td>
<td>( = 1 - 0.9726 ) ( = 0.0274 - 0.0276 )</td>
<td>A1</td>
</tr>
</tbody>
</table>

Notes

(a) B1 Writing or using \( \text{Po}(3) \) in either (i) or (ii)

(i) M1 writing or using \( P(X \leq 7) - P(X \leq 6) \quad \text{or} \quad \frac{e^{-3}3^7}{7!} \)

(ii) M1 writing or using \( 1 - P(X \leq 3) \). (Do not accept writing \( 1 - P(X < 4) \) unless they have used \( 1 - P(X \leq 3) \)).

(b) 1st M1 for writing or using a normal approximation

1st A1 for correct mean and sd (may be given if correct in standardisation formula)

2nd M1 Standardising using their mean and their sd and for finding correct area by doing \( 1 - P(Z \leq \text{their 1.92}) \) If they have not written down a mean and sd then these need to be correct here to award the mark

3rd M1 for attempting a continuity correction (19 ± 0.5 ) i.e. 18.5 or 19.5 only.

2nd A1 for \( \pm \frac{19.5 - 30}{\sqrt{30}} \) or awrt 1.9 or better.

3rd A1 awrt 0.0274, 0.0275 or 0.0276

SC using \( P(X < 20.5/19.5) - P(X < 19.5/18.5) \) can get M1A1 M0M1A0A0
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(a) (i)</td>
<td>$P(X &lt; 5) = 0.8424$ awrt 0.842</td>
<td>B1</td>
</tr>
<tr>
<td>(ii) P(X ≥ 7) = 1 − P(X ≤ 6)</td>
<td>$= 1 − 0.9857$ awrt 0.0143</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= 0.0143$</td>
<td></td>
</tr>
<tr>
<td>(b) P(X = 0) = $(1 − p)^{12}$</td>
<td>$(1 − p)^{12} = 0.05$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$(1 − p) = \sqrt[12]{0.05}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$p = 0.221$ awrt 0.221</td>
<td>A1</td>
</tr>
<tr>
<td>(c) Variance = $12p(1−p)$</td>
<td>$12p(1−p) = 1.92$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$12p − 12p^2 = 1.92$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$12p^2 − 12p + 1.92 = 0$ or $p^2 − p + 0.16 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p = \frac{12 \pm \sqrt{12^2 − 4 \times 12 \times 1.92}}{24}$ (5p − 1)(5p − 4) = 0</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$p = 0.2$ or $0.8$</td>
<td>A1,A1</td>
</tr>
</tbody>
</table>

**Notes**

(a) (ii) M1 writing or using $1 − P(X ≤ 6)$ Do not accept $1 − P(X < 7)$ unless $1 − P(X ≤ 6)$ has been used

(b) $1^\text{st}$ M1 $(1 − p)^n = 0.05$
$2^\text{nd}$ M1 taking $n$th root. If they have used logs they need to get to a correct expression for $1 − p$ for their equation.

(c) $1^\text{st}$ M1 $12p(1 − p) = 1.92$ o.e.
$2^\text{nd}$ M1 solving a quadratic either by factorising / completing the square / or formula.
Working must either be correct for their quadratic (they may use a quadratic from an incorrect rearrangement) or they must have written the appropriate formula down correctly and only made 1 error substituting into it. May be implied by a correct value of $p$.
$1^\text{st}$ A1 for 0.2
$2^\text{nd}$ A1 for 0.8

Total 10
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
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</tr>
</thead>
<tbody>
<tr>
<td>4 (a)</td>
<td>Mean = 1</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(b)</td>
<td>[ P(X \leq 2.4) = (2.4 - 4) \times \frac{1}{10} ] = 0.64 or [ \frac{16}{25} ]</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(c)</td>
<td>[ P(-3 &lt; X - 5 &lt; 3) = P(2 &lt; X &lt; 6) ] = 0.4</td>
<td>M1 A1</td>
</tr>
</tbody>
</table>
| (d)             | \[
\int_{a}^{4a} \frac{y^2}{4a-a} \, dy = \left[ \frac{y^3}{9a} \right]_{a}^{4a}
\] = \[ \frac{64a^3 - a^3}{9a} \] = \[ 7a^2 \] *AG | M1 M1 dep A1 |
| (e)             | Var(Y) = \[ \frac{1}{12} (4a - a)^2 \] or \[ \text{Var} (Y) = 7a^2 - \left( \frac{5}{2} \, a \right)^2 \] | M1 A1 cso |
| (f)             | \[ \frac{2}{3} = \frac{1}{3a} \left( \frac{8}{3} - a \right) \] \[ a = \frac{8}{9} \] | M1 A1 |

**Notes**

(b) M1 \((2.4 - 4) \times \frac{1}{10}\) or \(1 - (6 - 2.4) \times \frac{1}{10}\) o.e

(c) M1 finding \(P(2 < X < 6)\) or \(P(X > 2)\) or \(1 - P(X < 2)\). May be implied by a correct answer if there is no incorrect working. Do not ignore subsequent incorrect working.

**NB** if they change the distribution to \(U[-9,1]\) then M1 is for finding \(P(-3 < X < 1)\) or \(P(X > -3)\) or \(1 - P(X < -3)\). May be implied by a correct answer if there is no incorrect working. Do not ignore subsequent incorrect working.

(d) **NB remember the answer is given (AG) so they must show their working**

1st M1 writing or using \[ \int_{a}^{4a} y^2 f(y) \, dy \] with correct limits used at some point. Condone omission of \(dy\). \(f(y)\) does not need to be correct.

2nd M1 dependent on previous M being awarded. Attempting to integrate at \(y^n \rightarrow \frac{y^{n+1}}{n+1}\)

1st A1 correct expression - the correct limits must be substituted.

2nd A1 cso
(e) M1 either use of \( \frac{(b-a)^2}{12} \) or \( E(Y^2) - [E(Y)]^2 \): they may use their part (d) for \( E(Y^2) \)

(f) M1 using \( \frac{1}{3a} \left( \frac{8}{3} - a \right) \) = a probability or \( \frac{1}{3a} \left( 4a - \frac{8}{3} \right) \) = a probability

An answer of \( \frac{8}{9} \) with no incorrect working gains M1A1A1
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
</table>
| 5(a) | \[ P(T > t) = \frac{225}{(t+15)^2} \]  
\[ P(T \leq t) = 1 - P(T > t) \]  
\[ = 1 - \frac{225}{(t+15)^2} \]  
\[ F(t) = \begin{cases} 1 - \frac{225}{(t+15)^2} & t \geq 0 \\ 0 & \text{otherwise.} \end{cases} \] | B1 |
| (b) | \[ P(T < 3) = 1 - \frac{225}{(3+15)^2} \]  
\[ = \frac{11}{36} \text{ or } 0.30555... \]  
awrt 0.306 | M1/A1 |
| (c) | \[ P(T > 8|T > 3) = \frac{P(T > 8)}{P(T > 3)} \]  
\[ = \frac{225}{23^2} = \frac{225}{225} - \frac{182}{18^2} \]  
\[ = \frac{529}{324} \text{ or } 0.612.. \]  
awrt 0.612 | M1/A1 |
| (d) | \[ 1 - F(t) = 0.1 \]  
\[ \frac{225}{(t+15)^2} = 0.1 \]  
\[ \text{or } 1 - \frac{225}{(t+15)^2} = 0.9 \]  
\[ \frac{225}{0.1} = (t+15)^2 \]  
\[ t = \sqrt{\frac{225}{0.1}} - 15 \]  
\[ t = 32.4, \text{ also accept } 32/33 \] | M1/A1 |
| **Total** | **10** |       |
(a) B1 The line 
\[ P(T \leq t) = 1 - P(T > t) \text{ or } F(t) = 1 - P(T > t) \] or both of the following statements
\[ P(T > t) = \frac{225}{(t+15)^2} \text{ and } P(T \leq t) / F(t) = 1 - \frac{225}{(t+15)^2} \]
must be seen and no errors. Allow equivalent in words.

Condone use of < instead of \( \leq \) or > instead of \( \geq \) and vice versa.

**The cdf must be given.** Allow \( t > 0 \)

(b) M1 substituting 3 into \( F(t) \)

(c) 1st M1 The conditional probability must,
- be a quotient and
- have \( P(T > 3) \) or ‘their numerical equivalent’ for the denominator and
- have \( P(T > 8) \) or \( P(T > 5) \) or \( P(T > 8 \cap T > 3) \) or \( P(T > 5 \cap T > 3) \) or ‘their numerical equivalent’ for the numerator.

Allow \( \geq \) in place of >

2nd M1 writing or using \( P(T > 8) \) or \( P(T \geq 8) \).

**NB** This is independent of the first M mark.

(d) 1st M1 writing or using \( 1 - F(t) = 0.1 \) or \( P(T \geq t) = 0.1 \) May be implied by \( \frac{225}{(t+15)^2} = 0.1 \) o.e.

2nd M1 either square rooting or solving a quadratic either by factorising / completing the square / using the formula - must be correct for their quadratic.

A1 awrt 32.4 or 32 or 33. Do not accept \( 15\sqrt{10} - 15 \)
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td>A statement concerning a <strong>population parameter</strong></td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>A critical region is the range / set of values / answers or a test statistic or region/area or values (where the test is significant) that would lead to the rejection of H0 / acceptance of H1</td>
<td>B1 B1</td>
</tr>
<tr>
<td>(c)</td>
<td>( H_0 : p = 0.45 ) ( H_1 : p &lt; 0.45 ) (or ( p \neq 0.45 )) ( X \sim B(20, 0.45) ) ( P(X \leq 5) = 0.0553 ) <strong>CR</strong> ( X \leq 4 ) Accept H0. Not significant. 5 does not lie in the Critical region. There is no evidence that the proportion who voted for Mrs George is not 45% or there is evidence to support Mrs George’s claim</td>
<td>M1 A1 M1d A1cso</td>
</tr>
<tr>
<td>(d)</td>
<td>( B(8, 0.45) : P(0) = 0.0084 ) ( B(7, 0.45) : P(0) = 0.0152 ) Hence smallest value of ( n ) is 8 Alternative ( (0.55)^n &lt; 0.01 ) ( n \log 0.55 &lt; \log 0.01 ) ( n &gt; 7.7… ) Hence smallest value of ( n ) is 8</td>
<td>M1 A1 A1cso</td>
</tr>
</tbody>
</table>

Notes
(a) It must be a statement including the words **population parameter**.
(c) 1st M1 using \( B(20, 0.45) \) and finding \( P(X \leq 5) \) or \( P(X \geq 6) \) Using the normal approximation to the binomial is M0 A1 0.0553 (allow 0.9447) if not using CR or CR \( X \leq 4 \) or \( X < 5 \) 2nd M1 dependent on previous M being awarded. A correct statement (do not allow if there are contradicting non contextual statements nor award if 2 probabilities are given which would result in different conclusions) A1cso Conclusion must contain the words Mrs George. There must be no incorrect working seen. If there are no hypotheses you cannot award this mark. **NB** A correct contextual statement on it’s own will score M1 A1.
(d) M1 Attempt to find \( P(0) \) from \( B(n, 0.45) \) or \( (0.55)^n < 0.01 \) or \( (0.55)^n = 0.01 \) or \( (0.55)^n > 0.01 \) A1 \( P(0) = 0.0084 \) and \( P(0) = 0.0152 \) or getting 7.7 May be implied by correct answer. B1 cso. \( n = 8 \) should not come from incorrect working. **NB** An answer of 8 on its own with no working gains M1A1B1

**Total 10**
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>7(a)</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\int_0^5 a + bx , dx = 1$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\left[ ax + \frac{bx^2}{2} \right]_0^5 = 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$5a + \frac{25b}{2} = 1$</td>
<td>M1dep</td>
</tr>
<tr>
<td></td>
<td>$10a + 25b = 2$</td>
<td>A1cso</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>$\int_0^5 ax + bx^2 , dx = \frac{35}{12}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\left[ \frac{ax^2}{2} + \frac{bx^3}{3} \right]_0^5 = \frac{35}{12}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\frac{25a + 125b}{2} = \frac{35}{12}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$30a + 100b = 7$</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>$30a + 100b = 7$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$10a + 25b = 2$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$a = 0.1 \quad b = 0.04$</td>
<td>A1,A1</td>
</tr>
<tr>
<td>(d)</td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>$\int_0^m 0.1 + 0.04x , dx = 0.5$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\left[ 0.1x + \frac{0.04x^2}{2} \right]_0^m = 0.5$</td>
<td>A1ft</td>
</tr>
<tr>
<td></td>
<td>$0.1m + 0.02m^2 - 0.5 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m = \frac{-0.1 \pm \sqrt{0.1^2 + 4 \times 0.02 \times 0.5}}{2 \times 0.02}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$m = 3.09, -8.09$ therefore 3.09</td>
<td></td>
</tr>
<tr>
<td>(e)</td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>mean &lt; median (&lt; mode)</td>
<td>B1ft</td>
</tr>
<tr>
<td></td>
<td>negatively skewed</td>
<td>B1dep</td>
</tr>
</tbody>
</table>

Notes
(a) 1st M1 Attempting to integrate with correct limits or for an attempt to find area $0.5(a + b)h$ or Attempting to integrate and using $F(5) = 1$
1st A1 Correct integration or correct area
2nd M1 for using $=1$. This is dependent on the first M1 being awarded.
2nd A1 cso condone missing $dx$
(b) M1 using or writing (limits not needed) $\int_0^5 ax + bx^2 \, dx = \frac{35}{12}$
1st A1 correct integration
2nd A1 may be awarded for an unsimplified version $\frac{25a + 125b}{2} = \frac{35}{12}$
2nd A1 for using $=1$. This is dependent on the first A1 being awarded.
M1 attempting to solve “their equations” simultaneously – either using rearranging and substitution or making one of the coefficients the ‘same’ (ignore sign) and either adding or subtracting. May be implied by correct values for \(a\) and \(b\)

1st A1 for 0.1
2nd A1 for 0.04

M1 writing or using \(\int_0^a \text{"their } a\text{"} + \text{"their } b\text{"} x \, dx = 0.5\): limits not needed

1st A1 correct integration for their “\(a\)” and “\(b\)”

NB the correct equation simplifies to \(m^2 + m - 25 = 0\)

A1 3.09 only. If they have both roots then they must select 3.09

1st B1ft. They must compare their values for mean and median correctly. They only need to compare 2 of mean, median and mode. If they compare either the median or mean with the **mode only** then the value of the mode must be stated. They may draw a sketch that matches their values of ‘\(a\)’ and ‘\(b\)’ for \(0 \leq x \leq 5\). It must not go below the \(x\)-axis. This may be seen in part (a).

2nd B1 dependent f.t. on the previous B being awarded.
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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   • **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   • **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   • **B** marks are unconditional accuracy marks (independent of M marks)
   • Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   • bod – benefit of doubt
   • ft – follow through
   • the symbol $\sqrt{\cdot}$ will be used for correct ft
   • cao – correct answer only
   • cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   • isw – ignore subsequent working
   • awrt – answers which round to
   • SC: special case
   • oe – or equivalent (and appropriate)
   • dep – dependent
   • indep – independent
   • dp decimal places
   • sf significant figures
   • $\star$ The answer is printed on the paper
   • $\square$ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   • If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   • If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:
1. Factorisation
   \[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x =\]
   \[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x =\]

2. Formula
   Attempt to use correct formula (with values for \(a, b\) and \(c\)).

3. Completing the square
   Solving \(x^2 + bx + c = 0: \left( x \pm \frac{b}{2} \right)^2 = q \pm c, \quad q \neq 0, \quad \text{leading to } x = \ldots\)

Method marks for differentiation and integration:
1. Differentiation
   Power of at least one term decreased by 1. \((x^n \rightarrow x^{n-1})\)

2. Integration
   Power of at least one term increased by 1. \((x^n \rightarrow x^{n+1})\)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
<table>
<thead>
<tr>
<th>Question Number</th>
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</table>
| 1               | \[
\frac{7 + \sqrt{5}}{\sqrt{5} - 1} \times \frac{(\sqrt{5} + 1)}{(\sqrt{5} + 1)}
\] Multiplies top and bottom by a correct expression. This statement is sufficient. | M1 |
|                 | (Allow to multiply top and bottom by \( k(\sqrt{5} + 1) \)) | |
|                 | = \ldots \frac{4}{4} | A1cso |
|                 | Obtains a denominator of 4 or sight of \( (\sqrt{5} - 1)(\sqrt{5} + 1) = 4 \) | |
|                 | An attempt to multiply the numerator by \( (\pm \sqrt{5} \pm 1) \) and get 4 terms with at least 2 correct for their \( (\pm \sqrt{5} \pm 1) \). (May be implied) | M1 |
|                 | \((7 + \sqrt{5})(\sqrt{5} + 1) = 7\sqrt{5} + 5 + 7 + \sqrt{5}\) | |
|                 | Answer as written or \( a = 3 \) and \( b = 2 \). (Allow \( 2\sqrt{5} + 3 \)) | A1cso |
|                 | Correct answer with no working scores full marks | [4] |
| Way 2           | \[
\frac{7 + \sqrt{5}}{\sqrt{5} - 1} \times \frac{(-\sqrt{5} - 1)}{(-\sqrt{5} - 1)}
\] Multiplies top and bottom by a correct expression. This statement is sufficient. | M1 |
|                 | (Allow to multiply top and bottom by \( k(-\sqrt{5} - 1) \)) | |
|                 | = \ldots \frac{-4}{-4} | A1cso |
|                 | Obtains a denominator of -4 | |
|                 | \((7 + \sqrt{5})(-\sqrt{5} - 1) = -7\sqrt{5} - 5 - 7 - \sqrt{5}\) | M1 |
|                 | An attempt to multiply the numerator by \( (\pm \sqrt{5} \pm 1) \) and get 4 terms with at least 2 correct for their \( (\pm \sqrt{5} \pm 1) \). (May be implied) | |
|                 | \(3 + 2\sqrt{5}\) | A1cso |
|                 | Answer as written or \( a = 3 \) and \( b = 2 \) | |
|                 | Correct answer with no working scores full marks | [4] |
| Alternative using Simultaneous Equations: | \[
\frac{(7 + \sqrt{5})}{\sqrt{5} - 1} = a + b\sqrt{5} \Rightarrow 7 + \sqrt{5} = (a - b)\sqrt{5} + 5b - a
\] M1 |
<p>|                 | Multiplies and collects rational and irrational parts | |
|                 | ( a - b = 1, \ 5b - a = 7 ) A1 | |
|                 | Correct equations | |
|                 | ( a = 3, \ b = 2 ) | |
|                 | M1 for attempt to solve simultaneous equations A1 both answers correct | |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
</table>
| 2               | \[
\int \left( \frac{10x^5}{5} - \frac{4x^2}{2}, -\frac{3x^{\frac{1}{2}}}{2} \right) \, dx
\]

M1: Some attempt to integrate: 
\[x^n \rightarrow x^{n+1}\] on at least one term. (not for + c)

(If they think \[\frac{3}{\sqrt{x}}\] is \[3x^{\frac{1}{2}}\] you can still award the method mark for \[\frac{1}{x^{\frac{3}{2}}} \rightarrow x^{\frac{1}{2}}\]

A1: \[\frac{10x^5}{5}\] and \[-\frac{4x^2}{2}\] or better

A1: \[-\frac{3x^{\frac{1}{2}}}{2}\] or better

Each term correct and simplified and the + c all appearing together on the same line. Allow \[\sqrt{x}\] for \[x^{\frac{1}{2}}\]. Ignore any spurious integral or signs and/or dy/dx’s.

A1

Do not apply isw. If they obtain the correct answer and then e.g. divide by 2 they lose the last mark.

[4]
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>3(a)</td>
<td>[8^{\frac{1}{3}} = 2 \quad \text{or} \quad 8^5 = 32768] A correct attempt to deal with the [\frac{1}{3}] or the 5. [8^{\frac{1}{3}} = \sqrt[3]{8} \quad \text{or} \quad 8^5 = 8 \times 8 \times 8 \times 8 \times 8]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(\left(\frac{5}{8}\right)^3 = 32) Cao</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>A correct answer with no working scores full marks Alternative [8^3 = 8 \times 8^3 = 8 \times 2^2 = M1 \quad \text{(Deals with the 1/3)}] [= 32 \quad A1]</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>[\left(2x^2\right)^{\frac{1}{3}} = 2^3 \cdot x^{\frac{2}{3}}] One correct power either (2^\frac{1}{3}) or (x^{\frac{2}{3}}). [\left(2x^2\right)^{\frac{1}{3}} \times \left(2x^2\right)^{\frac{1}{3}} \times \left(2x^2\right)^{\frac{1}{3}}] on its own is not sufficient for this mark.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[\frac{8x^2}{4x^2} = 2x^{-2} \quad \text{or} \quad \frac{2}{\sqrt{x}}] M1: Divides coefficients of (x) and subtracts their powers of (x). <strong>Dependent on the previous M1</strong> dM1A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1: Correct answer Note that unless the power of (x) implies that they have subtracted their powers you would need to see evidence of subtraction. E.g. [\frac{8x^2}{4x^2} = 2x^{\frac{1}{2}}] would score dM0 unless you see some evidence that (3/2 - 2) was intended for the power of (x).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Note that there is a misconception that [\frac{\left(2x^2\right)^{\frac{1}{3}}}{4x^2} = \left(\frac{2}{4x^2}\right)^{\frac{1}{3}}] - this scores 0/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3)</td>
</tr>
</tbody>
</table>
For this question, mark (a) and (b) together and ignore labelling.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>4(a)</td>
<td>((a_2 = k(4+2) = 6k)) Any correct (possibly un-simplified) expression</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>(a_3 = k(\text{their } a_2 + 2) = 6k^2 + 2k) An attempt at (a_3). Can follow through their answer to (a) but (a_2) must be an expression in (k).</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(a_1 + a_2 + a_3 = 4 + (6k) + (6k^2 + 2k)) An attempt to find their (a_1 + a_2 + a_3)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(4 + (6k) + (6k^2 + 2k) = 2) A correct equation in any form.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Solves (6k^2 + 8k + 2 = 0) to obtain (k = \frac{-1}{3})</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(k = -1/3) Any equivalent fraction</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(k = -1) Must be from a correct equation. (Do not accept un-simplified)</td>
<td>B1</td>
</tr>
</tbody>
</table>

Note that it is quite common to think the sequence is an AP. Unless they find \(a_3\), this is likely only to score the M1 for solving their quadratic.

(6) [7]
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>5 (a)</strong></td>
<td>$6x + x &gt; 1 - 8$</td>
<td>Attempts to expand the bracket and collect $x$ terms on one side and constant terms on the other. Condone sign errors and allow one error in expanding the bracket. Allow $&lt;, \leq, \geq$, = instead of $&gt;$.</td>
</tr>
<tr>
<td></td>
<td>$x &gt; -1$</td>
<td>Cao</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Do not isw here, mark their final answer.</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$(x + 3)(3x - 1) = 0$</td>
<td>M1: Attempt to solve the quadratic to obtain two critical values</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow x = -3$ and $\frac{1}{3}$</td>
<td>A1: $x = -3$ and $\frac{1}{3}$ (may be implied by their inequality). Allow all equivalent fractions for -3 and 1/3. (Allow 0.333 for 1/3)</td>
</tr>
<tr>
<td></td>
<td>$-3 &lt; x &lt; \frac{1}{3}$</td>
<td>M1: Chooses “inside” region (The letter $x$ does not need to be used here)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1ft: Allow $x &lt; \frac{1}{3}$ and $x &gt; -3$ or $\left(-3, \frac{1}{3}\right)$ or $x &lt; \frac{1}{3} \cap x &gt; -3$. Follow through their critical values. (must be in terms of $x$ here) Allow all equivalent fractions for -3 and 1/3. Both ($x &lt; \frac{1}{3}$ or $x &gt; -3$) and ($x &lt; \frac{1}{3}$, $x &gt; -3$) as a final answer score A0.</td>
</tr>
<tr>
<td><strong>(4)</strong></td>
<td></td>
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</tr>
<tr>
<td><strong>[6]</strong></td>
<td></td>
<td>Note that use of $\leq$ or $\geq$ appearing in an otherwise correct answer in (a) or (b) should only be penalised once, the first time it occurs.</td>
</tr>
<tr>
<td>Question Number</td>
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<td>Marks</td>
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<td>-----------------</td>
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<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 3}{11 - (-1)} = \frac{3}{4}$</td>
<td>M1: Correct method for the gradient</td>
</tr>
<tr>
<td></td>
<td>A1: Any correct fraction or decimal</td>
<td>M1, A1</td>
</tr>
<tr>
<td></td>
<td>$y - 3 = \frac{3}{4} (x+1)$ or $y - 12 = \frac{3}{4} (x - 11)$</td>
<td>Correct straight line method using either of the given points and a numerical gradient.</td>
</tr>
<tr>
<td></td>
<td>or $y = \frac{3}{4} x + c$ with attempt at substitution to find $c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4y - 3x - 15 = 0$</td>
<td>Or equivalent with integer coefficients ($= 0$ is required)</td>
</tr>
<tr>
<td></td>
<td>This A1 should only be awarded in (a)</td>
<td></td>
</tr>
<tr>
<td>(a) Way 2</td>
<td></td>
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<tr>
<td></td>
<td>$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow y - 3 = \frac{x + 1}{12 - 3} = \frac{x + 1}{9} + 1$</td>
<td>M1: Use of a correct formula for the straight line</td>
</tr>
<tr>
<td></td>
<td>A1: Correct equation</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$12(y - 3) = 9(x + 1)$</td>
<td>Eliminates fractions</td>
</tr>
<tr>
<td></td>
<td>$4y - 3x - 15 = 0$</td>
<td>Or equivalent with integer coefficients ($= 0$ is required)</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solves their equation from part (a) and $L_2$ simultaneously to eliminate one variable</td>
<td>Must reach as far as an equation in $x$ only or in $y$ only. (Allow slips in the algebra)</td>
</tr>
<tr>
<td></td>
<td>$x = 3$ or $y = 6$</td>
<td>One of $x = 3$ or $y = 6$</td>
</tr>
<tr>
<td></td>
<td><strong>Both</strong> $x = 3$ and $y = 6$</td>
<td>Values can be un-simplified fractions.</td>
</tr>
<tr>
<td></td>
<td>Fully correct answers with no working can score 3/3 in (b)</td>
<td></td>
</tr>
<tr>
<td>(b) Way 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(-1, 3) \to -a + 3b + c = 0$</td>
<td>Substitutes the coordinates to obtain two equations</td>
</tr>
<tr>
<td></td>
<td>$(11,12) \to 11a + 12b + c = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\therefore a = -\frac{3}{4}, b = -\frac{4}{15}, c$</td>
<td>Obtains sufficient equations to establish values for $a, b$ and $c$</td>
</tr>
<tr>
<td></td>
<td>e.g. $c = 1 \Rightarrow b = -\frac{4}{15}, a = \frac{3}{15}$</td>
<td>Obtains values for $a, b$ and $c$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{15}x - \frac{4}{15}y + 1 = 0 \Rightarrow 4y - 3x - 15 = 0$</td>
<td>Correct equation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4)</td>
</tr>
<tr>
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<td>-----------------</td>
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</tr>
<tr>
<td>7(a)</td>
<td>(600 = 200 + (N - 1)20 \Rightarrow N = \ldots)</td>
<td>Use of 600 with a <strong>correct</strong> formula in an attempt to find (N). A correct formula could be implied by a correct answer. M1</td>
</tr>
<tr>
<td></td>
<td>(N = 21)</td>
<td>cso A1</td>
</tr>
<tr>
<td></td>
<td>Accept correct answer only.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(600 = 200 + 20N \Rightarrow N = 20) is M0A0 (wrong formula)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\frac{600 - 200}{20} = 20 \cdot N = 21) is M1A1 (correct formula implied)</td>
<td></td>
</tr>
<tr>
<td><strong>Listing:</strong></td>
<td>All terms must be listed up to 600 and 21 correctly identified. A solution that scores 2 if fully correct and 0 otherwise.</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>(b)</strong> Look for an AP first:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S = \frac{21}{2} (2 \times 200 + 20 \times 20)) or (\frac{21}{2} (200 + 600))</td>
<td>M1: Use of correct sum formula with their integer (n = N) or (N - 1) from part (a) where (3 &lt; N &lt; 52) and (a = 200) and (d = 20).</td>
<td>M1A1</td>
</tr>
<tr>
<td>(S = \frac{20}{2} (2 \times 200 + 19 \times 20)) or (\frac{20}{2} (200 + 580))</td>
<td>A1: Any correct un-simplified numerical expression with (n = 20) or (n = 21) (No follow through here)</td>
<td></td>
</tr>
<tr>
<td>(= 8400) or (7800))</td>
<td></td>
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<tr>
<td><strong>Then for the constant terms:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(600 \times (52 - &quot;N&quot;)) (= 18600))</td>
<td>M1: (600 \times k) where (k) is an integer and (3 &lt; k &lt; 52)</td>
<td>M1A1ft</td>
</tr>
<tr>
<td></td>
<td>A1: A correct un-simplified follow through expression with their (k) consistent with (n) so that (n + k = 52)</td>
<td></td>
</tr>
<tr>
<td>So total is 27000</td>
<td>Cao A1</td>
<td></td>
</tr>
<tr>
<td>Note that for the constant terms, they may correctly use an AP sum with (d = 0).</td>
<td></td>
<td></td>
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<tr>
<td>**There are no marks in (b) for just finding (S_{52})</td>
<td></td>
<td>(5)</td>
</tr>
<tr>
<td>If they obtain (N = 20) in (a) (0/2) and then in (b) proceed with, (\frac{20}{2} (2 \times 200 + 19 \times 20) + 32 \times 600 = 7800 + 19 \times 200 = 27000) allow them to ‘recover’ and score full marks in (b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Similarly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If they obtain (N = 22) in (a) (0/2) and then in (b) proceed with, (\frac{21}{2} (2 \times 200 + 20 \times 20) + 31 \times 600 = 8400 + 18 \times 600 = 27000) allow them to ‘recover’ and score full marks in (b)</td>
<td></td>
<td></td>
</tr>
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<td>Marks</td>
</tr>
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<tr>
<td>8</td>
<td>Horizontal translation – does <strong>not</strong> have to cross the y-axis on the right but must at least reach the x-axis.</td>
<td>B1</td>
</tr>
<tr>
<td>(a)</td>
<td>Touching at (-5, 0). This could be stated anywhere or -5 could be marked on the x-axis. Or (0, -5) <strong>marked in the correct place.</strong> Be fairly generous with ‘touching’ if the intention is clear. The right hand tail of their cubic shape crossing at (-1, 0). This could be stated anywhere or -1 could be marked on the x-axis. Or (0, -1) <strong>marked in the correct place.</strong> The curve must <strong>cross</strong> the x-axis and not stop at -1.</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>((x + 5)^2(x + 1)) Allow ((x + 3 + 2)^2(x - 1 + 2))</td>
<td>B1</td>
</tr>
<tr>
<td>(c)</td>
<td>When (x = 0, y = 25) M1: Substitutes (x = 0) into their expression in <strong>part (b)</strong> which is not (f(x)). This may be implied by their answer. Note that the question asks them to use part (b) but allow independent methods. A1: (y = 25) (Coordinates not needed) If they expand <strong>incorrectly</strong> prior to substituting (x = 0), score M1 A0 <strong>NB</strong> (f(x + 2) = x^3 + 11x^2 + 35x + 25)</td>
<td>M1 A1</td>
</tr>
</tbody>
</table>

<p>| (3)             | (1)               |       |
| (2)             | (6)               |       |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>9 (a)</td>
<td>$(3 - x^2)^2 = 9 - 6x^2 + x^4$</td>
<td>An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4, \quad p, q \neq 0$</td>
</tr>
<tr>
<td></td>
<td>$9x^2 + x^2$</td>
<td>Must come from $\frac{9 + x^4}{x^2}$</td>
</tr>
<tr>
<td></td>
<td>-6</td>
<td>Must come from $\frac{-6x^2}{x^2}$</td>
</tr>
<tr>
<td>Alternative 1:</td>
<td>$\frac{(3 - x^2)^2}{x^2}$ as $(3x^{-1} - x)^2$ and attempts to expand $= M1$ then $A1A1$ as in the scheme.</td>
<td></td>
</tr>
<tr>
<td>Alternative 2:</td>
<td>$(3 - x^2)^2 = 9 + Ax^2 + Bx^4$, expands $(3 - x^2)^2$ and compares coefficients $= M1$ then $A1A1$ as in the scheme.</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$-18x^{-3} + 2x$</td>
<td>M1: $x^n \rightarrow x^{n-1}$ on separate terms at least once. Do not award for $A \rightarrow 0$ (Integrating is $M0$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1ft: $-18x^{-3} + 2^nB^n x$ with a numerical $B$ and no extra terms. ($A$ may have been incorrect or even zero)</td>
</tr>
<tr>
<td>(c)</td>
<td>$f(x) = -9x^{-1} - 6x + \frac{x^3}{3} (+c)$</td>
<td>M1: $x^n \rightarrow x^{n+1}$ on separate terms at least once. (Differentiating is $M0$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1ft: $-9x^{-1} + Ax + \frac{Bx^3}{3} (+c)$ with numerical $A$ and $B$, $A, B \neq 0$</td>
</tr>
<tr>
<td></td>
<td>$10 = \frac{-9}{-3} - 6(-3) + \frac{(-3)^3}{3} + c$ so $c = \ldots$</td>
<td>Uses $x = -3$ and $y = 10$ in what they think is $f(x)$ (They may have differentiated here) but it must be a changed function i.e. not the original $f'(x)$, to form a linear equation in $c$ and attempts to find $c$. No $+ c$ gets $M0$ and $A0$ unless their method implies that they are correctly finding a constant.</td>
</tr>
<tr>
<td></td>
<td>$c = -2$</td>
<td>so</td>
</tr>
<tr>
<td></td>
<td>$(f(x) =) -9x^{-1} - 6x + \frac{x^3}{3} + \text{their } c$</td>
<td>Follow through their $c$ in an otherwise (possibly un-simplified) correct expression.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Allow $\frac{9}{x}$ for $-9x^{-1}$ or even $\frac{9x^{-1}}{-1}$.</td>
</tr>
</tbody>
</table>

Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>10(a)</td>
<td>(x^2 - 4k(1 - 2x) + 5k = 0)</td>
<td>Makes (y) the subject from the first equation and substitutes into the second equation (= 0 not needed here) or eliminates (y) by a correct method. M1</td>
</tr>
<tr>
<td></td>
<td>(x^2 + 8kx + k = 0) *</td>
<td>Correct completion to printed answer. There must be no incorrect statements. A1cso</td>
</tr>
<tr>
<td>(b)</td>
<td>((8k)^2 - 4k)</td>
<td>M1: Use of (b^2 - 4ac) (Could be in the quadratic formula or an inequality, = 0 not needed yet). There must be some correct substitution but there must be no (x)'s. No formula quoted followed by e.g. (8k^2 - 4k = 0) is M0. M1 A1</td>
</tr>
<tr>
<td></td>
<td>(k = \frac{1}{16}) (oe)</td>
<td>Cso (Ignore any reference to (k = 0)) but there must be no contradictory earlier statements. A1</td>
</tr>
<tr>
<td>(b) Way 2</td>
<td>(x^2 + 8kx + k = (x + \sqrt{k})^2)</td>
<td>M1: Correct strategy for equal roots M1A1</td>
</tr>
<tr>
<td>Equal roots</td>
<td>(8k = 2\sqrt{k})</td>
<td>A1: Correct equation</td>
</tr>
<tr>
<td></td>
<td>(k = \frac{1}{16}) (oe)</td>
<td>Cso (Ignore any reference to (k = 0)) A1</td>
</tr>
<tr>
<td>(b) Way 3</td>
<td>Completes the Square</td>
<td>M1: ((x \pm 4k)^2 \pm p \pm k, \ p \neq 0) M1A1</td>
</tr>
<tr>
<td></td>
<td>(x^2 + 8kx + k = (x + 4k)^2 - 16k^2 + k)</td>
<td>A1: Correct equation</td>
</tr>
<tr>
<td></td>
<td>(16k^2 - k = 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(k = \frac{1}{16}) (oe)</td>
<td>Cso (Ignore any reference to (k = 0)) A1</td>
</tr>
<tr>
<td>(c)</td>
<td>(x^2 + \frac{1}{2}x + \frac{1}{16} = 0) so ((x + \frac{1}{2})^2 = 0 \Rightarrow x =)</td>
<td>Substitutes their value of (k) into the given quadratic and attempt to solve their 2 or 3 term quadratic as far as (x =) (may be implied by substitution into the quadratic formula) or starts again and substitutes their value of (k) into the second equation and solves simultaneously to obtain a value for (x). M1</td>
</tr>
<tr>
<td></td>
<td>(x = -\frac{1}{4}, y = 1\frac{1}{2})</td>
<td>First A1 one answer correct, second A1 both answers correct. A1A1</td>
</tr>
<tr>
<td>Special Case:</td>
<td>(x^2 + \frac{1}{2}x + \frac{1}{16} = 0) \Rightarrow x = -\frac{1}{4}, \frac{1}{4} \Rightarrow y = 1\frac{1}{2}, \frac{1}{2}) allow M1A1A0</td>
<td>[8]</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>11(a)</strong></td>
<td>((-\frac{3}{4}, 0)). Accept (x = -\frac{3}{4})</td>
<td>B1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>(y = 4) (x = 0) or ‘(y)-axis’</td>
<td>B1: One correct asymptote B1B1</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>(\frac{dy}{dx} = -3x^{-2}) (\frac{dy}{dx} = kx^{-2}) (Allow (\frac{dy}{dx} = kx^{-2} + 4))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Cao (may be un-simplified but must be a fraction with no powers) e.g. (-3(-3)^{-2}) scores A0 unless evaluated as e.g. (-\frac{3}{9}) or is implied by their normal gradient.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Gradient of normal = (-1/m) Correct perpendicular gradient rule applied to a numerical gradient that must have come from substituting (x = -3) into their derivative.</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>Normal at (P) is ((y - 3) = 3(x + 3))</td>
<td>M1: Correct straight line method using ((-3, 3)) and a “changed” gradient. A wrong equation with no formula quoted is M0. Also dependent on the previous M1. dM1A1</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>((-4, 0)) and ((0, 12)). Both correct (May be seen on a sketch)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>So (AB) has length (\sqrt{160}) or (AB^2) has length 160</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>A1: (\sqrt{160}) or better e.g. (4\sqrt{10}) with no errors seen</td>
<td>(3)</td>
</tr>
</tbody>
</table>
Edexcel GCE
Core Mathematics C2
Advanced Subsidiary
Friday 24 May 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Total
1. The first three terms of a geometric series are 18, 12 and \( p \) respectively, where \( p \) is a constant.

Find

(a) the value of the common ratio of the series,

(b) the value of \( p \),

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.
2. (a) Use the binomial theorem to find all the terms of the expansion of

\[(2 + 3x)^4\]

Give each term in its simplest form. 

(b) Write down the expansion of

\[(2 - 3x)^4\]

in ascending powers of \(x\), giving each term in its simplest form.
Question 2 continued

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(Total 5 marks)
3. \[ f(x) = 2x^3 - 5x^2 + ax + 18 \]

where \( a \) is a constant.

Given that \((x - 3)\) is a factor of \( f(x) \),

(a) show that \( a = -9 \) \hspace{2cm} (2)

(b) factorise \( f(x) \) completely. \hspace{2cm} (4)

Given that

\[ g(y) = 2(3^y) - 5(2^y) - 9(3^y) + 18 \]

(c) find the values of \( y \) that satisfy \( g(y) = 0 \), giving your answers to 2 decimal places where appropriate. \hspace{2cm} (3)
Question 3 continued
Question 3 continued
Question 3 continued

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(Total 9 marks)
4. \[ y = \frac{5}{(x^2 + 1)} \]

(a) Complete the table below, giving the missing value of \( y \) to 3 decimal places.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>4</td>
<td>2.5</td>
<td>1</td>
<td>0.690</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

(b) Use the trapezium rule, with all the values of \( y \) from your table, to find an approximate value for the area of \( R \).

(c) Use your answer to part (b) to find an approximate value for

\[ \int_{0}^{3} \left( 4 + \frac{5}{(x^2 + 1)} \right) \, dx \]

giving your answer to 2 decimal places.
Figure 2 shows a plan view of a garden.
The plan of the garden $ABCDEA$ consists of a triangle $ABE$ joined to a sector $BCDE$ of a circle with radius 12 m and centre $B$.
The points $A$, $B$ and $C$ lie on a straight line with $AB = 23$ m and $BC = 12$ m.

Given that the size of angle $ABE$ is exactly 0.64 radians, find

(a) the area of the garden, giving your answer in m$^2$, to 1 decimal place, (4)

(b) the perimeter of the garden, giving your answer in metres, to 1 decimal place. (5)
Question 5 continued

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Question 5 continued
Figure 3 shows a sketch of part of the curve $C$ with equation

$$y = x(x + 4)(x - 2)$$

The curve $C$ crosses the $x$-axis at the origin $O$ and at the points $A$ and $B$.

(a) Write down the $x$-coordinates of the points $A$ and $B$.  

The finite region, shown shaded in Figure 3, is bounded by the curve $C$ and the $x$-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.
Question 6 continued
Question 6 continued

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(Total 8 marks)
7. (i) Find the exact value of \( x \) for which

\[
\log_2(2x) = \log_2(5x + 4) - 3
\]

(ii) Given that

\[
\log_a y + 3\log_a 2 = 5
\]

express \( y \) in terms of \( a \).
Give your answer in its simplest form.
Question 7 continued

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(Total 7 marks)
8. (i) Solve, for $-180^\circ \leq x < 180^\circ$,

$$\tan(x - 40^\circ) = 1.5$$

giving your answers to 1 decimal place. (3)

(ii) (a) Show that the equation

$$\sin \theta \tan \theta = 3\cos \theta + 2$$

can be written in the form

$$4\cos^2 \theta + 2\cos \theta - 1 = 0$$

(3)

(b) Hence solve, for $0 \leq \theta < 360^\circ$,

$$\sin \theta \tan \theta = 3\cos \theta + 2$$

showing each stage of your working. (5)
9. The curve with equation

\[ y = x^2 - 32\sqrt{x} + 20, \quad x > 0 \]

has a stationary point \( P \).

Use calculus

(a) to find the coordinates of \( P \),

(b) to determine the nature of the stationary point \( P \).
Question 9 continued
The circle $C$ has radius 5 and touches the $y$-axis at the point $(0, 9)$, as shown in Figure 4.

(a) Write down an equation for the circle $C$, that is shown in Figure 4.  

A line through the point $P(8, -7)$ is a tangent to the circle $C$ at the point $T$.

(b) Find the length of $PT$.  

Figure 4
Question 10 continued
Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 3 (6665/01)
Edexcel and BTEC Qualifications

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Alternatively, you can get in touch with us using the details on our contact us page at [www.edexcel.com/contactus](http://www.edexcel.com/contactus).

If you have any subject specific questions about this specification that require the help of a subject specialist, you can speak directly to the subject team at Pearson.

Their contact details can be found on this link: [www.edexcel.com/teachingservices](http://www.edexcel.com/teachingservices).

You can also use our online Ask the Expert service at [www.edexcel.com/ask](http://www.edexcel.com/ask). You will need an Edexcel username and password to access this service.

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \sqrt{\text{ }} \) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - \* The answer is printed on the paper
   - \[ \text{The second mark is dependent on gaining the first mark} \]

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

\[(x^2 + bx + c) = (x + p)(x + q)\], where \(|pq|=|c|\), leading to \(x = \ldots\)

\[(ax^2 + bx + c) = (mx + p)(nx + q)\], where \(|pq|=|c|\) and \(|mn|=|a|\), leading to \(x = \ldots\)

2. Formula

Attempt to use correct formula (with values for \(a\), \(b\) and \(c\)).

3. Completing the square

Solving \(x^2 + bx + c = 0\):

\[\left(x \pm \frac{b}{2}\right)^2 = q \pm c, \quad q \neq 0, \quad \text{leading to } x = \ldots\]

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. \((x^n \to x^{n-1})\)

2. Integration

Power of at least one term increased by 1. \((x^n \to x^{n+1})\)
Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1               | \[ \frac{3x^2 - 2x + 7}{x^2(0) - 4} \frac{3x^4 - 2x^3 - 5x^2 + (0) - 4}{3x^4 + 0x^3 - 12x^2} \]
|                 | \[-2x^3 + 7x^2 + 0x\] \[-2x^3 + 0x^2 + 8x\] \[7x^2 - 8x - 4\] \[7x^2 + 0x - 28\] \[-8x + 24\] \[a = 3\] | B1 |
|                 | Long division as far as \[ \frac{3x^2 - 2x^2}{x^2(0) - 4} \frac{3x^4 - 2x^3 - 5x^2 + (0) - 4}{3x^4 + 0x^3 - 12x^2} \]
|                 | \[-2x^3 + .................\] \[-2x^3 + .................\] | M1 |
|                 | Two of \( b = -2 \quad c = 7 \quad d = -8 \quad e = 24 \) | A1 |
|                 | All four of \( b = -2 \quad c = 7 \quad d = -8 \quad e = 24 \) | A1 |

**Notes for Question 1**

B1 Stating \( a = 3 \). This can also be scored by the coefficient of \( x^2 \) in \( 3x^2 - 2x + 7 \)

M1 Using long division by \( x^2 - 4 \) and getting as far as the ‘\( x \)’ term. The coefficients need not be correct.

Award if you see the whole number part as \( \ldots x^2 + \ldots x \) following some working. You may also see this in a table/grid.

Long division by \( (x + 2) \) will not score anything until \( (x - 2) \) has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.

A1 Achieving two of \( b = -2 \quad c = 7 \quad d = -8 \quad e = 24 \).

The answers may be embedded within the division sum and can be implied.

A1 Achieving all of \( b = -2 \quad c = 7 \quad d = -8 \) and \( e = 24 \).

Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0

Need to see \( a = \ldots \), \( b = \ldots \), or the values embedded in the rhs for all 4 marks.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| Alt 1 By Multiplication | * \(3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e\)  

Compares the \(x^4\) terms \(a = 3\)  

Compares coefficients to obtain a numerical value of one further constant  
\(-2 = b, \quad -5 = -4a + c \Rightarrow c = \ldots\)  

Two of \(b = -2 \quad c = 7 \quad d = -8 \quad e = 24\)  
All four of \(b = -2 \quad c = 7 \quad d = -8 \quad e = 24\) | B1 M1 A1 A1 |

(4 marks) |

Notes for Question 2

B1 Stating \(a = 3\). This can also be scored for writing \(3x^4 = ax^4\)

M1 Multiply out expression given to get *. Condone slips only on signs of either expression.

Then compare the coefficient of any term (other than \(x^4\)) to obtain a numerical value of one further constant. In reality this means a valid attempt at either \(b\) or \(c\). The method may be implied by a correct additional constant to \(a\).

A1 Achieving two of \(b = -2 \quad c = 7 \quad d = -8 \quad e = 24\)

A1 Achieving all of \(b = -2 \quad c = 7 \quad d = -8\) and \(e = 24\)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(i)</td>
<td>(\ln) graph crossing (x) axis at (1,0) and asymptote at (x=0)</td>
<td>B1</td>
</tr>
<tr>
<td>2(ii)</td>
<td>Shape including cusp</td>
<td>B1ft</td>
</tr>
<tr>
<td></td>
<td>Touches or crosses the (x) axis at (1,0)</td>
<td>B1ft</td>
</tr>
<tr>
<td></td>
<td>Asymptote given as (x=0)</td>
<td>B1</td>
</tr>
<tr>
<td>2(iii)</td>
<td>Shape</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Crosses at (5, 0)</td>
<td>B1ft</td>
</tr>
<tr>
<td></td>
<td>Asymptote given as (x=4)</td>
<td>B1</td>
</tr>
</tbody>
</table>

(7 marks)
Notes for Question 2

(i) B1  Correct shape, correct position and passing through (1, 0).
Graph must ‘start’ to the rhs of the y-axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone ‘slips’. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. Do not withhold this mark if (x=0) the asymptote is incorrect or not given.

(ii) B1ft Correct shape including the cusp wholly contained in quadrant 1.
The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum. The shape to the lhs of the cusp should not bend backwards past (1,0)
Tolerate a ‘linear’ looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items.
Follow through on an incorrect sketch in part (i) as long as it was above and below the x axis.

B1ft The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked ‘1’ on the x axis. Condone the point marked on the correct axis as (0, 1)
Follow through on an incorrect intersection in part (i).

B1 Award for the asymptote to the curve given/ marked as x = 0. Do not allow for it given/ marked as ‘the y axis’. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at x = 0. Accept if x=0 is drawn separately to the y axis.

(iii)

B1 Correct shape.
The gradient should always be negative and becoming less steep. It must be approximately infinite at the lh end and not have an obvious minimum. The lh end must not bend ‘forwards’ to make a C shape. The position is not important for this mark. See practice and qualification for clarification.

B1ft The graph crosses (or touches) the x axis at (5, 0). Allow for the curve passing through a point marked ‘5’ on the x axis. Condone the point marked on the correct axis as (0, 5)
Follow through on an incorrect intersection in part (i). Allow for ((i) + 4, 0)

B1 The asymptote is given/ marked as x = 4. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the y axis).

If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.
Examples of graphs in number 2

Part (i)

<table>
<thead>
<tr>
<th>Condoned</th>
<th>Not condoned</th>
<th>Condone</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Part (ii)

<table>
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<tr>
<th>1,1,1</th>
<th>0,1,1,</th>
<th>0,1,0,</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Graph" /></td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Example of follow through in part (ii) and (iii)

(i) B0
(ii) B1ftB1ftB0
(iii) B0B1ftB0
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 3(a)            | \[ 2 \cos x \cos 50 - 2 \sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40 \] \\
|                 | \[ \sin x(\cos 40 + 2 \sin 50) = \cos x(2 \cos 50 - \sin 40) \] \\
|                 | \[ + \cos x \Rightarrow \tan x(\cos 40 + 2 \sin 50) = 2 \cos 50 - \sin 40 \] \\
|                 | \[ \tan x = \frac{2 \cos 50 - \sin 40}{\cos 40 + 2 \sin 50}, \] (or numerical answer awrt 0.28) \\
|                 | States or uses \( \cos 50 = \sin 40 \) and \( \cos 40 = \sin 50 \) and so \( \tan x^o = \frac{1}{3} \tan 40^o \) cao | A1 * |
| (b)             | Deduces \( \tan 2\theta = \frac{1}{3} \tan 40 \) \\
|                 | \( 2\theta = 15.6 \) so \( \theta = \) awrt 7.8(1) One answer | A1 |
|                 | Also \( 2\theta = 195.6, 375.6, 555.6 \Rightarrow \theta = .. \) | M1 |
|                 | \( \theta = \) awrt 7.8, 97.8, 187.8, 277.8 All 4 answers | A1 |
| Alt 1 3(a)      | \[ 2 \cos x \cos 50 - 2 \sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40 \] \\
|                 | \[ 2 \cos x \sin 40 - 2 \sin x \cos 40 = \sin x \cos 40 + \cos x \sin 40 \] \\
|                 | \[ + \cos x \Rightarrow 2 \sin 40 - 2 \tan x \cos 40 = \tan x \cos 40 + \sin 40 \] \\
|                 | \[ \tan x = \frac{\sin 40}{3 \cos 40} \] ( or numerical answer awrt 0.28), \( \Rightarrow \tan x = \frac{1}{3} \tan 40 \) | A1,A1 |
| Alt 2 3(a)      | \[ 2 \cos(x + 50) = \sin(x + 40) \Rightarrow 2 \sin(40 - x) = \sin(x + 40) \] \\
|                 | \[ 2 \cos x \sin 40 - 2 \sin x \cos 40 = \sin x \cos 40 + \cos x \sin 40 \] \\
|                 | \[ + \cos x \Rightarrow 2 \sin 40 - 2 \tan x \cos 40 = \tan x \cos 40 + \sin 40 \] \\
|                 | \[ \tan x = \frac{\sin 40}{3 \cos 40} \] ( or numerical answer awrt 0.28), \( \Rightarrow \tan x = \frac{1}{3} \tan 40 \) | A1,A1 |
Notes for Question 3

(a)

M1 Expand both expressions using \( \cos(x + 50) = \cos x \cos 50 - \sin x \sin 50 \) and 
\( \sin(x + 40) = \sin x \cos 40 + \cos x \sin 40 \). Condone a missing bracket on the lhs. The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions.
Allow if written separately and not in a connected equation.

M1 Divide by \( \cos x \) to reach an equation in \( \tan x \).
Below is an example of M1M1 with incorrect sign on left hand side
\[
2 \cos x \cos 50 + 2 \sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40
\Rightarrow 2 \cos 50 + 2 \tan x \sin 50 = \tan x \cos 40 + \sin 40
\]
This is independent of the first mark.

A1 \( \tan x = \frac{2 \cos 50 - \sin 40}{\cos 40 + 2 \sin 50} \)
Accept for this mark \( \tan x = \text{awrt} 0.28... \) as long as M1M1 has been achieved.

A1* States or uses \( \cos 50 = \sin 40 \) and \( \cos 40 = \sin 50 \) leading to showing
\[
\tan x = \frac{2 \cos 50 - \sin 40}{\cos 40 + 2 \sin 50} = \frac{\sin 40}{3 \cos 40} = \frac{1}{3} \tan 40
\]
This is a given answer and all steps above must be shown. The line above is acceptable.
Do not allow from \( \tan x = \text{awrt} 0.28... \)

(b)

M1 For linking part (a) with (b). Award for writing \( \tan 2 \theta = \frac{1}{3} \tan 40 \)

A1 Solves to find one solution of \( \theta \) which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of \( \theta \). It must be a full method but can be implied by any correct answer.
\[
\theta = \frac{180 + \text{their} \alpha}{2}, (or) \frac{360 + \text{their} \alpha}{2}, (or) \frac{540 + \text{their} \alpha}{2}
\]

A1 Obtains all four answers awrt 1dp. \( \theta = 7.8, 97.8, 187.8, 277.8 \).
Ignore any extra solutions outside the range.
Withhold this mark for extras inside the range.
Condone a different variable. Accept \( x = 7.8, 97.8, 187.8, 277.8 \)

Answers fully given in radians, loses the first A mark.
Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85
Mixed units can only score the first M1
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<tr>
<td>4(a)</td>
<td>$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe. Puts $f'(x) = 0$ to give $x = -1$ and $x = 0$ or one coordinate Obtains $(0, -16)$ and $(-1, 25e^2 - 16)$ CSO</td>
<td>M1A1 dM1A1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm \frac{4}{5}e^{-x}$</td>
<td>B1*</td>
</tr>
<tr>
<td>(c)</td>
<td>Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt} 0.485$ $\Rightarrow x_2 = \text{awrt} 0.492, x_3 = \text{awrt} 0.489$</td>
<td>M1A1 A1</td>
</tr>
<tr>
<td>(d)</td>
<td>$\alpha = 0.49$ $f(0.485) = -0.487, f(0.495) = (+)0.485$ , sign change and deduction</td>
<td>B1 B1</td>
</tr>
</tbody>
</table>

**Notes for Question 4**

No marks can be scored in part (a) unless you use differentiation as required by the question.

(a) M1 Uses $vu' + uv'$. If the rule is quoted it must be correct.

If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x} + Bxe^{2x}$

A1 $f'(x) = 50x^2e^{2x} + 50xe^{2x}$.

Allow un simplified forms such as $f'(x) = 25x^2 \times 2e^{2x} + 50x \times e^{2x}$

M1 Sets $f'(x) = 0$ , factorises out/ or cancels the $e^{2x}$ leading to at least one solution of $x$

This is dependent upon the first M1 being scored.

A1 Both $x = -1$ and $x = 0$ or one complete coordinate . Accept $(0, -16)$ and $(-1, 25e^2 - 16)$ or $(-1, \text{awrt} -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.

$x = -1, 0 \quad y = \frac{25}{e^2} - 16, -16$ or linked together by coordinate pairs $(0, -16)$ and $(-1, 25e^2 - 16)$ but the ‘pairs’ must be correct and exact.
Notes for Question 4 Continued

(b)
B1 This is a show that question and all elements must be seen
Candidates must 1) State that \( f(x) = 0 \) or writes \( 25x^2e^{3x} - 16 = 0 \) or \( 25x^2e^{2x} = 16 \)
2) Show at least one intermediate (correct) line with either
\[ x^2 \text{ or } x \text{ the subject. Eg } x^2 = \frac{16}{25}e^{-2x}, \quad x = \sqrt[5]{\frac{16}{25}}e^{-2x} \text{ oe} \]
or square rooting \( 25x^2e^{2x} = 16 \) \( \Rightarrow 5xe^x = \pm 4 \)
or factorising by DOTS to give \( (5xe^x + 4)(5xe^x - 4) = 0 \)
3) Show the given answer \( x = \pm \frac{4}{5}e^{-x} \).
Condone the minus sign just appearing on the final line.
A ‘reverse’ proof is acceptable as long as there is a statement that \( f(x) = 0 \)

(c)
M1 Substitutes \( x_0 = 0.5 \) into \( x = \frac{4}{5}e^{-x} \Rightarrow x_1 = ... \)
This can be implied by \( x_i = \frac{4}{5}e^{-0.5} \), or awrt 0.49
A1 \( x_1 = \text{awrt 0.485} \) 3dp. Mark as the first value given. Don’t be concerned by the subscript.
A1 \( x_2 = \text{awrt 0.492} \), \( x_3 = \text{awrt 0.489} \) 3dp. Mark as the second and third values given.

(d)
B1 States \( \alpha = 0.49 \)
B1 Justifies by

either calculating correctly \( f(0.485) \) and \( f(0.495) \) to awrt 1sf or 1dp,
\( f(0.485) = -0.5, f(0.495) = (+)0.5 \) rounded
\( f(0.485) = -0.4, f(0.495) = (+)0.4 \) truncated
giving a reason – accept change of sign, \( >0 <0 \) or \( f(0.485) \times f(0.495) < 0 \)
and giving a minimal conclusion. Eg. Accept hence root or \( \alpha = 0.49 \)
A smaller interval containing the root may be used, eg \( f(0.49) \) and \( f(0.495) \). Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of \( x_4 = \text{awrt 0.491} \) and stating (or seeing each value round to) 0.49
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<tr>
<td>5(a)</td>
<td>$\frac{dx}{dy} = 2 \times 3 \sec 3y \sec 3y \tan 3y = (6 \sec^2 3y \tan 3y)$</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6 \sec^2 3y \tan 3y}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\tan^2 3y = \sec^2 3y - 1 = x - 1$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just $x$.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^\frac{3}{2}}$</td>
<td>CSO</td>
</tr>
<tr>
<td></td>
<td>$\frac{d^2 y}{dx^2} = \frac{0 - [6(x-1)^{\frac{3}{2}} + 3x(x-1)^{\frac{1}{2}}]}{36x^2(x-1)^{\frac{5}{2}}}$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$\frac{d^2 y}{dx^2} = \frac{6 - 9x}{36x^2(x-1)^{\frac{5}{2}}} = \frac{2 - 3x}{12x^2(x-1)^{\frac{3}{2}}}$</td>
<td>dM1A1</td>
</tr>
<tr>
<td>(c)</td>
<td>$x = (\cos 3y)^{-2}$ \Rightarrow $\frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3 \sin 3y$</td>
<td>M1A1</td>
</tr>
<tr>
<td>Alt 1 to 5(a)</td>
<td>$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3 \sec 3y \tan 3y + 3 \sec 3y \times \sec 3y \tan 3y$</td>
<td>M1A1</td>
</tr>
<tr>
<td>Alt 2 to 5(a)</td>
<td>$\frac{d^2 y}{dx^2} = \frac{\frac{1}{6}[x^{-\frac{1}{2}}(-\frac{1}{2})(x-1)^{-\frac{1}{2}} + (-1)x^{-2}(x-1)^{-\frac{3}{2}}]}{x^2(x-1)^{\frac{3}{2}}}$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{\frac{1}{6}}{x^2(x-1)^{\frac{3}{2}}}[x(-\frac{1}{2}) + (-1)(x-1)]$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{12} x^2(x-1)^{-\frac{3}{2}}[2 - 3x]$</td>
<td>oe</td>
</tr>
<tr>
<td>(10 marks)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Alt 1 To 5(c)   | | |
| Alt 1 To 5(c)   | | |
Notes for Question 5

(a) 
M1 Uses the chain rule to get \( A \sec^3 y \sec^3 y \tan^3 y = \left( A \sec^2 3y \tan 3y \right) \).

There is no need to get the lhs of the expression. Alternatively could use the chain rule on \((\cos 3y)^2 \Rightarrow A(\cos 3y)^{-3} \sin 3y\)
or the quotient rule on \(\frac{1}{(\cos 3y)^2} \Rightarrow \pm A \cos 3y \sin 3y \over (\cos 3y)^4\)

A1 \(\frac{dx}{dy} = 2 \times 3 \sec^3 y \sec^3 y \tan 3y \) or equivalent. There is no need to simplify the rhs but both sides must be correct.

(b) 
M1 Uses \(\frac{dy}{dx} = \frac{1}{dx} \) to get an expression for \(\frac{dy}{dx} \). Follow through on their \(\frac{dx}{dy}\)

Allow slips on the coefficient but not trig expression.

B1 Writes \(\tan^2 3y = \sec^2 3y - 1\) or an equivalent such as \(\tan 3y = \sqrt{\sec^2 3y - 1}\) and uses \(x = \sec^2 3y\) to obtain either \(\tan^2 3y = x - 1\) or \(\tan 3y = -1\) \(x - 1\) \(\frac{1}{2}\)

All elements must be present.

Accept \[\sqrt{x}
\]

\[\begin{array}{c}
\sqrt{x} \\
\sqrt{x - 1} \\
y \\
\cos 3y = \frac{1}{\sqrt{x}} \\
\tan 3y = \sqrt{x - 1}
\end{array}\]

If the differential was in terms of \(\sin 3y, \cos 3y\) it is awarded for \(\sin 3y = \frac{\sqrt{x - 1}}{\sqrt{x}}\)

M1 Uses \(\sec^2 3y = x\) and \(\tan^2 3y = \sec^2 3y - 1 = x - 1\) or equivalent to get \(\frac{dy}{dx}\) in just \(x\). Allow slips on the signs in \(\tan^2 3y = \sec^2 3y - 1\).

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. \(\frac{dx}{dy} = 6 \sec^2 3y \tan 3y = 6x(x - 1)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{6x(x - 1)^{\frac{1}{2}}}\)

Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0
Example 1- Scores 0 marks in part (b)

\[ \frac{dx}{dy} = 6 \sec^2 3y \tan 3y \Rightarrow \frac{dy}{dx} = \frac{1}{6 \sec^2 3x \tan 3x} = \frac{1}{6 \sec^2 3x \sqrt{\sec^2 3x - 1}} = \frac{1}{6x(x-1)^{\frac{1}{2}}} \]

Example 2- Scores M1B1M1A0

\[ \frac{dx}{dy} = 2 \sec^2 3y \tan 3y \Rightarrow \frac{dy}{dx} = \frac{1}{2 \sec^2 3y \tan 3y} = \frac{1}{2 \sec^2 3y \sqrt{\sec^2 3y - 1}} = \frac{1}{2x(x-1)^{\frac{1}{2}}} \]

(c) Using Quotient and Product Rules

M1 Uses the quotient rule \( \frac{vu' - uv'}{v^2} \) with \( u = 1 \) and \( v = 6x(x-1)^{\frac{1}{2}} \) and achieving

\[ u' = 0 \text{ and } v' = A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}. \]

If the formulae are quoted, both must be correct. If they are not quoted nor implied by their working allow expressions of the form

\[
\frac{d^2y}{dx^2} = \frac{0 - [A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}
\]

or

\[
\frac{d^2y}{dx^2} = \frac{0 - A(x-1)^{\frac{1}{2}} \pm Bx(x-1)^{-\frac{1}{2}}}{C(x-1)^2}
\]

A1 Correct un simplified expression

\[
\frac{d^2y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)} \quad \text{oe}
\]

dM1 Multiply numerator and denominator by \((x-1)^{\frac{1}{2}}\) producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of \((x-1)^{-\frac{1}{2}}\) from the numerator and collect like terms from the linear expression.

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression

\[
\frac{d^2y}{dx^2} = \frac{2 - 3x}{12x^2(x-1)^{\frac{1}{2}}} \quad \text{oe}
\]
(c) Using Product and Chain Rules

M1 Writes \( \frac{dy}{dx} = \frac{1}{6x(x-1)^2} \) and uses the product rule with \( u = A \) and \( v = A^{-1} \) and

\[
\frac{1}{3} \quad \frac{1}{2} \qquad (x-1) \quad (x-1)^{\frac{1}{2}} \quad \text{If any rule is quoted it must be correct.}
\]

If the rules are not quoted nor implied then award if you see an expression of the form

\[
(x-1)^{\frac{1}{2}} \times Bx^{-1} \pm C(x-1)^{\frac{1}{2}} \times x^{-2}
\]

A1 \( \frac{d^2y}{dx^2} = \frac{1}{6} [(-\frac{1}{2})(x-1)^{\frac{3}{2}} + (-1)x^{-2}(x-1)^{\frac{1}{2}}] \)

dM1 Factorises out / uses a common denominator of \( x^{-2}(x-1)^{\frac{1}{2}} \) producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression \( \frac{d^2y}{dx^2} = \frac{1}{12x^2} (2 - 3x) \quad oe \)

(c) Using Quotient and Chain rules Rules

M1 Uses the quotient rule \( \frac{vu' - uv'}{v^2} \) with \( u = (x-1)^{-\frac{1}{2}} \) and \( v = 6x \) and achieving

\[
u' = A(x-1)^{-\frac{3}{2}} \quad \text{and} \quad v' = B.
\]

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

\[
\left( \frac{d^2y}{dx^2} \right) = \frac{Cx(x-1)^{\frac{3}{2}} - D(x-1)^{\frac{1}{2}}}{Ex^{2}}
\]

A1 Correct un simplified expression \( \frac{d^2y}{dx^2} = \frac{6x^2 - \frac{1}{2}(x-1)^{\frac{3}{2}} - (x-1)^{\frac{1}{2}} \times 6}{(6x)^2} \)

dM1 Multiply numerator and denominator by \((x-1)^{\frac{3}{2}} \) producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of \( x^{-\frac{1}{2}} \) from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression \( \frac{d^2y}{dx^2} = \frac{2 - 3x}{12x^2(x-1)^{\frac{1}{2}}} \quad oe \quad \frac{d^2y}{dx^2} = \frac{(2 - 3x)x^{-2}(x-1)^{\frac{1}{2}}}{12} \)
(c) Using just the chain rule

M1 Writes \( \frac{dy}{dx} = \frac{1}{6x(x-1)^2} = \frac{1}{(36x^3 - 36x^2)^{\frac{1}{2}}} \) and proceeds by the chain rule to

\[ A(36x^3 - 36x^2)^{\frac{3}{2}} (Bx^2 - Cx). \]

M1 Would automatically follow under this method if the first M has been scored.
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<td><strong>6(a)</strong></td>
<td>$\ln(4-2x)(9-3x) = \ln(x+1)^2$</td>
<td>M1, M1</td>
</tr>
<tr>
<td></td>
<td>$\text{So } 36-30x+6x^2 = x^2+2x+1 \text{ and } 5x^2-32x+35 = 0$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Solve $5x^2-32x+35 = 0 \text{ to give } x = \frac{7}{5} \text{ oe (Ignore the solution } x = 5)$</td>
<td>M1A1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Take logs' to give $\ln 2^x + \ln e^{3x+1} = \ln 10$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x \ln 2 + (3x+1) \ln e = \ln 10$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x(\ln 2 + 3 \ln e) = \ln 10 - \ln e \Rightarrow x = ..$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>and uses $\ln e = 1$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{-1+\ln 10}{3 + \ln 2}$</td>
<td>A1</td>
</tr>
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Note that the 4th M mark may occur on line 2 (10 marks)

Notes for Question 6

(a) M1 Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.

M1 Uses power rule for logs write the $2 \ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets

A1 Undoes the logs to obtain the 3TQ =0. $5x^2 - 32x + 35 = 0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.

M1 Solves a quadratic by any allowable method. The quadratic cannot be a version of $(4-2x)(9-3x) = 0$ however.

A1 Deduces $x = 1.4$ or equivalent. Accept both $x=1.4$ and $x=5$. Candidates do not have to eliminate $x = 5$. You may ignore any other solution as long as it is not in the range $-1 < x < 2$. Extra solutions in the range scores A0.
(b)

M1 Takes logs of both sides and splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides and using the subtraction law.

M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1

\[ \ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x \ln 2 \times (3x + 1) \ln e = \ln 10 \]

dM1 This is dependent upon both previous two M’s being scored. It can be awarded for a full method to solve their linear equation in \( x \). The terms in \( x \) must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to \( x = \ldots \)

M1 Uses \( \ln e = 1 \). This could appear in line 2, but it must be part of their equation and not just a statement.

Another example where it could be awarded is

\[ e^{3x+1} = \frac{10}{2^x} \Rightarrow 3x + 1 = \ldots \]

A1 Obtains answer

\[ x = \frac{-1 + \ln 10}{3 + \ln 2} = \left( \frac{\ln 10 - 1}{3 + \ln 2} \right) = \left( \frac{\log_{10} 10 - 1}{3 + \log_{10} 2} \right) \text{ OE}. \text{ DO NOT ISW HERE} \]

Note 1: If the candidate takes \( \log_{10} \)'s of both sides can score M1M1dM1M0A0 for 3 out of 5.

\[ \text{Answer} = x = \frac{-\log e + \log 10}{3 \log e + \log 2} = \left( \frac{-\log e + 1}{3 \log e + \log 2} \right) \]

Note 2: If the candidate writes \( x = \frac{-1 + \log 10}{3 + \log 2} \) without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.
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<td>Alt 1 to 6(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>W</strong>rites l<strong>h</strong>s in e’s</td>
<td>$2^x e^{3x+1} = 10 \Rightarrow e^{\ln 2} e^{3x+1} = 10$</td>
<td>$1st$ M1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow e^{\ln 2+3x+1} = 10, \quad x \ln 2 + 3x + 1 = \ln 10$</td>
<td>$2nd$ M1, $4th$ M1</td>
</tr>
<tr>
<td></td>
<td>$x(\ln 2+3) = \ln 10 - 1 \Rightarrow x = ..$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{-1 + \ln 10}{3 + \ln 2}$</td>
<td>A1 (5)</td>
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</table>

**Notes for Question 6 Alt 1**

- M1: Writes the lhs of the expression in e’s. Seeing $2^x = e^{\ln 2}$ in their equation is sufficient
- M1: Uses the addition law on the lhs to produce a single exponential
- dM1: Takes ln’s of both sides to produce and attempt to solve a linear equation in $x$
  You may condone slips in signs for this mark but the process must be correct leading to $x = ..$
- M1: Uses ln $e = 1$. This could appear in line 2
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<tr>
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<th>Marks</th>
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<tbody>
<tr>
<td>7(a)</td>
<td>$0 \leq f(x) \leq 10$</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(b)</td>
<td>$ff(0) = f(5), = 3$</td>
<td>B1,B1</td>
</tr>
<tr>
<td>(c)</td>
<td>$y = \frac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow 5y-4 = xy+3x$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \frac{5y-4}{y+3}$</td>
<td>A1</td>
</tr>
<tr>
<td>(d)</td>
<td>$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4\text{ oe}$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$f(x) = 4 \Rightarrow x = 6$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4\text{ oe}$</td>
<td>M1A1</td>
</tr>
<tr>
<td>Alt 1 to 7(d)</td>
<td>$gf(x) = 16 \Rightarrow \frac{4+3(ax+b)}{5-(ax+b)} = 16$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$ax+b = x-2 \text{ or } 5-2.5x$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow x = 6$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\frac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = ...$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow x = 0.4\text{ oe}$</td>
<td>A1 (5)</td>
</tr>
</tbody>
</table>
Notes for Question 7

(a)  
**B1**  Correct range. Allow $0 \leq f(x) \leq 10$, $0 \leq f(x) \leq 10$, $0 \leq y \leq 10$, $0 \leq \text{range} \leq 10$, $[0,10]$  
Allow $f(x) \geq 0$ and $f(x) \leq 10$ but not $f(x) \geq 0$ or $f(x) \leq 10$  
Do Not Allow $0 \leq x \leq 10$. The inequality must include BOTH ends.

(b)  
**B1**  For correct one application of the function at $x=0$  
Possible ways to score this mark are  
$f(0) = 5$, $f(5) \rightarrow 5 \rightarrow ...$  
**B1:** 3  (‘3’ can score both marks as long as no incorrect working is seen.)

(c)  
**M1**  For an attempt to make $x$ or a replaced $y$ the subject of the formula. This can be scored for putting $y = g(x)$, multiplying across, expanding and collecting $x$ terms on one side of the equation. Condone slips on the signs.

**dM1**  Take out a common factor of $x$ (or a replaced $y$) and divide, to make $x$ subject of formula. Only allow one sign error for this mark.

**A1**  Correct answer. No need to state the domain. Allow $g^{-1}(x) = \frac{5x-4}{3+x}$  
$y = \frac{5x-4}{3+x}$  
Accept alternatives such as $y = \frac{4-5x}{-3-x}$ and $y = \frac{5-4}{3+x}$

(d)  
**M1**  Stating or implying that $f(x) = g^{-1}(16)$. For example accept $\frac{4+3f(x)}{5-f(x)} = 16 \Rightarrow f(x) = ...$  
**A1**  Stating $f(x) = 4$ or implying that solutions are where $f(x) = 4$  
**B1**  $x = 6$ and may be given if there is no working  
**M1**  Full method to obtain other value from line $y = 5 - 2.5x$  
$5 - 2.5x = 4 \Rightarrow x = ...$  
Alternatively this could be done by similar triangles. Look for $\frac{2}{5} = \frac{2-x}{4} (oe) \Rightarrow x = ...$  
**A1** 0.4 or 2/5  
**Alt 1 to (d)**  
**M1**  Writes $gf(x) = 16$ with a linear $f(x)$. The order of $gf(x)$ must be correct.  
Condone invisible brackets. Even accept if there is a modulus sign.

**A1**  Uses $f(x) = x - 2$ or $f(x) = 5 - 2.5x$ in the equation $gf(x) = 16$  
**B1**  $x = 6$ and may be given if there is no working  
**M1**  Attempt at solving $\frac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = ...$. The bracketing must be correct and there must be no more than one error in their calculation  
**A1**  $x = 0.4, \frac{2}{5}$ or equivalent
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<th>Question Number</th>
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<tbody>
<tr>
<td>8(a)</td>
<td>$R = \sqrt{7^2 + 24^2} = 25$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\tan \alpha = \frac{24}{7}$, $\Rightarrow \alpha = \text{awrt} 73.74^\circ$</td>
<td>M1A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Maximum value of $24\sin x + 7\cos x = 25$ so $V_{\text{min}} = \frac{21}{25} = (0.84)$</td>
<td>M1A1</td>
</tr>
<tr>
<td>(c)</td>
<td>Distance $AB = \frac{7}{\sin \theta}$, with $\theta = \alpha$</td>
<td>M1, B1</td>
</tr>
<tr>
<td></td>
<td>So distance = $7.29\text{m} = \frac{175}{24}\text{m}$</td>
<td>A1</td>
</tr>
<tr>
<td>(d)</td>
<td>$R \cos(\theta - \alpha) = \frac{21}{1.68} \Rightarrow \cos(\theta - \alpha) = 0.5$</td>
<td>M1, A1</td>
</tr>
<tr>
<td></td>
<td>$\theta - \alpha = 60 \Rightarrow \theta = .., \theta - \alpha = -60 \Rightarrow \theta = ..$</td>
<td>dM1, dM1</td>
</tr>
<tr>
<td></td>
<td>$\theta = \text{awrt} 133.7, 13.7$</td>
<td>A1, A1</td>
</tr>
</tbody>
</table>

Notes for Question 8

(a)
- B1 25. Accept 25.0 but not $\sqrt{625}$ or answers that are not exactly 25. Eg 25.0001
- M1 For $\tan \alpha = \pm \frac{24}{7}$, $\tan \alpha = \pm \frac{7}{24}$.
  If the value of R is used only accept $\sin \alpha = \pm \frac{24}{R}$, $\cos \alpha = \pm \frac{7}{R}$
- A1 Accept answers which round to 73.74 – must be in degrees for this mark

(b)
- M1 Calculates $V = \frac{21}{\text{their } R}$ NOT - R
- A1 Obtains correct answer. $V = \frac{21}{25}$ Accept 0.84
  Do not accept if you see incorrect working- ie from $\cos(\theta - \alpha) = -1$ or the minus just disappearing from a previous line.
  Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate $V$ by the quotient rule (or similar), set $V' = 0$ to find $\theta$ and then sub this back into $V$ to find its value.
(c) M1 Uses the trig equation $\sin \theta = \frac{7}{AB}$ with a numerical $\theta$ to find $AB = ...$

B1 Uses $\theta$ = their value of $\alpha$ in a trig calculation involving sin. ($\sin \alpha = \frac{AB}{7}$ is condoned)

A1 Obtains answer $\frac{175}{24}$ or awrt 7.29

(d) M1 Substitutes $V = 1.68$ and their answer to part (a) in $V = \frac{21}{24 \sin \theta + 7 \cos \theta}$ to get an equation of the form $R \cos(\theta \pm \alpha) = \frac{21}{1.68}$ or $1.68 R \cos(\theta \pm \alpha) = 21$ or $\cos(\theta \pm \alpha) = \frac{21}{1.68 R}$.
    Follow through on their $R$ and $\alpha$

A1 Obtains $\cos(\theta \pm \alpha) = 0.5$ oe. Follow through on their $\alpha$. It may be implied by later working.

dM1 Obtains one value of $\theta$ in the range $0 < \theta < 150$ from inverse cos +their $\alpha$
    It is dependent upon the first M being scored.

dM1 Obtains second angle of $\theta$ in the range $0 < \theta < 150$ from inverse cos +their $\alpha$
    It is dependent upon the first M being scored.

A1 one correct answer awrt $\theta = 133.7$ or 13.7 $1dp$

A1 both correct answers awrt $\theta = 133.7$ and 13.7 $1dp$.

Extra solutions in the range loses the last A1.
Answers in radians, lose the first time it occurs. Answers must be to 3dp
For your info $\alpha = 1.287, \theta_1 = 2.334, \theta_2 = 0.240$
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme
**General Principles for Core Mathematics Marking**

(But note that specific mark schemes may sometimes override these general principles).

**Method mark for solving 3 term quadratic:**

1. **Factorisation**
   
   \[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \]

   \[ax^2 + bx + c = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \]

2. **Formula**
   
   Attempt to use correct formula (with values for \(a\), \(b\) and \(c\)).

3. **Completing the square**

   Solving \(x^2 + bx + c = 0\):
   
   \[
   \left( x \pm \frac{b}{2} \right)^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to } x = ...
   \]

**Method marks for differentiation and integration:**

1. **Differentiation**
   
   Power of at least one term decreased by 1. \((x^n \rightarrow x^{n-1})\)

2. **Integration**
   
   Power of at least one term increased by 1. \((x^n \rightarrow x^{n+1})\)

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:

- **Method mark** for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
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<tr>
<td>1. (a)</td>
<td>$\int x^2e^x , dx$, 1st Application: $\begin{cases} u = x^2 \Rightarrow \frac{du}{dx} = 2x \ v = e^x \Rightarrow \frac{dv}{dx} = e^x \end{cases}$, 2nd Application: $\begin{cases} u = x \Rightarrow \frac{du}{dx} = 1 \ v = e^x \Rightarrow \frac{dv}{dx} = e^x \end{cases}$</td>
<td>[ x^2e^x - \int \lambda xe^x , dx, \lambda &gt; 0 ] M1 [ x^2e^x - \int 2xe^x , dx ] A1 oe [ = x^2e^x - 2 \left( xe^x - \int e^x , dx \right) ] [ = x^2e^x - 2(xe^x - e^x) { + c } ] [ \left{ \left[ x^2e^x - 2(xe^x - e^x) \right]_0^1 \right} ] [ = (1^2e^1 - 2(1e^1 - e^1)) - (0^2e^0 - 2(0e^0 - e^0)) ] [ = e - 2 ] [5] [ \text{Either } \pm Ax^2e^x \pm Bxe^x \pm C \int e^x , dx ] M1 [ \text{or for } \pm K \int xe^x , dx \rightarrow \pm K \left( xe^x - \int e^x , dx \right) ] M1 [ \pm Ax^2e^x \pm Bxe^x \pm Ce^x ] A1 [ \text{Correct answer, with/without } + c ] [2] [ \text{Applying limits of 1 and 0 to an expression of the form } \pm Ax^2e^x \pm Bxe^x \pm Ce^x, \ A \neq 0, \ B \neq 0 \text{ and } C \neq 0 \text{ and subtracts the correct way round.} ] [5] [ \text{Correct answer, with/without } + c ] A1 oe [ \text{Evidence of a proper consideration of the limit of 0 (as detailed above) is needed for M1.} ] [5] [ \text{So, just subtracting zero is M0.} ] [5] [ \text{A1: } e - 2 \text{ or } e^1 - 2 \text{ or } -2 + e. \text{ Do not allow } e - 2e^0 \text{ unless simplified to give } e - 2. ] [2] [ \text{Note: } 0.718... \text{ without seeing } e^2 \text{ is A0.} ] [2] [ \text{WARNING: } \text{Please note that this A1 mark is for correct solution only.} ] [2] [ \text{So incorrect } [\ldots...]_0^1 \text{ leading to } e - 2 \text{ is A0.} ] [2] [ \text{Note: } 0.718... \text{ from no working is M0A0} ] [2]</td>
</tr>
<tr>
<td>Question Number</td>
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<td>Marks</td>
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<tr>
<td>2. (a)</td>
<td>[ \left( \frac{1+x}{1-x} \right)^{\frac{1}{2}} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} ] ( (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>[ = \left( 1 + \frac{1}{2} \right) x + \frac{1}{8} x^2 + ... ] ( x ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 )</td>
<td>See notes</td>
</tr>
<tr>
<td></td>
<td>[ = \left( 1 + \frac{1}{2} \right) x - \frac{1}{8} x^2 + ... ] ( x ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 )</td>
<td>See notes</td>
</tr>
<tr>
<td></td>
<td>[ = 1 + \frac{1}{2} x + \frac{3}{8} x^2 + \frac{1}{2} x + \frac{1}{4} x^2 - \frac{1}{8} x^2 + ... ] ( x ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[ = 1 + x + \frac{1}{2} x^2 ] ( x ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 )</td>
<td>Answer is given in the question.</td>
</tr>
<tr>
<td></td>
<td>( \left( 1 + \frac{1}{2} x \right)^{\frac{1}{2}} ) ( 1 + \frac{1}{2} x ) ( \frac{1}{2} ) ( 2! ) ( x^2 ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ = 1 + \left( \frac{1}{26} \right) + \left( \frac{1}{2} \right)^2 ] ( 1 ) ( \frac{1}{26} ) ( \frac{1}{2} ) ( 2! ) ( x^2 ) or ( \frac{1}{4} x^2 ) ( 2! ) ( x^2 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{3} = \frac{3\sqrt{3}}{5} = 1405 ) ( 1352 ) ( 5 ) ( 3 \sqrt{3} )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{3} = \frac{7025}{4056} )</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Notes for Question 2**

(a) **B1:** \( (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} \) or \( \sqrt{(1+x)(1-x)^{-\frac{1}{2}}} \) seen or implied. (Also allow \( (1+x)(1-x)^{-\frac{1}{2}} \)).

**M1:** Expands \( (1+x)^{\frac{1}{2}} \) to give any 2 out of 3 terms simplified or un-simplified,

Eg: \( 1 + \frac{1}{2} x \) or \( 1 + \left( \frac{1}{2} \right) x + \frac{1}{8} x^2 \) or \( 1 + \left( \frac{1}{2} \right) x + \frac{1}{8} x^2 \)

or expands \( (1-x)^{-\frac{1}{2}} \) to give any 2 out of 3 terms simplified or un-simplified,

Eg: \( 1 + \left( \frac{1}{2} \right) \left( -x \right) \) or \( 1 + \left( \frac{1}{2} \right) \left( -x \right) + \frac{1}{8} \left( -x \right)^2 \) or \( 1 + \left( \frac{1}{2} \right) \left( -x \right) + \frac{1}{8} \left( -x \right)^2 \)

Also allow: \( 1 + \left( \frac{1}{2} \right) \left( -x \right) \left( -\frac{3}{2} \right) \) \( x^2 \) for M1.

**A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore \( x^3 \) and \( x^4 \) terms)

**A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore \( x^3 \) and \( x^4 \) terms)

**Note:** Candidates can give decimal equivalents when expanding out their binomial expansions.

**M1:** Multiplies out to give 1, exactly two terms in \( x \) and exactly three terms in \( x^2 \).

**A1:** Candidate achieves the result on the exam paper. Make sure that their working is sound.

**Special Case:** Award SC FINAL M1A1 for a correct \( \left( 1 + \frac{1}{2} x - \frac{1}{8} x^2 + ... \right) \times \left( \frac{1}{2} x + \frac{3}{8} x^2 + ... \right) \) multiplied out with no errors to give either \( 1 + x + \frac{3}{8} x^3 + \frac{1}{4} x^2 - \frac{1}{8} x^2 \) \( 1 + \frac{1}{2} x + \frac{3}{8} x^2 + \frac{1}{2} x + \frac{1}{8} x^2 \) or \( 1 + \frac{1}{2} x + \frac{1}{4} x^2 + \frac{1}{2} x + \frac{1}{4} x^2 \) leading to the correct answer of \( 1 + x + \frac{1}{2} x^2 \).
Notes for Question 2 Continued

2. (a) ctd

Note: If a candidate writes down either \((1 + x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots\) or \((1 - x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \ldots\) with no working then you can award 1st M1, 1st A1.

Note: If a candidate writes down both correct binomial expansions with no working, then you can award 1st M1, 1st A1, 2nd A1.

(b) M1: Substitutes \(x = \frac{1}{26}\) into both sides of \(\sqrt{\frac{1 + x}{1 - x}}\) and \(1 + x + \frac{1}{2}x^2\)

B1: For sight of \(\frac{27}{25}\) (or better) and \(\frac{1405}{1352}\) or equivalent fraction

Eg: \(\frac{3\sqrt{3}}{5}\) and \(\frac{1405}{1352}\) or \(0.6\sqrt{3}\) and \(\frac{1405}{1352}\) or \(\frac{3\sqrt{3}}{5}\) and \(1\frac{53}{1352}\) or \(\sqrt{3}\) and \(\frac{5}{3}(\frac{1405}{1352})\) are fine for B1.

A1: \(\frac{7025}{4056}\) or any equivalent fraction, eg: \(\frac{14050}{8112}\) or \(\frac{182650}{105456}\) etc.

Special Case: Award SC: M1B1A0 for \(\sqrt{3} \approx 1.732001972\ldots\) or truncated 1.732001 or awrt 1.732002.

Note that \(\frac{7025}{4056} = 1.732001972\ldots\) and \(\sqrt{3} = 1.732050808\ldots\)

Aliter

2. (a) Way 2

\[
\begin{align*}
\left\{ \sqrt{\frac{1 + x}{1 - x}} \right\} & = \frac{(1 + x)(1 - x)^{\frac{1}{2}}}{(1 + x)(1 - x)^{\frac{1}{2}}} = (1 - x^2)^{\frac{1}{2}}(1 - x)^{-1} \quad \text{B1} \\
& = \left(1 + \frac{1}{2}(-x^2) + \ldots\right) \times \left(1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2 + \ldots\right) \quad \text{See notes M1A1A1} \\
& = \left(1 - \frac{1}{2}x^2 + \ldots\right) \times \left(1 + x + x^2 + \ldots\right) \quad \text{See notes M1} \\
& = 1 + x + x^2 - \frac{1}{2}x^2 \\
& = 1 + x + \frac{1}{2}x^2 \\
& \quad \text{Answer is given in the question. A1 *}
\end{align*}
\]

[6]

Aliter

2. (a) Way 2

B1: \((1 - x^2)^{\frac{1}{2}}(1 - x)^{-1}\) seen or implied.

M1: Expands \((1 - x^2)^{\frac{1}{2}}\) to give both terms simplified or un-simplified, \(1 + \left(\frac{1}{2}\right)(-x^2)\)

or expands \((1 - x)^{-1}\) to give any 2 out of 3 terms simplified or un-simplified,

Eg: \(1 + (-1)(-x) + \ldots + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2 + \ldots\) or \(1 + \ldots + \frac{(-1)(-2)}{2!}(-x)^2\)

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore \(x^3\) and \(x^4\) terms)

A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore \(x^3\) and \(x^4\) terms)

M1: Multiplies out to give 1, exactly one term in \(x\) and exactly two terms in \(x^2\).

A1: Candidate achieves the result on the exam paper. Make sure that their working is sound.
Notes for Question 2 Continued

| Aliter 2. (a) | \( \begin{align*} 
\sqrt{\frac{1 + x}{1 - x}} = & (1 + x)(1 - x)^{-1/2} \\
\frac{(1 + x)(1 + x)}{(1 - x)(1 + x)} = & (1 + x)(1 - x)^{-1/2} \\
= & (1 + x) \left[ 1 + \frac{1}{2} x^2 + \ldots \right] \\
= & 1 + x + \frac{1}{2} x^2 
\end{align*} \) | B1

Note: The final M1 mark is dependent on the previous method mark for Way 3.

| Aliter 2. (a) | \( \begin{align*} 
\sqrt{\frac{1 + x}{1 - x}} = & \frac{\sqrt{1 + x}}{\sqrt{1 - x}} = 1 + x + \frac{1}{2} x^2 \\
\Rightarrow & (1 + x)^{\frac{1}{2}} = \left( 1 + x + \frac{1}{2} x^2 \right) (1 - x)^{\frac{1}{2}} \\
(1 + x)^{\frac{1}{2}} = & 1 + \left( \frac{1}{2} \right) x + \frac{\left( \frac{1}{2} \right)(-\frac{1}{2})}{2!} x^2 + \ldots \\
& \left\{ = 1 + \frac{1}{2} x - \frac{1}{8} x^2 + \ldots \right\} , \\
(1 - x)^{\frac{1}{2}} = & 1 + \left( \frac{1}{2} \right) (-x) + \frac{\left( \frac{1}{2} \right)(-\frac{1}{2})}{2!} (-x)^2 + \ldots \\
& \left\{ = 1 - \frac{1}{2} x - \frac{1}{8} x^2 + \ldots \right\} \\
\text{RHS} = & \left( 1 + x + \frac{1}{2} x^2 \right) (1 - x)^{\frac{1}{2}} = \left( 1 + x + \frac{1}{2} x^2 \right) \left( 1 - \frac{1}{2} x - \frac{1}{8} x^2 + \ldots \right) \\
& = 1 - \frac{1}{2} x - \frac{1}{8} x^2 + x - \frac{1}{2} x^2 + \frac{1}{2} x^2 \\
& = 1 + \frac{1}{2} x - \frac{1}{8} x^2 \\
\text{So, LHS} = & 1 + \frac{1}{2} x - \frac{1}{8} x^2 = \text{RHS} 
\end{align*} \) | B1, M1, A1

B1: \( (1 + x)^{\frac{1}{2}} = \left( 1 + x + \frac{1}{2} x^2 \right) (1 - x)^{\frac{1}{2}} \) seen or implied.

M1: For Way 4, this M1 mark is dependent on the first B1 mark.

Expands \( (1 + x)^{\frac{1}{2}} \) to give any 2 out of 3 terms simplified or un-simplified,

Eg: \( 1 + \frac{1}{2} x \) or \( + \left( \frac{1}{2} \right) x + \frac{\left( \frac{1}{2} \right)(-\frac{1}{2})}{2!} x^2 \) or \( 1 + \ldots + \frac{\left( \frac{1}{2} \right)(-\frac{1}{2})}{2!} x^2 \)

or expands \( (1 - x)^{\frac{1}{2}} \) to give any 2 out of 3 terms simplified or un-simplified,

Eg: \( 1 + \left( \frac{1}{2} \right) (-x) \) or \( + \left( \frac{1}{2} \right) (-x) + \frac{\left( \frac{1}{2} \right)(-\frac{1}{2})}{2!} (-x)^2 \) or \( 1 + \ldots + \frac{\left( \frac{1}{2} \right)(-\frac{1}{2})}{2!} (-x)^2 \)

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore \( x^3 \) and \( x^4 \) terms)

A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore \( x^3 \) and \( x^4 \) terms)

M1: For Way 4, this M1 mark is dependent on the first B1 mark.

Multiplies out RHS to give 1, exactly two terms in \( x \) and exactly three terms in \( x^2 \).

A1: Candidate achieves the result on the exam paper. Candidate needs to have correctly processed both

the LHS and RHS of \( (1 + x)^{\frac{1}{2}} = \left( 1 + x + \frac{1}{2} x^2 \right) (1 - x)^{\frac{1}{2}} \).
3. (a) 1.154701

(b) \( \text{Area} \approx \frac{1}{2} \times \pi \times 6.794168 = 1.778709023... = 1.7787 \) (4 dp)

(c) \[ V = \pi \int_0^\frac{\pi}{2} \left( \sec \left( \frac{x}{2} \right) \right)^2 \, dx \]

\[ = \{ \pi \} \left[ 2 \tan \left( \frac{x}{2} \right) \right]_0^{\frac{\pi}{2}} \]

\[ = 2\pi \]

**Notes for Question 3**

(a) **B1:** 1.154701 correct answer only. Look for this on the table or in the candidate’s working.

(b) **B1:** Outside brackets \( \frac{1}{2} \times \frac{\pi}{6} \) or \( \frac{\pi}{12} \) or awrt 0.262

**M1:** For structure of trapezium rule [ ............. ]

**A1:** anything that rounds to 1.7787

**Note:** It can be possible to award: (a) B0 (b) B1M1A1 (awrt 1.7787)

**Note:** Award B1M1A1 for \( \frac{\pi}{12} (1 + 1.414214 + \frac{\pi}{6} (1.035276 + their 1.154701) = 1.778709023... \)

**Bracketing mistake:** Unless the final answer implies that the calculation has been done correctly,

Award B1M0A0 for \( \frac{1}{2} \times \frac{\pi}{6} + 1 + 2\left(1.035276 + their 1.154701\right)+1.414214 \) (nb: answer of 7.0596...).

Award B1M0A0 for \( \frac{1}{2} \times \frac{\pi}{6} (1 + 1.414214) + 2(1.035276 + their 1.154701) \) (nb: answer of 5.01199...).

**Alternative method for part (b): Adding individual trapezia**

Area \( \approx \frac{\pi}{6} \times \left[ \frac{\pi}{6} \left( \frac{1+1.035276}{2} \right) + \frac{1.035276+1.154701}{2} + \frac{1.154701+1.414214}{2} \right] = 1.778709023... \)

**B1:** \( \frac{\pi}{6} \) and a divisor of 2 on all terms inside brackets.

**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.

**A1:** anything that rounds to 1.7787
### 3. (e)

**B1:** For a correct statement of \( \pi \int \left( \sec \left( \frac{x}{2} \right) \right)^2 \) or \( \pi \int \sec^2 \left( \frac{x}{2} \right) \) or \( \pi \int \frac{1}{\left( \cos \left( \frac{x}{2} \right) \right)^2} \{dx\} \).

Ignore limits and \( dx \). Can be implied.

Note: Unless a correct expression stated \( \pi \int \sec \left( \frac{x^2}{4} \right) \) would be B0.

**M1:** \( \pm \lambda \tan \left( \frac{x}{2} \right) \) from any working.

**A1:** \( 2 \tan \left( \frac{x}{2} \right) \) or \( \frac{1}{(\lambda)} \tan \left( \frac{x}{2} \right) \) from any working.

**A1:** \( 2\pi \) from a correct solution only.

**Note:** The \( \pi \) in the volume formula is only required for the B1 mark and the final A1 mark.

**Note:** Decimal answer of 6.283... without correct exact answer is A0.

**Note:** The B1 mark can be implied by later working – as long as it is clear that the candidate has applied \( \pi \int y^2 \) in their working.

**Note:** Writing the correct formula of \( V = \pi \int y^2 \{dx\} \), but incorrectly applying it is B0.
4. \( x = 2 \sin t, \quad y = 1 - \cos 2t \quad \{ = 2 \sin^2 t \}, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \)

(a) \( \frac{dx}{dt} = 2 \cos t, \quad \frac{dy}{dt} = 2 \sin 2t \) or \( \frac{dy}{dt} = 4 \sin t \cos t \)

So, \( \frac{dy}{dx} = \frac{2 \sin 2t}{2 \cos t} = \frac{4 \cos t \sin t}{2 \cos t} = 2 \sin t \).

At \( t = \frac{\pi}{6} \), \( \frac{dy}{dx} = \frac{2 \sin \left( \frac{2\pi}{6} \right)}{2 \cos \left( \frac{\pi}{6} \right)} = \frac{1}{1} = 1 \)

(b) \( y = 1 - \cos 2t = 1 - (1 - 2 \sin^2 t) = 2 \sin^2 t \)

So, \( y = 2 \left( \frac{x^2}{2} \right) \) or \( y = \frac{x^2}{2} \) or \( y = 2 - 2 \left( 1 - \left( \frac{x^2}{2} \right) \right) \)

Either \( k = 2 \) or \( -2 \leq x \leq 2 \)

(c) Range: \( 0 \leq f(x) \leq 2 \) or \( 0 \leq y \leq 2 \) or \( 0 \leq f \leq 2 \)

Notes for Question 4

(a)

B1: At least one of \( \frac{dx}{dt} \) or \( \frac{dy}{dt} \) correct. Note: that this mark can be implied from their working.

B1: Both \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \) are correct. Note: that this mark can be implied from their working.

M1: Applies their \( \frac{dy}{dx} \) divided by their \( \frac{dx}{dt} \) and attempts to substitute \( t = \frac{\pi}{6} \) into their expression for \( \frac{dy}{dx} \).

This mark may be implied by their final answer.

Ie. \( \frac{dy}{dx} = \frac{\sin 2t}{2 \cos t} \) followed by an answer of \( \frac{1}{2} \) would be M1 (implied).

A1: For an answer of \( 1 \) by correct solution only.

Note: Don’t just look at the answer! A number of candidates are finding \( \frac{dy}{dx} = 1 \) from incorrect methods.

Note: Applying \( \frac{dx}{dt} \) divided by their \( \frac{dy}{dt} \) is M0, even if they state \( \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} \).

Special Case: Award SC: B0B0M1A1 for \( \frac{dx}{dt} = -2 \cos t, \quad \frac{dy}{dt} = -2 \sin 2t \) leading to \( \frac{dy}{dx} = \frac{-2 \sin 2t}{-2 \cos t} \)

which after substitution of \( t = \frac{\pi}{6} \), yields \( \frac{dy}{dx} = 1 \)

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!
4. (b) **M1:** Uses the **correct** double angle formula \( \cos 2t = 1 - 2\sin^2 t \) or \( \cos 2t = 2\cos^2 t - 1 \) or \( \cos 2t = \cos^2 t - \sin^2 t \) in an attempt to get \( y \) in terms of \( \sin^2 t \) or get \( y \) in terms of \( \cos^2 t \) or get \( y \) in terms of \( \sin^2 t \) and \( \cos^2 t \). Writing down \( y = 2\sin^2 t \) is fine for M1.

**A1:** Achieves \( y = \frac{x^2}{2} \) or un-simplified equivalents **in the form** \( y = f(x) \). For example:

\[
y = \frac{2x^2}{4} \quad \text{or} \quad y = 2 \left( \frac{x}{2} \right)^2 \quad \text{or} \quad y = 2 - 2 \left( 1 - \left( \frac{x}{2} \right)^2 \right) \quad \text{or} \quad y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}
\]

and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation.

**IMPORTANT:** Please check working as this result can be fluked from an incorrect method.

**Award A0** if there is a \( +c \) added to their answer.

**B1:** Either \( k = 2 \) or a candidate writes down \( -2 \leq x \leq 2 \). Note: \( -2 \leq k \leq 2 \) unless \( k \) stated as 2 is B0.

**Note:** The values of 0 and/or 2 need to be evaluated in this part

**B1:** Achieves an inclusive upper or lower limit, using acceptable notation. Eg: \( f(x) \geq 0 \) or \( f(x) \leq 2 \)

**Special Case:** SC: B1B0 for either \( 0 < f(x) < 2 \) or \( 0 < f < 2 \) or \( 0 < y < 2 \) or \( 0, 2 \)

**Special Case:** SC: B1B0 for \( 0 \leq x \leq 2 \).

**IMPORTANT:** Note that: Therefore candidates can use either \( y \) or \( f \) in place of \( f(x) \)

**Examples:**

- \( 0 \leq x \leq 2 \) is SC: B1B0
- \( 0 < x < 2 \) is B0B0
- \( x \geq 0 \) is B0B0
- \( x \leq 2 \) is B0B0
- \( f(x) > 0 \) is B0B0
- \( f(x) < 2 \) is B0B0
- \( x > 0 \) is B0B0
- \( x < 2 \) is B0B0
- \( 0 \leq f(x) \geq 2 \) is B0B0
- \( 0 < f(x) \leq 2 \) is B1B0
- \( 0 \leq f(x) < 2 \) is B1B0.
- \( f(x) \geq 0 \) is B1B0
- \( f(x) \leq 2 \) is B1B0
- \( f(x) < 0 \) and \( f(x) \leq 2 \) is B1B1. Must state AND \{or\} \( \cap \)
- \( 2 \leq f(x) \leq 2 \) is B0B0
- \( f(x) \geq 0 \) or \( f(x) \leq 2 \) is B1B0.
- \( |f(x)| \leq 2 \) is B1B0
- \( |f(x)| \geq 2 \) is B0B0
- \( 1 \leq f(x) \leq 2 \) is B1B0
- \( 1 < f(x) < 2 \) is B0B0
- \( 0 \leq f(x) \leq 4 \) is B1B0
- \( 0 < f(x) < 4 \) is B0B0
- \( 0 \leq \text{Range} \leq 2 \) is B1B0
- \( \text{Range} \geq 0 \) is B1B0
- \( \text{Range} \leq 2 \) is B1B0
- \( \text{Range} \geq 0 \) and \( \text{Range} \leq 2 \) is B1B0.
- \( [0, 2] \) is B1B1
- \( (0, 2) \) is SC B1B0

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**Aliter**

4. (a) **Way 2**

\[
\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t
\]

At \( t = \frac{\pi}{6} \):

\[
\frac{dx}{dt} = 2\cos \left( \frac{\pi}{6} \right) = \sqrt{3}, \quad \frac{dy}{dt} = 2\sin \left( \frac{2\pi}{6} \right) = \sqrt{3}
\]

Hence \( \frac{dy}{dx} = 1 \)

So B1, B1.

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Notes for Question 4 Continued

<table>
<thead>
<tr>
<th>Aliter 4. (a)</th>
<th>Way 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \frac{1}{2} x^2 \Rightarrow \frac{dy}{dx} = x )</td>
<td>Correct differentiation of their Cartesian equation. B1ft</td>
</tr>
<tr>
<td>At ( t = \frac{\pi}{6} ), ( \frac{dy}{dx} = 2 \sin \left( \frac{\pi}{6} \right) )</td>
<td>Finds ( \frac{dy}{dx} = x ), using the correct Cartesian equation only. B1</td>
</tr>
<tr>
<td>= 1</td>
<td>Finds the value of “( x )” when ( t = \frac{\pi}{6} ) and substitutes this into their ( \frac{dy}{dx} ) Correct value for ( \frac{dy}{dx} ) of 1 A1</td>
</tr>
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<table>
<thead>
<tr>
<th>Aliter 4. (b)</th>
<th>Way 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 1 - \cos 2t = 1 - (2 \cos^2 t - 1) )</td>
<td>M1</td>
</tr>
<tr>
<td>( y = 2 - 2 \cos^2 t \Rightarrow \cos^2 t = \frac{2 - y}{2} \Rightarrow 1 - \sin^2 t = \frac{2 - y}{2} )</td>
<td>(Must be in the form ( y = f(x) )).</td>
</tr>
<tr>
<td>( 1 - \left( \frac{x}{2} \right)^2 = \frac{2 - y}{2} )</td>
<td>A1</td>
</tr>
<tr>
<td>( y = 2 - 2 \left( 1 - \left( \frac{x}{2} \right)^2 \right) )</td>
<td></td>
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<thead>
<tr>
<th>Aliter 4. (b)</th>
<th>Way 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 2 \sin t \Rightarrow t = \sin^{-1} \left( \frac{x}{2} \right) )</td>
<td>Rearranges to make ( t ) the subject and substitutes the result into ( y ).</td>
</tr>
<tr>
<td>So, ( y = 1 - \cos \left( 2 \sin^{-1} \left( \frac{x}{2} \right) \right) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>A1 oe</td>
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</tbody>
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<thead>
<tr>
<th>Aliter 4. (b)</th>
<th>Way 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 1 - \cos 2t \Rightarrow \cos 2t = 1 - y \Rightarrow t = \frac{1}{2} \cos^{-1}(1 - y) )</td>
<td>Rearranges to make ( t ) the subject and substitutes the result into ( y ).</td>
</tr>
<tr>
<td>So, ( x = \pm 2 \sin \left( \frac{1}{2} \cos^{-1}(1 - y) \right) )</td>
<td>M1</td>
</tr>
<tr>
<td>So, ( y = 1 - \cos \left( 2 \sin^{-1} \left( \frac{x}{2} \right) \right) )</td>
<td>A1 oe</td>
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<thead>
<tr>
<th>Aliter 4. (b)</th>
<th>Way 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} = 2 \sin t = x \Rightarrow y = \frac{1}{2} x^2 + c )</td>
<td>Full method of finding ( y = \frac{1}{2} x^2 ) using a value of ( t: -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} )</td>
</tr>
<tr>
<td>Eg: when eg: ( t = 0 ) (nb: ( -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} )), ( x = 0, y = 1 - 1 = 0 \Rightarrow c = 0 \Rightarrow y = \frac{1}{2} x^2 )</td>
<td>M1</td>
</tr>
<tr>
<td>Note: ( \frac{dy}{dx} = 2 \sin t = x \Rightarrow y = \frac{1}{2} x^2 ), with no attempt to find ( c ) is M1A0.</td>
<td>A1</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
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<tr>
<td>-----------------</td>
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<tr>
<td>5. (a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \left{ \begin{array}{l} x = u^2 \Rightarrow \frac{dx}{du} = 2u \quad \text{or} \quad \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \quad \text{or} \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} \ \int \frac{1}{x(2\sqrt{x} - 1)} , dx = \int \frac{1}{u^2(2u - 1)} , 2du \ \int \frac{2}{u(2u - 1)} , du \end{array} \right. )</td>
</tr>
<tr>
<td></td>
<td>( \frac{2}{u(2u - 1)} \equiv \frac{A}{u} + \frac{B}{(2u - 1)} \Rightarrow 2 \equiv A(2u - 1) + Bu )</td>
</tr>
<tr>
<td></td>
<td>( u = 0 \Rightarrow 2 = -A \Rightarrow A = -2 )</td>
</tr>
<tr>
<td></td>
<td>( u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4 )</td>
</tr>
<tr>
<td></td>
<td>So, ( \int \frac{2}{u(2u - 1)} , du = \int \left( -\frac{2}{u} + \frac{4}{(2u - 1)} \right) , du )</td>
</tr>
<tr>
<td></td>
<td>Integrates ( \frac{M}{u} + \frac{N}{(2u - 1)} ), ( M \neq 0 ), ( N \neq 0 ) to obtain any one of ( \pm \lambda \ln u ) or ( \pm \mu \ln(2u - 1) )</td>
</tr>
<tr>
<td></td>
<td>At least one term correctly followed through</td>
</tr>
<tr>
<td></td>
<td>So, ( \left[ -2\ln u + 2\ln(2u - 1) \right]_1^3 )</td>
</tr>
<tr>
<td></td>
<td>= ( (2\ln3 + 2\ln(3 - 1)) - (2\ln1 + 2\ln(2(1) - 1)) ) ( = -2\ln3 + 2\ln5 - (0) )</td>
</tr>
<tr>
<td></td>
<td>= ( 2\ln\left( \frac{5}{3} \right) )</td>
</tr>
<tr>
<td></td>
<td>2\ln\left( \frac{5}{3} \right)</td>
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**Notes for Question 5**

(a) **B1:** \( \frac{dx}{du} = 2u \) or \( dx = 2udu \) or \( \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \) or \( \frac{du}{dx} = \frac{1}{2\sqrt{x}} \) or \( du = \frac{dx}{2\sqrt{x}} \)

**M1:** A full substitution producing an integral in \( u \) only (including the \( du \)) (Integral sign not necessary).

The candidate needs to deal with the “\( x \)”, the “\( 2\sqrt{x} - 1 \)” and the “\( dx \)” and converts from an integral term in \( x \) to an integral in \( u \). (Remember the integral sign is not necessary for M1).

**A1**: leading to the result printed on the question paper (including the \( du \)). (Integral sign is needed).

(b) **M1:** Writing \( \frac{2}{u(2u - 1)} \equiv \frac{A}{u} + \frac{B}{(2u - 1)} \) or writing \( \frac{1}{u(2u - 1)} \equiv \frac{P}{u} + \frac{Q}{(2u - 1)} \) and a complete method for finding the value of at least one of their \( A \) or their \( B \) (or their \( P \) or their \( Q \)).

**A1:** Both their \( A = -2 \) and their \( B = 4 \). (Or their \( P = -1 \) and their \( Q = 2 \) with the multiplying factor of 2 in front of the integral sign).

**M1:** Integrates \( \frac{M}{u} + \frac{N}{(2u - 1)} \), \( M \neq 0 \), \( N \neq 0 \) (i.e. a two term partial fraction) to obtain any one of \( \pm \lambda \ln u \) or \( \pm \mu \ln(2u - 1) \) or \( \pm \mu \ln\left( u - \frac{1}{2} \right) \)

**A1ft:** At least one term correctly followed through from their \( A \) or from their \( B \) (or their \( P \) and their \( Q \)).

**A1:** \(-2\ln u + 2\ln(2u - 1)\)

**Notes for Question 5 Continued**

5. (b) ctd **M1:** Applies limits of 3 and 1 in \( u \) or 9 and 1 in \( x \) in their (i.e. any) changed function and subtracts the
correct way round.

Note: If a candidate just writes \((-2 \ln 3 + 2 \ln(2(3) - 1)) \text{ oe}, this is ok for M1.

A1: \(2 \ln \left(\frac{5}{3}\right)\) correct answer only. (Note: \(a = 5, b = 3\).)

Important note: Award M0A0M1A1A0 for a candidate who writes

\[
\int \frac{2}{u(2u - 1)} \, du = \int \frac{2}{u} + \frac{2}{(2u - 1)} \, du = 2 \ln u + \ln(2u - 1)
\]

AS EVIDENCE OF WRITING \(\frac{2}{u(2u - 1)}\) AS PARTIAL FRACTIONS IS GIVEN.

Important note: Award M0A0M0A0A0 for a candidate who writes down either

\[
\int \frac{2}{u(2u - 1)} \, du = 2 \ln u + 2 \ln(2u - 1) \quad \text{or} \quad \int \frac{2}{u(2u - 1)} \, du = 2 \ln u + \ln(2u - 1)
\]

WITHOUT ANY EVIDENCE OF WRITING \(\frac{2}{u(2u - 1)}\) as partial fractions.

Important note: Award M1A1M1A1A1 for a candidate who writes down

\[
\int \frac{2}{u(2u - 1)} \, du = -2 \ln u + 2 \ln(2u - 1)
\]

WITHOUT ANY EVIDENCE OF WRITING \(\frac{2}{u(2u - 1)}\) as partial fractions.

Note: In part (b) if they lose the “2” and find \(\int \frac{1}{u(2u - 1)} \, du\) we can allow a maximum of M1A0 M1A1ftA0 M1A0.
6. \( \frac{d\theta}{dt} = \lambda (120 - \theta), \ \theta \leq 100 \)

(a) \[ \int \frac{1}{120 - \theta} \ d\theta = \int \lambda \ dt \quad \text{or} \quad \int \frac{1}{\lambda (120 - \theta)} \ d\theta = \int \ dt \]

\(-\ln(120 - \theta); = \lambda t + c \quad \text{or} \quad -\frac{1}{\lambda} \ln(120 - \theta); = t + c \)

\( \{ t = 0, \ \theta = 20 \Rightarrow -\ln(120 - 20) = \lambda (0) + c \) See notes M1 A1

\( c = -\ln 100 \Rightarrow -\ln(120 - \theta) = \lambda t - \ln 100 \)

then either...

\[ \begin{array}{|c|c|}
\hline
-\lambda t = \ln(120 - \theta) - \ln 100 & \lambda t = \ln 100 - \ln(120 - \theta) \\
\hline
-\lambda t = \ln \left( \frac{120 - \theta}{100} \right) & \lambda t = \ln \left( \frac{100}{120 - \theta} \right) \\
\hline
e^{-\lambda t} = \frac{120 - \theta}{100} & e^{\lambda t} = \frac{100}{120 - \theta} \\
\hline
100e^{-\lambda t} = 120 - \theta & \Rightarrow 120 - \theta = 100e^{-\lambda t} \\
\hline
\end{array} \]

leading to \( \theta = 120 - 100e^{-\lambda t} \)

or...

\[ \begin{array}{|c|c|}
\hline
\{ \lambda = 0.01, \ \theta = 100 \Rightarrow \} \quad 100 = 120 - 100e^{-0.01t} \\
\hline
\Rightarrow 100e^{-0.01t} = 120 - 100 \Rightarrow -0.01t = \ln \left( \frac{120 - 100}{100} \right) \\
\hline
t = \frac{1}{-0.01} \ln \left( \frac{120 - 100}{100} \right) \\
\hline
\end{array} \]

Uses correct order of operations by moving from 100 = 120 - 100e^{-0.01t} to give \( t = \ldots \) and \( t = A \ln B, \) where \( B > 0 \)

\[ [8] \]

(b) \[ \{ \lambda = 0.01, \ \theta = 100 \Rightarrow \} \quad 100 = 120 - 100e^{-0.01t} \]

\[ \Rightarrow 100e^{-0.01t} = 120 - 100 \Rightarrow -0.01t = \ln \left( \frac{120 - 100}{100} \right) \]

\[ t = \frac{1}{-0.01} \ln \left( \frac{120 - 100}{100} \right) \]

\[ \left\{ t = \frac{1}{-0.01} \ln \left( \frac{1}{5} \right) = 100 \ln 5 \right\} \]

\( t = 160.94379... = 161 \) (s) (nearest second) awrt 161 A1

\[ [3] \]
Notes for Question 6

(a)\[ \begin{align*}
\text{B1: } & \text{ Separates variables as shown. } d\theta \text{ and } dt \text{ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.} \\
\text{Either} & \\
\text{M1: } & \int \frac{1}{120-\theta} d\theta \rightarrow \pm A \ln(120-\theta) \quad \text{or} \quad \int \frac{1}{\lambda(120-\theta)} d\theta \rightarrow \pm A \ln(120-\theta), \ A \text{ is a constant.} \\
\text{A1: } & \int \frac{1}{120-\theta} d\theta \rightarrow -\ln(120-\theta) \quad \text{or} \quad \int \frac{1}{\lambda(120-\theta)} d\theta \rightarrow -\frac{1}{\lambda} \ln(120-\theta) \quad \text{or} \quad -\frac{1}{\lambda} \ln(120\lambda-\lambda\theta), \\
\text{M1: } & \int \lambda dt \rightarrow \lambda t \\
\text{A1: } & \int \lambda dt \rightarrow \lambda t + c \quad \text{or} \quad \int dt \rightarrow t + c \quad \text{The } c \text{ can appear on either side of the equation.} \\
\text{IMPORTANT: } & + c \text{ can be on either side of their equation for the } 2\text{nd A1 mark.} \\
\text{M1: } & \text{Substitutes } t = 0 \AND \theta = 20 \text{ in an integrated or changed equation containing } c \text{ (or } A \text{ or } \ln A). \\
\text{Note: } & \text{that this mark can be implied by the correct value of } c. \ {\text{Note that } -\ln100 = -4.60517...}. \\
\end{align*} \]

\[\begin{align*}
\text{dM1: } & \text{Uses their value of } c \text{ which must be a } \ln \text{ term, and uses fully correct method to eliminate their logarithms. } \text{Note: } \text{This mark is dependent on all three previous method marks being awarded.} \\
\text{A1*: } & \text{This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate’s working. } \text{Do not accept huge leaps in working at the end. } \text{So a minimum of } \text{either:} \\
\text{(1): } & e^{\lambda t} = \frac{120-\theta}{100} \Rightarrow 100e^{\lambda t} = 120-\theta \Rightarrow \theta = 120-100e^{-\lambda t} \\
\text{or (2): } & e^{\lambda t} = \frac{100}{120-\theta} \Rightarrow (120-\theta)e^{\lambda t} = 100 \Rightarrow 120-\theta = 100e^{-\lambda t} \Rightarrow \theta = 120-100e^{-\lambda t} \\
\text{is required for A1.} \\
\text{Note: } & \int \frac{1}{(120\lambda-\lambda\theta)} d\theta \rightarrow -\frac{1}{\lambda} \ln(120\lambda-\lambda\theta) \text{ is ok for the first M1A1 in part (a).}
\end{align*}\]

(b)\[\begin{align*}
\text{M1: } & \text{Substitutes } \lambda = 0.01 \AND \theta = 100 \text{ into the printed equation or one of their earlier equations connecting } \\
\theta \text{ and } t. \quad \text{This mark can be implied by subsequent working.} \\
\text{dM1: } & \text{Candidate uses correct order of operations by moving from } 100 = 120 - 100e^{-0.01t} \text{ to } t = ... \\
\text{Note: } & \text{that the } 2\text{nd Method mark is dependent on the } 1\text{st Method mark being awarded in part (b).} \\
\text{A1: } & \text{awrt 161 or “awrt” 2 minutes 41 seconds. } \text{ (Ignore incorrect units).}
\end{align*}\]

Alter 6. (a) Way 2 \[\begin{align*}
\int \frac{1}{120-\theta} d\theta &= \int \lambda dt \\
-\ln(120-\theta) &= \lambda t + c \\
-\ln(120-\theta) &= \lambda t + c \\
\ln(120-\theta) &= -\lambda t + c \\
120-\theta &= Ae^{-\lambda t} \\
\theta &= 120 - Ae^{-\lambda t} \\
\{t = 0, \theta = 20 \Rightarrow \} 20 &= 120 - Ae^0 \\
A &= 120 - 20 = 100 \\
\text{So, } \theta &= 120 - 100e^{-\lambda t}
\end{align*}\]

\[\text{B1 M1 A1; M1 A1} \]

\[\text{M1} \]

\[\text{ddM1 A1*} \]
### Notes for Question 6 Continued

(a) **B1M1A1M1A1**: Mark as in the original scheme.

**M1**: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be $c$ or $A$. **Note** that this mark can be implied by the correct value of $c$ or $A$.

**dddM1**: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **Note**: This mark is dependent on all three previous method marks being awarded.

**A1***: Same as the original scheme.

**Note**: The jump from \( \ln(120 - \theta) = -\lambda t + c \) to \( 120 - \theta = Ae^{-\lambda t} \) with no incorrect working is condoned in part (a).

<table>
<thead>
<tr>
<th>Aliter 6. (a) Way 3</th>
<th>B1</th>
<th>M1</th>
<th>A1</th>
<th>A1</th>
<th>M1</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \int \frac{1}{120-\theta} , d\theta = \int \lambda , dt ) { \Rightarrow \int \frac{-1}{\theta-120} , d\theta = \int \lambda , dt }</td>
<td>B1</td>
<td>M1</td>
<td>A1</td>
<td>A1</td>
<td>M1</td>
<td>A1</td>
</tr>
<tr>
<td>(-\ln</td>
<td>\theta-120</td>
<td>= \lambda t + c)</td>
<td>Modulus required for 1st A1.</td>
<td>Modulus not required here!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{t = 0, \theta = 20 \Rightarrow -\ln</td>
<td>20-120</td>
<td>= \lambda(0) + c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Rightarrow c = -\ln100 \Rightarrow -\ln</td>
<td>\theta-120</td>
<td>= \lambda t - \ln100)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>then either...</td>
<td>or...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-\lambda t = \ln</td>
<td>\theta-120</td>
<td>-\ln100)</td>
<td>(\lambda t = \ln100-\ln</td>
<td>\theta-120</td>
<td>)</td>
<td></td>
</tr>
<tr>
<td>(-\lambda t = \ln\left(\frac{\theta-120}{100}\right))</td>
<td>(\lambda t = \ln\left(\frac{100}{\theta-120}\right))</td>
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</tr>
<tr>
<td>As $\theta \leq 100$</td>
<td>Understanding of modulus is required here!</td>
<td>dddM1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(e^{-\lambda t} = \frac{120-\theta}{100})</td>
<td>(e^{\lambda t} = \frac{100}{120-\theta})</td>
<td></td>
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</tr>
<tr>
<td>(100e^{-\lambda t} = 120-\theta)</td>
<td>((120-\theta)e^{\lambda t} = 100)</td>
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<td></td>
</tr>
<tr>
<td>leading to $\theta = 120 - 100e^{-\lambda t}$</td>
<td>(\Rightarrow 120 - \theta = 100e^{-\lambda t})</td>
<td></td>
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</tbody>
</table>

**B1**: Mark as in the original scheme.

**M1**: Mark as in the original scheme ignoring the modulus.

**A1**: \(\int \frac{1}{120-\theta} \, d\theta \rightarrow -\ln|\theta-120|\). **(The modulus is required here)**.

**M1A1**: Mark as in the original scheme.

**M1**: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be $c$ or $A$. Mark as in the original scheme ignoring the modulus.

**dddM1**: Mark as in the original scheme AND the candidate must demonstrate that they have converted \(\ln|\theta-120|\) to \(\ln(120 - \theta)\) in their working. **Note**: This mark is dependent on all three previous method marks being awarded.

**A1**: Mark as in the original scheme.
<table>
<thead>
<tr>
<th>Aliter 6. (a) Way 4</th>
<th><strong>Use of an integrating factor</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{d\theta}{dt} = \lambda(120 - \theta) \Rightarrow \frac{d\theta}{dt} + \lambda \theta = 120\lambda )</td>
<td>B1</td>
</tr>
<tr>
<td>IF = ( e^{\lambda t} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{d}{dt}(e^{\lambda t}\theta) = 120\lambda e^{\lambda t} ), ( e^{\lambda t}\theta = 120\lambda e^{\lambda t} + k )</td>
<td>M1A1</td>
</tr>
<tr>
<td>( \theta = 120 + K e^{-\lambda t} )</td>
<td>M1</td>
</tr>
<tr>
<td>( {t = 0, \theta = 20 \Rightarrow } - 100 = K )</td>
<td></td>
</tr>
<tr>
<td>( \theta = 120 - 100e^{-\lambda t} )</td>
<td>M1A1</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
</tr>
<tr>
<td>7. (a)</td>
<td>[ x^2 + 4xy + y^2 + 27 = 0 ]</td>
</tr>
<tr>
<td></td>
<td>[ \begin{align*} 2x + (4y + 4x \frac{dy}{dx}) + 2y \frac{dy}{dx} &amp;= 0 \ 2x + 4y + (4x + 2y)\frac{dy}{dx} &amp;= 0 \ \frac{dy}{dx} &amp;= \frac{-2x - 4y}{4x + 2y} \end{align*} ]</td>
</tr>
<tr>
<td>(b)</td>
<td>[ 4x + 2y = 0 ]</td>
</tr>
<tr>
<td></td>
<td>[ y = -2x ]</td>
</tr>
<tr>
<td></td>
<td>[ x^2 + 4x(-2x) + (-2x)^2 + 27 = 0 ]</td>
</tr>
<tr>
<td></td>
<td>[ \left( -\frac{1}{2}y \right)^2 + \left( -\frac{1}{2}y \right)y + y^2 + 27 = 0 ]</td>
</tr>
<tr>
<td></td>
<td>[ -3x^2 + 27 = 0 ]</td>
</tr>
<tr>
<td></td>
<td>[ x = -3 ]</td>
</tr>
<tr>
<td>When ( x = -3 ), ( y = -2(-3) )</td>
<td>When ( y = 6 ), ( x = \frac{1}{2}(6) )</td>
</tr>
<tr>
<td></td>
<td>[ y = 6 ]</td>
</tr>
</tbody>
</table>

**Notes for Question 7**

(a) **M1**: Differentiates implicitly to include either \( 4x \frac{dy}{dx} \) or \( \pm \frac{dy}{dx} \). (Ignore \( \frac{dy}{dx} = 0 \)).

**A1**: \( (x^2) \rightarrow (2x) \) and \( \ldots + y^2 + 27 = 0 \rightarrow + 2y \frac{dy}{dx} = 0 \).

**Note**: If an extra term appears then award A0.

**Note**: The " = 0" can be implied by rearrangement of their equation.

i.e.: \( 2x + 4y + 4x \frac{dy}{dx} + 2y \frac{dy}{dx} \) leading to \( 4x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 4y \) will get A1 (implied).

(b) **B1**: \( 4y + 4x \frac{dy}{dx} \) or \( 4 \left( y + x \frac{dy}{dx} \right) \) or equivalent

**dM1**: An attempt to factorise out \( \frac{dy}{dx} \) as long as there are at least two terms in \( \frac{dy}{dx} \).

i.e. \( \ldots + (4x + 2y) \frac{dy}{dx} = \ldots \) or \( \ldots + 2(2x + y) \frac{dy}{dx} = \ldots \)

**Note**: This mark is dependent on the previous method mark being awarded.

**A1**: For \( -2x - 4y \) or equivalent. Eg: \( \frac{+2x + 4y}{4x - 2y} \) or \( \frac{-2(x + 2y)}{4x + 2y} \) or \( \frac{-x - 2y}{2x + y} \)

**cso**: If the candidate’s solution is not completely correct, then do not give this mark.
### Notes for Question 7 Continued

| (b) | **M1:** Sets the denominator of their \( \frac{dy}{dx} \) equal to zero (or the numerator of their \( \frac{dx}{dy} \) equal to zero) oe.  
\[
A1: \text{Rearranges to give either } y = -2x \text{ or } x = -\frac{1}{2}y. \text{ (correct solution only).}
\]
\[
The first two marks can be implied from later working, i.e. for a correct substitution of either \( y = -2x \) into \( y^2 \) or for \( x = -\frac{1}{2}y \) into \( 4xy \).
\]
**M1:** Substitutes \( y = \pm \lambda x \) or \( x = \pm \mu y \) or \( y = \pm \lambda x \pm a \) or \( x = \pm \mu y \pm b \) (\( \lambda \neq 0, \mu \neq 0 \)) into \( x^2 + 4xy + y^2 + 27 = 0 \) to form an equation in one variable.

| **dM1:** leading to at least either \( x^2 = A, A > 0 \) or \( y^2 = B, B > 0 \)  
**Note:** This mark is dependent on the previous method mark (M1*) being awarded.  
**A1:** For \( x = -3 \) (ignore \( x = 3 \)) or if \( y \) was found first, \( y = 6 \) (ignore \( y = -6 \)) (correct solution only).  
**ddM1** Substitutes their value of \( x \) into \( y = \pm \lambda x \) to give \( y = \text{value} \)  
Alternatively, substitutes their value of \( y \) into \( x = \pm \mu y \) to give \( x = \text{value} \)  
**Note:** This mark is dependent on the two previous method marks (M1* and dM1*) being awarded.  
**A1:** \((-3, 6)\) eso.  
**Note:** If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. **DO NOT APPLY ISW ON THIS OCCASION.**  
**Note:** \( x = -3 \) followed later in working by \( y = 6 \) is fine for A1.  
**Note:** \( y = 6 \) followed later in working by \( x = -3 \) is fine for A1.  
**Note:** \( x = -3, 3 \) followed later in working by \( y = 6 \) is A0, unless candidate indicates that they are rejecting \( x = 3 \)  
**Note:** Candidates who set the numerator of \( \frac{dy}{dx} \) equal to 0 (or the denominator of their \( \frac{dx}{dy} \) equal to zero) can only achieve a maximum of 3 marks in this part. They can only achieve the 2\(^{nd}\), 3\(^{rd}\) and 4\(^{th}\) Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find \((-6, 3)\) \{ or even \((6, -3)\)\}.  
**Note:** Candidates who set the numerator or the denominator of \( \frac{dy}{dx} \) equal to \( \pm k \) (usually \( k = 1 \)) can only achieve a maximum of 3 marks in this part. They can only achieve the 2\(^{nd}\), 3\(^{rd}\) and 4\(^{th}\) Method marks to give a marking profile of M0A0M1M1A0M1A0.  
**Special Case:** It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for \( \frac{dy}{dx} \)) to gain all 7 marks in part (b).  
**Eg:** An incorrect part (a) answer of \( \frac{dy}{dx} = \frac{2x - 4y}{4x + 2y} \) can lead to a correct \((-3, 6)\) in part (b) and 7 marks. |
### Question 8

**Scheme**

8. \( l: \mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}, \ A(3, -2, 6), \ \overrightarrow{OP} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} \)

(a) \( \{ \overrightarrow{PA} \} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} \) \( \{ \overrightarrow{AP} \} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix} \)

\[
\begin{pmatrix} 3 + p \\ -2 \\ 6 - 2p \end{pmatrix} = \begin{pmatrix} -3 - p \\ 2 \\ 2p - 6 \end{pmatrix}
\]

\[
\begin{pmatrix} 3 + p \\ -2 \\ 6 - 2p \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 6 + 2p - 4 - 6 + 2p = 0
\]

\( p = 1 \)

(b) \( |\overrightarrow{AP}| = \sqrt{4^2 + (-2)^2 + 4^2} \) or \( |\overrightarrow{AP}| = \sqrt{(-4)^2 + 2^2 + (-4)^2} \)

So, \( \overrightarrow{PA} \) or \( \overrightarrow{AP} = 3\overrightarrow{36} \) or \( \overrightarrow{6} \) \( \text{cao} \)

It follows that, \( AB = "6" \) \( \{ = \overrightarrow{PA} \} \) or \( PB = "6\sqrt{2}" \) \( \{ = \sqrt{2} \overrightarrow{PA} \} \)

\{Note that \( AB = "6" = 2 \text{(the modulus of the direction vector of} \ l)\}\)

\[
\overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \text{ or } \overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - 7 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}
\]

\[
\begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix} \text{ and } \begin{pmatrix} -6 \\ -6 \\ 8 \end{pmatrix}
\]

Uses a correct method in order to find both possible sets of coordinates of \( B \).

Both coordinates are correct. \( \text{A1 cao} \)

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td></td>
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</tbody>
</table>

#### Notes for Question 8

**M1**: Finds the difference between \( \overrightarrow{OA} \) and \( \overrightarrow{OP} \). Ignore labelling.

If no “subtraction” seen, you can award M1 for 2 out of 3 correct components of the difference.

**A1**: Accept any of \( \begin{pmatrix} 3 + p \\ -2 \\ 6 - 2p \end{pmatrix} \) or \( (3 + p)i - 2j + (6 - 2p)k \) or \( \begin{pmatrix} -3 - p \\ 2 \\ 2p - 6 \end{pmatrix} \) or \( (-3 - p)i + 2j + (2p - 6)k \)

[4] \( \text{cao} \)

[5] \( \text{ft} \)
8. (a) M1: Applies the formula \( \overrightarrow{PA} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \) or \( \overrightarrow{AP} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \) correctly to give a linear equation in \( p \) which is set equal to zero. **Note:** The dot product can also be with \( \pm k \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \). Eg: Some candidates may find
\[
\begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \\ -5 \end{pmatrix},
\]
for instance, and use this in their dot product which is fine for M1.

A1: Finds \( p = 1 \) from a correct solution only.

**Note:** The direction of subtraction is not important in part (a).

(b) M1: Uses their value of \( p \) and Pythagoras to obtain a numerical expression for either \( AP \) or \( PA \) or \( 2AP \) or \( 2PA \).

Eg: \( PA = \sqrt{4^2 + (-2)^2 + 2^2} \) or \( \sqrt{(-4)^2 + 2^2 + 4^2} \) or \( \sqrt{4^2 + 2^2 + 4^2} \) or \( \sqrt{(-4)^2 + 2^2 + 4^2} \) or \( \sqrt{4^2 + 2^2 + 4^2} \) or \( \sqrt{4^2 + 2^2 + 4^2} \). 

A1: \( AP = PA = 36 \) or \( 6 \) \( \text{cao} \) or \( AP^2 = 36 \) \( \text{cao} \)

B1ft: States or it is clear from their working that \( AB = "6" \{ \text{their evaluated } PA \} \) or \( PB = "6" \sqrt{2} \{ \text{their evaluated } PA \} \).

**Note:** So a correct follow length is required here for either \( AB \) or \( PB \) using their evaluated \( PA \).

**Note:** This mark may be found on a diagram.

**Note:** If a candidate states that \( \overrightarrow{AP} = \overrightarrow{AB} \) and then goes on to find \( \overrightarrow{AP} = 6 \) then the B1 mark can be implied.

**IMPORTANT:** This mark may be implied as part of expressions such as:
\[
\{ AB = \sqrt{(10 + 2\lambda)^2 + (10 + 2\lambda)^2 + (-5 - \lambda)^2} = 6 \} \text{ or } \{ AB^2 = (10 + 2\lambda)^2 + (10 + 2\lambda)^2 + (-5 - \lambda)^2 = 36 \}
\]
or
\[
\{ PB = \sqrt{(14 + 2\lambda)^2 + (8 + 2\lambda)^2 + (-1 - \lambda)^2} = 6\sqrt{2} \} \text{ or } \{ PB^2 = (14 + 2\lambda)^2 + (8 + 2\lambda)^2 + (-1 - \lambda)^2 = 72 \}
\]

M1: Uses a full method in order to find both possible sets of coordinates of B:

Eg 1: \( \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \pm 2 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \)

Eg 2: \( \overrightarrow{OB} = \begin{pmatrix} 13 \\ 2 \\ 1 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \) and \( \overrightarrow{OB} = \begin{pmatrix} 13 \\ 2 \\ 1 \end{pmatrix} - 7 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \)

**Note:** If a candidate achieves at least one of the correct \((7, 2, 4)\) or \((-1, -6, 8)\) then award SC M1 here.

**Note:** \( \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \) and \( \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} - 7 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \) is M0.

A1: For both \((7, 2, 4)\) and \((-1, -6, 8)\). Accept vector notation or \( i, j, k \) notation.

**Note:** All the marks are accessible in part (b) if \( p = 1 \) is found from incorrect working in part (a).

**Note:** Imply M1A1B1 and award M1 for candidates who write: \( \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \pm 2 \begin{pmatrix} 2 \\ -1 \end{pmatrix} \), with little or no earlier working.
8. \( \text{Helpful Diagram!} \)

\[
\begin{align*}
|AB|^2 &= 9\lambda^2 + 90\lambda + 225 \\
\overrightarrow{AB} &= \begin{pmatrix} 10 + 2\lambda \\ 10 + 2\lambda \\ -5 - \lambda \end{pmatrix} \\
\lambda &= -3 \\
\overrightarrow{PA} &= \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} \\
\lambda &= -5 \\
\overrightarrow{PB} &= \begin{pmatrix} 14 + 2\lambda \\ 8 + 2\lambda \\ -1 - \lambda \end{pmatrix} \\
|\overrightarrow{PB}|^2 &= 9\lambda^2 + 90\lambda + 261
\end{align*}
\]

8. (b) \textbf{Way 2:} Setting \( AB = "6" \) or \( AB^2 = "36" \) \textbf{Note:} It is possible for you to apply the main scheme for Way 2.

\[
\begin{align*}
\{ AB = "6" \Rightarrow AB^2 = "36" \Rightarrow \} \quad (10 + 2\lambda)^2 + (10 + 2\lambda)^2 + (-5 - \lambda)^2 &= "36" \\
9\lambda^2 + 90\lambda + 225 &= 36 \\
9\lambda^2 + 90\lambda + 189 &= 0 \\
\lambda^2 + 10\lambda + 21 &= 0 \\
(\lambda + 3)(\lambda + 7) &= 0 \\
\lambda &= -3, -7
\end{align*}
\]

Then apply final M1 A1 as in the original scheme. | ... M1 A1

8. (b) \textbf{Way 3:} Setting \( PB = "6\sqrt{2}" \) or \( PB^2 = "72" \) \textbf{Note:} It is possible for you to apply the main scheme for Way 3.

\[
\begin{align*}
\{ PB = "6\sqrt{2}" \Rightarrow PB^2 = "72" \Rightarrow \} \quad (14 + 2\lambda)^2 + (8 + 2\lambda)^2 + (-1 - \lambda)^2 &= "72" \\
9\lambda^2 + 90\lambda + 261 &= 72 \\
9\lambda^2 + 90\lambda + 189 &= 0 \\
\lambda^2 + 10\lambda + 21 &= 0 \\
(\lambda + 3)(\lambda + 7) &= 0 \\
\lambda &= -3, -7
\end{align*}
\]

Then apply final M1 A1 as in the original scheme. | ... M1 A1
## Notes for Question 8 Continued

### 8. (b) (You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).

**Way 4:** Using the dot product formula between \( \overrightarrow{PA} \) and \( \overrightarrow{PB} \), ie: \[ \cos 45° = \frac{\overrightarrow{PA} \cdot \overrightarrow{PB}}{\overrightarrow{PA} \overrightarrow{PB}}. \]

\[ \overrightarrow{PA} \cdot \overrightarrow{PB} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 14 + 2\lambda \\ 8 + 2\lambda \\ -1 - \lambda \end{pmatrix} = 56 + 8\lambda - 16 - 4\lambda - 4 - 4\lambda = 36 \]

\[ \{ \cos 45° \} = \frac{1}{\sqrt{2}} = \frac{36}{6\sqrt{9\lambda^2 + 90\lambda + 261}} \]

\[ \frac{1}{2} = \frac{36}{9\lambda^2 + 90\lambda + 261} \]

\( 9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0 \)

\( \lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda + 3)(\lambda + 7) = 0 \)

\( \lambda = -3, -7 \)

Then apply final M1 A1 as in the original scheme.

### 8. (b) (You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 5).

**Way 5:** Using the dot product formula between \( \overrightarrow{AB} \) and \( \overrightarrow{PB} \), ie: \[ \cos 45° = \frac{\overrightarrow{AB} \cdot \overrightarrow{PB}}{\overrightarrow{AB} \overrightarrow{PB}}. \]

\[ \cos 45° = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{9\lambda^2 + 90\lambda + 225}} \]

\[ \{ \cos 45° \} = \frac{1}{\sqrt{2}} = \frac{140 + 20\lambda + 28\lambda + 4\lambda^2 + 80 + 20\lambda + 16\lambda + 4\lambda^2 + 5 + 5\lambda + \lambda + \lambda^2}{\sqrt{9\lambda^2 + 90\lambda + 225} \sqrt{9\lambda^2 + 90\lambda + 261}} \]

\[ \frac{1}{2} = \frac{9\lambda^2 + 90\lambda + 225}{\sqrt{9\lambda^2 + 90\lambda + 225} \sqrt{9\lambda^2 + 90\lambda + 261}} \]

\[ \frac{1}{2} = \frac{(9\lambda^2 + 90\lambda + 225)^2}{(9\lambda^2 + 90\lambda + 225)(9\lambda^2 + 90\lambda + 261)} \]

\[ 9\lambda^2 + 90\lambda + 261 = 2(9\lambda^2 + 90\lambda + 225) \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0 \]

\( \lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda + 3)(\lambda + 7) = 0 \)

\( \lambda = -3, -7 \)

Then apply final M1 A1 as in the original scheme.
Way 6:  
\[ \overrightarrow{PA} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \]  
and direction vector of \( l \) is \( \mathbf{d} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \)

So,  
\[ |\overrightarrow{PA}| = 2|\mathbf{d}| \quad \text{or} \quad PA = 2|\mathbf{d}| \quad \text{A correct statement relating these distances (and not vectors)} \]

Apply final M1 A1 as in the original scheme. | ... M1 A1

Note: \( \overrightarrow{PA} = 2\mathbf{d} \) with no other creditable working is M0A0B0...

\textbf{Note:} \( \overrightarrow{PA} = 2\mathbf{d} \), followed by  
\[ \overrightarrow{OB} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm 2 \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \]  
is M1A1B1M1 and the final A1 mark is for both sets of correct coordinates.
Mark Scheme (Results)

Summer 2013

GCE Decision Mathematics 1 (6689/01)
Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \(\checkmark\) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
1. (a) Bipartite (graph) 
   e.g. (see below for alternatives) 
   First alternating path: B – 4 = L – 3 = H – 2 
   Change status to give B = 4 – L = 3 – H = 2 
   Improved matching: A = 1, B = 4, H = 2, (I unmatched), L = 3 R = 5 
   Second alternating path: I – 1 = A – 3 = L – 5 = R – 6 
   Changing status to give: I = 1 – A = 3 – L = 5 – R = 6 
   Complete matching: A = 3, B = 4, H = 2, I = 1, L = 5, R = 6 

Notes for Question 1

<table>
<thead>
<tr>
<th>Possible 1st paths</th>
<th>A</th>
<th>B</th>
<th>H</th>
<th>I</th>
<th>L</th>
<th>R</th>
<th>Subsequent 2nd paths</th>
</tr>
</thead>
<tbody>
<tr>
<td>B – 4 – L – 5 – R – 6</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>-</td>
<td>5</td>
<td>6</td>
<td>I – 1 – A – 3 – H – 2</td>
</tr>
<tr>
<td>I – 1 – A – 3 – H – 2</td>
<td>3</td>
<td>-</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>B – 4 – L – 5 – R – 6</td>
</tr>
</tbody>
</table>

a1B1: CAO, but be charitable on spelling, award if phonetically close.
b1M1: An alternating path (e.g. letter – number – letter – ...) from either B to 2 or 6 or from I to 2 – or vice versa
b1A1: CAO – a correct path including change status either stated (only accept ‘change (of) status’ or ‘c.s.’) or shown (all symbols e.g. (...=...=...) interchanged (...=...=...)). Chosen path clear.
b2A1: CAO must follow from the correct stated path. Accept on a clear diagram (with five arcs only).
b2M1: A second alternating path from the remaining (unused) I or B to the remaining (unused) 6 or 2 - or vice versa.
b3A1: CAO including change status (stated or shown), chosen path clear
b4A1: CAO must follow from two correct stated paths (so both previous M marks must have been awarded).
Accept on a clear diagram (with six arcs only).
### Question 2

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2. (a)</strong></td>
<td>Bin 1: 0.6 0.2 0.4 0.5 0.1</td>
<td>Bin 3: 1.6</td>
</tr>
<tr>
<td></td>
<td>Bin 2: 1.5 0.3</td>
<td>Bin 4: 0.7 0.9</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>0.6 1.5 1.6 0.2 0.4 0.5 0.7 0.1 0.9 0.3 pivot 0.5</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>0.6 1.5 0.7 0.9 0.5 0.2 0.4 0.1</td>
<td>pivots 1.6 0.1</td>
</tr>
<tr>
<td></td>
<td>1.6 0.6 1.5 0.7 0.9 0.5 0.2 0.4 0.3 0.1</td>
<td>pivots 0.7 0.4</td>
</tr>
<tr>
<td></td>
<td>1.6 1.5 0.9 0.7 0.6 0.5 0.4 0.2 0.3 0.1</td>
<td>pivots 0.9 0.3 (0.6)</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Bin 1: 1.6 0.4</td>
<td>Bin 2: 1.5 0.5</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>e.g. 6.8/2 = 3.4 so yes a minimum of 4 bins is needed</td>
<td>B1 (1)</td>
</tr>
</tbody>
</table>

### Notes for Question 2

- **a1M1**: First four items placed correctly and at least six values put in bins. (Condone cumulative totals here only.)
- **a1A1**: Bin 1 correct
- **a2A1**: CSO All correct
- **b1M1**: Quick sort – pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0) and first pass gives >p, p, <p. So after the first pass the list should read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing 1 pivot per iteration M1 only**
- **b1A1**: First pass correct, next two pivots chosen consistently for second pass.
- **b2A1ft**: second and third passes correct (ft from their first pass and choice of pivots) – need not be choosing pivots for the fourth pass for this mark.
- **b3A1**: CSO (correct solution only – all previous marks in this part must have been awarded) including ‘sort complete’ – this could be shown by the final list being re-written or ‘sorted’ statement or each item being used as a pivot.

- **c1M1**: Must be using ‘sorted’ list in descending order. First four items placed correctly and at least six values put in bins. (Condone cumulative totals here only.)
- **c1A1**: First seven items placed correctly (so Bin 1 and 2 correct, Bin 3 containing 0.9 and 0.7 and Bin 4 containing 0.6)
- **c2A1**: CSO

**SC for part (c)** If ‘sorted’ list is wrong from part (b) (i.e. one error e.g. a missing number, an extra number or one number incorrectly placed) then award M1 only in (c) for their first seven items correctly placed.

- **d1B1**: A conclusion based on their answer to part (c) together with either a correct lower bound calculation or based on the total > 6 or full bins (three of the bins are full in part (c)).
### Part (b) Using middle left as pivot

<table>
<thead>
<tr>
<th>0.6</th>
<th>1.5</th>
<th>1.6</th>
<th>0.2</th>
<th>0.4</th>
<th>0.5</th>
<th>0.7</th>
<th>0.1</th>
<th>0.9</th>
<th>0.3</th>
<th>pivot 0.4</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1.5</td>
<td>1.6</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>pivots 1.6 0.1</td>
<td>A1</td>
</tr>
<tr>
<td>1.6</td>
<td>0.6</td>
<td>1.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>0.4</td>
<td>0.2</td>
<td>0.3</td>
<td>0.1</td>
<td>pivots 0.5 0.2</td>
<td>A1</td>
</tr>
<tr>
<td>1.6</td>
<td>0.6</td>
<td>1.5</td>
<td>0.7</td>
<td>0.9</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>pivots 1.5 (0.3)</td>
<td>A1ft</td>
</tr>
<tr>
<td>1.6</td>
<td>1.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.9</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>pivot 0.7</td>
<td>A1</td>
</tr>
<tr>
<td>1.6</td>
<td>1.5</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>sort complete</td>
<td>A1cso</td>
</tr>
</tbody>
</table>

### Misreads

- If they have misread a number **at the start of part (a), so genuinely miscopied** and got say 1.0 instead of 0.1 then mark the whole question as a misread – removing the last two A or B marks earned. This gives a maximum total of 9.
- If they have used the correct numbers in part (a) and then use incorrect numbers in part (b) (say 1.0 instead of 0.1) from the beginning of the sort or misread their own numbers during part (b) then count it as an **error in part (b)** but mark part (c) as a misread – giving a maximum of 8 or maybe 7 marks depending on how many marks they lose in (b).

### Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverses the list **in part (b)** then they can score full marks.
- If the list is not reversed in part (b) then mark as a misread (so remove the last two A marks if earned in part (b)). If the list is reversed at the start of part (c) but not in part (b) then still treat this as a misread. If the list is still in ascending order in part (c) award no marks for first fit increasing. If the candidate says that the list needs reversing in part (b) but doesn’t actually show the reversed list in part (b) then remove the final A mark.

#### Ascending (middle left)

<table>
<thead>
<tr>
<th>0.6</th>
<th>1.5</th>
<th>1.6</th>
<th>0.2</th>
<th>0.4</th>
<th>0.5</th>
<th>0.7</th>
<th>0.1</th>
<th>0.9</th>
<th>0.3</th>
<th>(0.4)</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>1.5</td>
<td>1.6</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>(0.1, 0.6)</td>
<td>A1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>1.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>1.6</td>
<td>(0.2, 0.5)</td>
<td>A1ft</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>1.5</td>
<td>0.7</td>
<td>0.9</td>
<td>1.5</td>
<td>1.6</td>
<td>((0.3), 1.5)</td>
<td>A1ft</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.9</td>
<td>1.5</td>
<td>1.6</td>
<td>(0.7)</td>
<td>A1cso+complete</td>
</tr>
</tbody>
</table>

#### Ascending (middle right)

<table>
<thead>
<tr>
<th>0.6</th>
<th>1.5</th>
<th>1.6</th>
<th>0.2</th>
<th>0.4</th>
<th>0.5</th>
<th>0.7</th>
<th>0.1</th>
<th>0.9</th>
<th>0.3</th>
<th>(0.5)</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.4</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.6</td>
<td>1.5</td>
<td>1.6</td>
<td>0.7</td>
<td>0.9</td>
<td>(0.1, 1.6)</td>
<td>A1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
<td>0.6</td>
<td>1.5</td>
<td>0.7</td>
<td>0.9</td>
<td>1.6</td>
<td>(0.4, 0.7)</td>
<td>A1ft</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>1.5</td>
<td>0.9</td>
<td>1.6</td>
<td>(0.3, (0.6), 0.9)</td>
<td>A1ft</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.9</td>
<td>1.5</td>
<td>1.6</td>
<td>A1cso+complete</td>
<td></td>
</tr>
</tbody>
</table>
3. (a) AC, CD, CE; EF; BC

(b) B1

(c) B1 B1

(d) EF, AC, CD, reject AD, CE, reject DE, CB

(e) Time = 40 (days)
Notes for Question 3

Accept the **weight** of each arc to represent the arcs (as each value is unique).

a1M1: Prim’s – first three arcs correctly chosen or first four nodes correctly chosen {A, C, D, E, ...}. Any rejections seen during selection M0. Order of nodes may be seen at the top of the matrix {1, - , 2, 3, 4, -}

a1A1: First four arcs correctly chosen or all six nodes correctly chosen {A, C, D, E, F, B}. Order of nodes may be seen at the top of the matrix {1, 6, 2, 3, 4, 5}

a2A1: CSO (must be considering arcs for this final mark).

**Misread:** Starting at a node other than A scores **M1 only** – **must** have the first three arcs (or four nodes or numbers) correct.

<table>
<thead>
<tr>
<th>Starting at</th>
<th>Minimum arcs required for M1</th>
<th>Nodes</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>AC CD CE</td>
<td>ACDE(FB)</td>
<td>1(6)234(5)</td>
</tr>
<tr>
<td>B</td>
<td>BC AC CD</td>
<td>BCAD(EF)</td>
<td>3124(56)</td>
</tr>
<tr>
<td>C</td>
<td>AC CD CE</td>
<td>CADE(FB)</td>
<td>2(6)134(5)</td>
</tr>
<tr>
<td>D</td>
<td>CD AC CE</td>
<td>DCAE(FB)</td>
<td>3(6)214(5)</td>
</tr>
<tr>
<td>E</td>
<td>EF CE AC</td>
<td>EFCA(DB)</td>
<td>4(6)3(5)12</td>
</tr>
<tr>
<td>F</td>
<td>EF CE AC</td>
<td>FECA(DB)</td>
<td>4(6)3(5)21</td>
</tr>
</tbody>
</table>

b1B1: CAO (weights not required)
c1B1: Any four arcs added correctly (weights not required)
c2B1: CAO (including weights) – bod if arcs ‘appear’ to be crossed out (they may be using the network diagram to answer part (d)).
d1M1: Kruskal’s – first three arcs correctly chosen and **at least one rejection seen at some point**.  
d1A1: All five arcs selected correctly EF, AC, CD, CE, CB.
d2A1: CAO All selections and rejections correct (in correct order and at the correct time).

- Listing all the arcs in order and then listing those arcs in the tree in the correct order is fine for **full marks** (this implies that rejections are correct and at the correct time)
- Listing all the arcs in order and just drawing the MST is **M0**

**SC for part (d):** If the network diagram is incorrect in part (c) **and it is clear that the candidate has used part (c) (instead of the original table) to answer part (d)** then award **M1 only** for the first three arcs correctly chosen and at least one rejection seen at some point provided their network is connected and contains at least nine arcs.

e1B1: CAO (ignore lack/incorrect units)
Question 4

(a) Shortest path S to T: SAGEDHT
Length of shortest path S to T: 30 (miles)

(b) Shortest path S to T via F: SCBFEHT
Length is 31 (miles)

Notes for Question 4

a1M1: A larger value replaced by a smaller value at least once in the working values at either B or E or F or H or T.

a1A1: All values in A, B, C and D correct. The working values at B must be in the correct order.

a2A1: All values in E, F and G correct and the working values in the correct order. Penalise order of labelling only once per question (F, G and E labelled in that order and F must be labelled after A, B, C and D).

a3A1ft: All values in H and T ft correct and the working values in the correct order. Penalise order of labelling only once per question (H and T labelled in that order and H labelled after all other nodes).

a4A1: Route CAO.

a5A1ft: ft on their final value (if answer is not 30 ft their final value at T).

b1B1: Route CAO

b2B1: Length CAO (condone lack of (or incorrect) units throughout).
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 5. (a) | \(AB + DE = 44 + 30 = 74^*\)  
\(AD + BE = 42 + 35 = 77\)  
\(AE + BD = 39 + 38 = 77\)  
Repeat arcs AC, BC and DE | M1 |
| | E.g. ABCADCBEDFGDEGHECA (18 nodes)  
Length: 344 + 74 = 418 | B1  
B1ft |
| (b) | One of AB (44), AD (42) or BD (38) will still have to be repeated.  
**BD(38) is the shortest**  
So start at E and **finish at A**, route length now is 344 + 38 = **382** | M1  
A1  
DA1 |

### Notes for Question 5

**a1**M1: Three distinct pairings of their four odd nodes  
**a1A1:** Any one row correct including pairing **and** total  
**a2A1:** Any two rows correct including pairing **and** total  
**a3A1:** All three rows correct including pairing **and** total  
**a4A1:** CAO correct **arcs** identified AC, BC and DE. Accept ACB or AB via C (check to see if via C appears in working) but **do not** accept AB for this mark  
**b1B1:** Any correct route (checks: eighteen nodes (or seventeen arcs), the route starts and ends at A, pairings AC, BC and DE appear twice in the route and that every letter (A to H inclusive) appears at least once).  
**b2B1ft:** correct answer of 418 or 344 + their least out of a choice of at least two totals given in part (a)  
**c1M1:** Either identifies the need to repeat one pairing which does not include E (could list potential repeats) **or** identifies the need to repeat BD (or 38).  
**c1A1:** Identifies the need to repeat one pairing which does not include E **and** this is BD (38) **because it is the least**. To score the first two marks the candidate must make it clear that they need to repeat **BD because it has the least weight of those pairings that do not include E**.  
**c2DA1:** correct finishing point (A) and length (382). This mark is dependent on them identifying BD (38) as the repeat.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. (a)</td>
<td>He must buy at least 90 boats in total ((x + y \geq 90))</td>
<td>B1 (1)</td>
</tr>
</tbody>
</table>
| (b) | E.g. The number of 2-seater boats \((x)\) must be less than or equal to 1.5 times the number of 4-seater boats \((y)\). \((2x \leq 3y)\)  
E.g. The number of 4-seater boats \((y)\) must be greater than or equal to \(2/3\) the number of 2-seater boats \((x)\). \((\text{check}: y = 2, x = 3, 2, 1, ...)\) | B1 B1 |
| (c) | The correct 3 lines added; \(x + y = 90\); \(3y = 2x\); \(y = x + 30\)  
Region, R labelled | B1; B1; B1 B1 (4) |
| (d) | \((\text{minimise } C = 100x + 300y)\) | B1 (1) |
| (e) | Method clear – either at least 2 vertices tested or objective line drawn \((54, 36), \text{ so } 54 \text{ 2-seater and } 36 \text{ 4-seater}\)  
At a cost of £16 200 | M1 A1 B1 B1 (4)  
12 marks |
Notes for Question 6

a1B1: CAO (must have ‘boats’, ‘least’, ‘90’, must be talking about boats not cost)
b1B1: For a statement in context with either the ratio of coefficients correct (the 2 with the 2-seater and the 3 with the 4-seater) or inequality correct with correct numbers present but not in the correct ratio.
b2B1: Clear accurate correct statement in context.
c1B1: x + y = 90 correctly drawn. Must pass within one small square of the points of intersection with the axes
c2B1: 3y = 2x correctly drawn. Must pass within one small square of the origin and (90, 60).
c3B1: y = x + 30 correctly drawn. Must pass within one small square of (0, 30) and (60, 90).
c4B1: Region, R, correctly labelled – not just implied by shading – must have scored all three previous marks in this part.
d1B1: CAO (isw if 100x + 300y simplified’ to k(100x + 300y) but if 100x + 300y not stated then B0)
e1M1: Line must be correct to within one small square if extended from axis to axis OR attempting to find two vertices of their R (or the correct R) by either reading off their graph or using simultaneous equations and testing using their objective function.
e1A1: Correct objective line (same condition that the line must be correct to within one small square if extended from axis to axis) OR testing (30, 60) correctly (giving 21 000) and testing (54, 36) correctly (giving 16 200).
e1B1: Correct point identified. (Condone in terms of x and y rather than in terms of boats.)
e2B1: CAO – condone lack of/incorrect units on the cost.

Examples for part (b) scoring B1 B1 (useful check: when y = 2, x = 3, 2, 1, … or when x = 3, y = 2, 3, 4, …)

- Twice the number of 2-seater boats must be at most three times the number of 4-seater boats
- Three times the number of 4-seater must be at least twice the number of 2-seater boats
- For every three 2-seater boats there must be at least two 4-seater boats (or multiple of this ratio)
- For every two 4-seater boats there must be at most three 2-seater boats (or multiple of this ratio)
- At most 60% of the total boats are 2-seater
- At least 40% of the total boats are 4-seater

Examples of B1 B0 – in each case either the inequality is the correct way round OR the 2 is with 2-seater boats and the 3 is with the 4-seater boats (accept multiples of 2 and 3) (useful numbers: when y = 2, x = 3, 4, 5,… when x = 3, y = 2, 1, …, when y = 3, x = 2, 1, …, when x = 2, y = 3, 4, 5, …)

- Twice the number of 2-seater boats must be at least three times the number of 4-seater boats
- Three times the number of 4-seater must be at most twice the number of 2-seater boats
- Three times the number of 2-seater must be at least twice the number of 4-seater boats
- For every three 2-seater boats there must be at most two 4-seater boats (or multiple of this ratio)
- For every two 4-seater boats there must be at least three 2-seater boats (or multiple of this ratio)
- For every two 2-seater boats there must be at least three 4-seater boats (or multiple of this ratio)
- For every three 4-seater boats there must be at most two 2-seater boats (or multiple of this ratio)
- At least 60% of the total boats are 2-seater
- At most 40% of the total boats are 4-seater
- At least 60% of the total boats are 4-seater
- At most 40% of the total boats are 2-seater
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7.(a)</strong></td>
<td><img src="image" alt="Diagram" /></td>
<td><strong>M1 A1 (4)</strong></td>
</tr>
<tr>
<td><strong>(b)</strong> Float on M = 42 – 26 – 8 = 8</td>
<td>M1 A1</td>
<td><strong>(2)</strong></td>
</tr>
<tr>
<td><strong>(c)(i)</strong> 2 day delay on P – no effect on the project completion date (float on P is 4)</td>
<td>B1</td>
<td><strong>(2)</strong></td>
</tr>
<tr>
<td><strong>(c)(ii)</strong> 2 day delay on Q – project finishes 2 days late (Q is a critical activity)</td>
<td>B1</td>
<td><strong>(2)</strong></td>
</tr>
<tr>
<td><strong>(d)</strong> (172/53 = 3.245, so) a minimum of 4 workers needed</td>
<td>B1</td>
<td><strong>(1)</strong></td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>(e)</td>
<td><img src="image" alt="Gantt Chart" /></td>
<td>M1 A1 (any 6 more)</td>
</tr>
<tr>
<td>(f)</td>
<td>E.g. Activities H, I, J, K and L together with $22 &lt; \text{time} &lt; 26$ stated. So 5 workers needed</td>
<td>M1 A1 (all 11) (4)</td>
</tr>
<tr>
<td>(g)</td>
<td>The cascade gives a higher lower bound, so (f) is better.</td>
<td>M1 A1 (2)</td>
</tr>
</tbody>
</table>

**17 marks**
Notes for Question 7

Notes:
a1M1: All top boxes complete, values generally increasing left to right, condone one ‘rogue’ (if values do not increase from left to right then if one value is ignored and then the values do increase from left to right then this is considered to be only one rogue value)
a1A1: CAO.
a2M1: All bottom boxes complete, values generally decreasing right to left, condone one ‘rogue’.
a2A1: CAO
b1M1: Correct calculation seen – all three numbers correct (ft), float 0.
b1A1: Float correct (no ft on this mark)
c1B1: CAO
c2B1: CAO
d1B1: 4 with (or without) working scores this mark
e1M1: At least six activities added including six floats. Scheduling diagram scores M0.
e1A1: Six activities including their floats dealt with correctly.
e2M1: All remaining eleven activities including all eleven floats.
e2A1: CAO.

Examples for part (f):

Example 1: Activities H, I, J, K and L with 22 < time < 26 so 5 workers needed.
Example 2: At 10 < time < 14, F, D, E and H must be happening. Activity G must be happening 7 < time < 18 but its duration is 5 so it must also occur at some point in the interval 10 < time < 14 so 5 workers needed.

f1M1: Example 1: A statement with the correct number of workers (5) and the correct activities (H, I, J, K and L) with some mention of time, or
Example 2: A statement with the correct number of workers (5), the correct activities (F, D, E and H) with some mention of time and an indication that G must be happening with the other four activities at some point - give bod but e.g. ‘at time 11 F, D, E, G and H must be happening’ is M0. Scheduling the activities only scores M0.

f1A1: A correct, complete full statement with details of both time and activities. Candidates only need to give a time within the intervals stated.

Please note strict inequalities for the time. Allow e.g. on ‘day 23’ as equivalent to 22 < time < 23.

g1M1: Must have attempted both parts (d) and (f). Their higher lower bound chosen + attempt at a reason.

Allow for the M mark a reason which argues that e.g. the cascade chart gives a better lower bound (e.g. it takes into account exactly when activities must be taking place) or e.g. the calculation gives a better lower bound (e.g. as it takes into account the sum of all the activities) but without specifically answering the question of which of the two bounds is better. Give bod on an attempt at a reason.

g1A1: CAO plus a correct reason given. Acceptable reasons e.g. the cascade gives a larger value or the bound for the cascade shows that the project cannot be done with fewer workers, etc.
Mark Scheme (Results)

Summer 2013

GCE Decision Mathematics 2 (6690/01)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \sqrt{ } \) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - □ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td>M1A1  (2)</td>
</tr>
<tr>
<td></td>
<td>e.g. starting from A: AB, BD, BC, CE or AB, BC, CE, BD</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$2 \times 56 = 112$</td>
<td>B1  (1)</td>
</tr>
<tr>
<td>(c)</td>
<td>$15 \begin{array}{cccc} A &amp; B &amp; C &amp; E \ 15 &amp; 15 &amp; 11 &amp; 18 \end{array}$ $25 = 84$</td>
<td>M1 A1 A1  (3)</td>
</tr>
<tr>
<td></td>
<td>$15 \begin{array}{cccc} D &amp; A \ 18 &amp; 25 \end{array}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>and $15 \begin{array}{cccc} A &amp; B &amp; D &amp; E &amp; C &amp; A \ 15 &amp; 15 &amp; 18 &amp; 11 &amp; 19 \end{array}$ $= 78$</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>78 is the better upper bound</td>
<td>B1ft  (1)</td>
</tr>
<tr>
<td>(e)</td>
<td><img src="image2.png" alt="Diagram" /></td>
<td>1M1A1  (4)</td>
</tr>
</tbody>
</table>

Lower bound = $48 + 15 + 15 = 78$
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>The route is ABDECA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(The optimal route length is 78, since upper bound = lower bound)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a1M1</td>
<td>First three arcs (or all 5 nodes / or numbers across the top of the matrix) selected correctly (may start from any node). Award M1 only for a correct tree with no working.</td>
</tr>
<tr>
<td></td>
<td>a1A1</td>
<td>CAO (order of arc selection clear)</td>
</tr>
<tr>
<td></td>
<td>b1B1</td>
<td>112 CAO</td>
</tr>
<tr>
<td></td>
<td>c1M1</td>
<td>Nearest Neighbour either A-B-C-E-D- or A-B-D-E-C- (condone lack of return to start). Accept 12354 or 12534 across the top of the matrix.</td>
</tr>
<tr>
<td></td>
<td>c1A1</td>
<td>1 route and length CAO (Do not ISW if route length is doubled)</td>
</tr>
<tr>
<td></td>
<td>c2A1</td>
<td>both routes and lengths CAO (Do not ISW if route lengths are doubled)</td>
</tr>
<tr>
<td></td>
<td>d1B1 ft</td>
<td>their stated shortest (must be a number)</td>
</tr>
<tr>
<td></td>
<td>e1M1</td>
<td>Finding correct RMST (maybe implicit) 48 sufficient, or correct numbers. 3 arcs.</td>
</tr>
<tr>
<td></td>
<td>e1A1</td>
<td>CAO; tree or 48 or 11 + 18 + 19 seen.</td>
</tr>
<tr>
<td></td>
<td>e2M1</td>
<td>Adding 2 least arcs to B; 15 and 15 or two out of BA, BC or BD or 30 only</td>
</tr>
<tr>
<td></td>
<td>e2A1</td>
<td>CAO 78</td>
</tr>
<tr>
<td></td>
<td>f1B1</td>
<td>CAO, accept any start point for the correct tour, but must return to start. Dependent on their answer to part (d) = their answer to part (e).</td>
</tr>
</tbody>
</table>

B1 (1)

Total 12
2(a)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18</td>
<td></td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>5</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>C</td>
<td>13</td>
<td>8</td>
<td></td>
<td>21</td>
</tr>
<tr>
<td>D</td>
<td>12</td>
<td></td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>27</td>
<td>18</td>
<td>20</td>
<td>65</td>
</tr>
</tbody>
</table>

B1 (1)

(b)

<table>
<thead>
<tr>
<th>A</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>9 - θ</td>
</tr>
<tr>
<td>C</td>
<td>13 - θ</td>
</tr>
<tr>
<td>D</td>
<td>θ</td>
</tr>
</tbody>
</table>

M1A1 (2)

(c)

<table>
<thead>
<tr>
<th>Shadow costs</th>
<th>10</th>
<th>22</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>X</td>
<td>-11</td>
</tr>
<tr>
<td>-15</td>
<td>B</td>
<td>20</td>
<td>X</td>
</tr>
<tr>
<td>-7</td>
<td>C</td>
<td>21</td>
<td>X</td>
</tr>
<tr>
<td>-1</td>
<td>D</td>
<td>X</td>
<td>0</td>
</tr>
</tbody>
</table>

1M1A1

<table>
<thead>
<tr>
<th>A</th>
<th>18 - θ</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>4 - θ</td>
<td>17 + θ</td>
</tr>
<tr>
<td>D</td>
<td>9 + θ</td>
<td>3 - θ</td>
</tr>
</tbody>
</table>

2M1A1

(θ = 3) entering cell A2, exiting cell D3

(4)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d)</td>
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<table>
<thead>
<tr>
<th>Shadow costs</th>
<th>10</th>
<th>11</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>-4</td>
<td>B</td>
<td>9</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>10</td>
<td>X</td>
</tr>
<tr>
<td>-1</td>
<td>D</td>
<td>X</td>
<td>11</td>
</tr>
</tbody>
</table>

Optimal since no negative improvement indices.

a1B1 CAO

b1M1 A valid route, only one empty square, D1, used, 0’s balance.
b1A1 Correct route, up to an improved solution (six numbers no zeros)

c1M1 Finding 7 shadow costs and 6 Improvement indices
c1A1 Shadow costs [Alt: A(10), B(-5), C(3), D(9), 1(0), 2(12), 3(9)] and improvement indices CAO
c2M1 A valid route, their most negative II chosen, only one empty square used, 0’s balance.
c2A1 CSO (entering A2, and exiting D3 stated)

d1M1 Finding 7 shadow costs and all 6 IIs or at least 1 negative II found.
d1A1 CAO for the shadow costs [Alt: A(10), B(6), C(14), D(9), 1(0), 2(1), 3(-2)] and 6 positive IIs
d2A1 CSO (for part (d)) + optimal.

Marks

M1 A1

A1 (3)

Total 10
3(a) Initial flow = 44

(b) Value of cut = 12+7+4+10+2+5+31 = 71

(c) e.g. SACFHT – 3; SADGIT – 4; SBEDFHT – 2
    e.g. SACFHT – 3; SADFHT – 2; SADGIT – 2; SBEDGHT - 2

(d) e.g.

(e) Maximum flow = minimum cut
    e.g. cut through CH, CF, AD, BD, DE, EG and EI

    a1B1 CAO
    b1B1 CAO
    c1M1 One valid flow augmenting route found and a value stated.
    c1A1 Flow increased by at least 2
    c2A1 A second correct flow route (and value at least 2) correct
    c3A1 CSO Flow increased by 9 and no more.
    d1M1 Consistent flow pattern > 50 (check each node, must have exactly 1
    number per arc)
    d1A1 CAO, showing flow of 53, must follow from their routes.
    e1DM1 Must have attempted (d) and made an attempt at a cut.
    e1A1 cut correct – may be drawn. Refer to max flow-min cut theorem all
    four words (alternative cut: CH, CF, AD, BD, BE).

 Guidance for 3(c)
 SA +7 SB +2 AC +3 AD +4 BD none BE +2
 ED +2 CH none CF +3 EG none EI none
 (DF+2 DG+2 FH +5 FT none FI none GI +4 HT +5 IT +4)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(a)</td>
<td>$\begin{bmatrix} 4 &amp; -6 \ -2 &amp; 3 \ -1 &amp; 2 \end{bmatrix}$ column 2 dominates column 1</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(b)</td>
<td>$\begin{bmatrix} -4 &amp; 2 &amp; 1 \ 6 &amp; -3 &amp; -2 \end{bmatrix}$</td>
<td>B1 B1 (2)</td>
</tr>
</tbody>
</table>
| (c)             | Let $p =$ probability that B plays new row 1  
If A plays 1: B’s expected winnings = $-4p + 6(1 - p) = 6 - 10p$  
If A plays 2: B’s expected winnings = $2p - 3(1 - p) = -3 + 5p$  
If A plays 3: B’s expected winnings = $p - 2(1 - p) = -2 + 3p$ | 1M1A1 (2) |

![Graph showing the expected winnings for different strategies] 

$6 - 10p = -2 + 3p$  
$8 = 13p$  
$p = \frac{8}{13}$  
B should play 1: never,  
play 2 with probability $\frac{8}{13}$ and play 3 with probability $\frac{5}{13}$  
The value of the game is $-\frac{2}{13}$ to B  
B1 B1 (2)  
Total 11
<table>
<thead>
<tr>
<th>Question Number</th>
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<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1B1</td>
<td>CAO (accept reduced matrix or ‘column 2 dominates column 1’ or column crossed out). Allow recover in part (b)</td>
<td></td>
</tr>
<tr>
<td>b1B1</td>
<td>either $3 \times 2$ matrix with correct values (including signs) or $2 \times 3$ matrix with correct values (condone incorrect signs)</td>
<td></td>
</tr>
<tr>
<td>b2B1</td>
<td>CAO</td>
<td></td>
</tr>
<tr>
<td>c1M1</td>
<td>Setting up three probability expressions, implicit definition of ‘p’.</td>
<td></td>
</tr>
<tr>
<td>c1A1</td>
<td>CAO (condone incorrect simplification)</td>
<td></td>
</tr>
<tr>
<td>c1B1ft</td>
<td>Attempt at three lines (correct gradients and intersection with ‘axes’), accept $p &gt; 1$ or $p &lt; 0$ here. Must be functions of p.</td>
<td></td>
</tr>
<tr>
<td>c2B1</td>
<td>CAO $0 \leq p \leq 1$, scale clear (or 1 line = 1), condone lack of labels. Rulers used.</td>
<td></td>
</tr>
<tr>
<td>c2M1</td>
<td>Finding their correct optimal point, must have three lines and set up an equation to find $0 \leq p \leq 1$. Dependent on first B mark in part (c). Must have three intersection points. Solving all three simultaneous equations only is M0.</td>
<td></td>
</tr>
<tr>
<td>c3B1</td>
<td>All three options listed must fit from their p, check page 1 for B should never play 1. $0 \leq$ probabilities $\leq 1$.</td>
<td></td>
</tr>
<tr>
<td>c4B1</td>
<td>-2/13 CAO (accept awrt 0.154)</td>
<td></td>
</tr>
</tbody>
</table>

**SC1:** If column 2 deleted in (a) candidates can earn a maximum of

(a) B0  (b) B1 B0  (c) M1 A0 B1 B0 M1 A0 B1 B1 (max. of 6) – the final B mark is for the value of the game being $-4/3$

**SC2:** If column 3 is deleted in (a) candidates can earn a maximum of

(a) B0  (b) B1 B0  (c) M1 A0 B1 B0 M0 A0 B0 B0
Variable $z$ was increased first, since it has become a basic variable.

<table>
<thead>
<tr>
<th>b.v</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>r</th>
<th>s</th>
<th>t</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>s</td>
<td>-1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>z</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>P</td>
<td>2</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b.v</th>
<th>X</th>
<th>y</th>
<th>z</th>
<th>r</th>
<th>s</th>
<th>t</th>
<th>value</th>
<th>row ops</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>$-\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>4</td>
<td>$R_i \rightarrow 2$</td>
</tr>
<tr>
<td>s</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3}{2}$</td>
<td>1</td>
<td>$-\frac{1}{2}$</td>
<td>10</td>
<td>$R_2 \rightarrow R_1$</td>
</tr>
<tr>
<td>z</td>
<td>$-\frac{3}{2}$</td>
<td>0</td>
<td>1</td>
<td>$-\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>7</td>
<td>$R_1 \rightarrow R_1$</td>
</tr>
<tr>
<td>P</td>
<td>$-\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{5}{2}$</td>
<td>0</td>
<td>3</td>
<td>35</td>
<td>$R_4 \rightarrow 5R_1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b.v</th>
<th>X</th>
<th>y</th>
<th>z</th>
<th>r</th>
<th>s</th>
<th>t</th>
<th>value</th>
<th>row ops</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>14</td>
<td>$R_1 \rightarrow R_2$</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-3</td>
<td>2</td>
<td>-1</td>
<td>20</td>
<td>$R_2 \rightarrow R_2$</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-5</td>
<td>3</td>
<td>-1</td>
<td>37</td>
<td>$R_3 \rightarrow R_2$</td>
</tr>
<tr>
<td>P</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$\frac{5}{2}$</td>
<td>45</td>
<td>$R_4 \rightarrow 5R_2$</td>
</tr>
</tbody>
</table>

$P = 45; x = 20; y = 14; z = 37; r = s = t = 0.$

**Question Number**

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>5(a)</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
</tr>
</tbody>
</table>

**Total 11**
\begin{itemize}

\item \textbf{6 (a)} Since maximising subtract all elements from some \( n \geq 257 \), say 260.

\[
\begin{bmatrix}
9 & 17 & 3 \\
16 & 13 & 5 \\
11 & 8 & 14 \\
\end{bmatrix}
\quad n = 257
\hspace{1cm}
\begin{bmatrix}
6 & 14 & 0 \\
13 & 10 & 2 \\
8 & 5 & 11 \\
\end{bmatrix}
\quad n = 258
\hspace{1cm}
\begin{bmatrix}
7 & 15 & 1 \\
14 & 11 & 3 \\
9 & 6 & 12 \\
\end{bmatrix}
\]

\item \textbf{6 (b)}

\[ x_{ij} = \begin{cases} 
1 & \text{if worker } i \text{ does task } j \\
0 & \text{otherwise} 
\end{cases} \]

Where \( x_{ij} \) indicates worker \( i \) being assigned to task \( j \)

\( i \in \{H, K, J\} \text{ and } j \in \{1, 2, 3\} \)

\begin{itemize}

\item E.g.

\text{Minimise}

\[ P = 9x_{H1} + 17x_{H2} + 3x_{H3} + 16x_{J1} + 13x_{J2} + 5x_{J3} + 11x_{K1} + 8x_{K2} + 14x_{K3} \]

\[ (P = 6x_{H1} + 14x_{H2} + 13x_{J1} + 10x_{J2} + 2x_{J3} + 8x_{K1} + 5x_{K2} + 11x_{K3}) \]

\[ (P = 7x_{H1} + 15x_{H2} + x_{H3} + 14x_{J1} + 11x_{J2} + 3x_{J3} + 9x_{K1} + 6x_{K2} + 12x_{K3}) \]

\text{OR maximise}

\[ P = 251x_{H1} + 243x_{H2} + 257x_{H3} + 244x_{J1} + 247x_{J2} + 255x_{J3} + 249x_{K1} + 252x_{K2} + 246x_{K3} \]

\end{itemize}

Subject to:

\begin{align*}
\begin{array}{ll}
\text{or} & \sum x_{ij} = 1 \\
\text{or} & \sum x_{ij} = 1 \\
\text{or} & \sum x_{ij} = 1 \\
\text{or} & \sum x_{ij} = 1 \\
\text{or} & \sum x_{ij} = 1 \\
\text{or} & \sum x_{ij} = 1 \\
\text{or} & \sum x_{ij} = 1 \\
\end{array}
\end{align*}

\text{a1B1 CAO (o.e.)}

\text{b1B1 possible values of } x_{ij} \text{ defined}

\text{b2B1 Defining } x_{ij} \text{ including the set of values for } i \text{ and } j

\text{b3B1 Objective function}

\text{b4B1 Minimise/Maximise but consistent with objective function}

\text{b1M1 Three equations, unit coefficients, } \sum = 1

\text{b1A1 Any three equations CAO (condone inconsistent notation)}

\end{itemize}
<table>
<thead>
<tr>
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<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>b2A1</td>
<td>All six equations CAO (consistent notation required)</td>
<td></td>
</tr>
</tbody>
</table>
The actions Nigel should take are:
Keep, Keep, Replace, Keep in years 1, 2, 3 and 4 respectively
His income will be £32 000.
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{\text{ }}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper
   - $\square$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

**Method mark for solving 3 term quadratic:**

1. **Factorisation**
   
   \[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = \left| c \right|, \text{ leading to } x = \]
   
   \[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = \left| c \right| \text{ and } |mn| = \left| a \right|, \text{ leading to } x = \]

2. **Formula**
   
   Attempt to use correct formula (with values for \(a\), \(b\) and \(c\)).

3. **Completing the square**
   
   Solving \(x^2 + bx + c = 0\):
   
   \[\left( x \pm \frac{b}{2} \right)^2 \pm q \pm c, \quad q \neq 0, \text{ leading to } x = \ldots \]

**Method marks for differentiation and integration:**

1. **Differentiation**
   
   Power of at least one term decreased by 1. (\(x^n \rightarrow x^{n-1}\))

2. **Integration**
   
   Power of at least one term increased by 1. (\(x^n \rightarrow x^{n+1}\))

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:

**Method mark** for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
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<tr>
<th>Question Number</th>
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<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$M = \begin{pmatrix} x &amp; x-2 \ 3x-6 &amp; 4x-11 \end{pmatrix}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\det M = x(4x-11) - (3x-6)(x-2)$</td>
<td>Correct attempt at determinant</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>$x^2 + x - 12 (= 0)$</td>
<td>Correct 3 term quadratic</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>$(x+4)(x-3) (= 0) \rightarrow x = ...$</td>
<td>Their 3TQ = 0 and attempts to solve relevant quadratic using factorisation or completing the square or correct quadratic formula leading to $x =$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>$x = -4, x = 3$</td>
<td>Both values correct</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>

Notes

$x(4x-11) = (3x-6)(x-2)$ award first M1
$\pm(x^2 + x - 12)$ seen award first M1A1

Method mark for solving 3 term quadratic:

1. Factorisation
   $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x =$
   $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x =$

2. Formula
   Attempt to use correct formula (with values for $a$, $b$ and $c$).

3. Completing the square
   Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x =$...

Both correct with no working 4/4, only one correct 0/4

Total 4
<table>
<thead>
<tr>
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<th>Scheme</th>
<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( f(x) = \cos(x^2) - x + 3 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a)  
\( f(2.5) = 1.499 \ldots \)  
\( f(3) = -0.9111 \ldots \)  
Either any one of \( f(2.5) = \text{awrt } 1.5 \) or \( f(3) = \text{awrt } -0.91 \)  
M1

Sign change (positive, negative) (and \( f(x) \) is continuous) therefore root or equivalent.  
Both \( f(2.5) = \text{awrt } 1.5 \) and \( f(3) = \text{awrt } -0.91 \), sign change and conclusion.  
A1

Use of degrees gives \( f(2.5) = 1.494 \) and \( f(3) = 0.988 \) which is awarded M1A0  
(2)

(b)  
\[
\alpha = \frac{3 - \alpha}{"0.91113026188"} = \frac{\alpha - 2.5}{"1.4994494182"}
\]
Correct linear interpolation method – accept equivalent equation - ensure signs are correct.  
M1 A1ft

\[
\alpha = \frac{3 \times 1.499\ldots + 2.5 \times 0.9111\ldots}{1.499\ldots + 0.9111\ldots}
\]
\( \alpha = 2.81 \) (2d.p.)  
A1

(3)

Notes  
Alternative (b)  
Gradient of line is \( \frac{1.499\ldots - 0.9111\ldots}{0.5} = -4.82 \) (3sf). Attempt to find equation of straight line and equate \( y \) to 0 award M1 and A1ft for their gradient awrt 3sf.
<table>
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<tr>
<th>Question Number</th>
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<th>Notes</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(a)</td>
<td>Ignore part labels and mark part (a) and part (b) together.</td>
<td>Attempts $f(0.5)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 9\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - 13$</td>
<td>Sets $f(0.5) = 0$ and leading to $k = \ldots$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{4} = \frac{9}{4} + k \cdot -13 = 0 \Rightarrow k = \ldots$</td>
<td>$k = 30$</td>
<td>cao A1</td>
</tr>
<tr>
<td><strong>Alternative using long division:</strong></td>
<td>2$x^3 - 9x^2 + kx - 13 = (2x - 1)$</td>
<td>Full method to obtain a remainder as a function of $k$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= x^2 - 4x + \frac{1}{2}k - 2$ (Quotient)</td>
<td></td>
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<tr>
<td></td>
<td>Remainder $\frac{1}{2}k - 15$</td>
<td>Their remainder = 0</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}k - 15 = 0$</td>
<td></td>
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<tr>
<td></td>
<td>$k = 30$</td>
<td></td>
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<tr>
<td><strong>Alternative by inspection:</strong></td>
<td>$(2x - 1)(x^2 - 4x + 13) = 2x^3 - 9x^2 + 30x - 13$</td>
<td>First M for $(2x - 1)(x^2 + bx + c)$ or $(x - \frac{1}{2})(2x^2 + bx + c)$</td>
<td>M1dM1</td>
</tr>
<tr>
<td></td>
<td>Second M1 for $ax^2 + bx + c$ where $(b = -4$ or $c = 13)$ or $(b = -8$ or $c = 26)$</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$k = 30$</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>$f(x) = (2x - 1)(x^2 - 4x + 13)$</td>
<td>Uses inspection or long division or compares coefficients and $(2x - 1)$ or $(x - \frac{1}{2})$ to obtain a quadratic factor of this form.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>or $\left(x - \frac{1}{2}\right)(2x^2 - 8x + 26)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x^2 - 4x + 13$ or $2x^2 - 8x + 26$</td>
<td>$(x^2 - 4x + 13)$ or $(2x^2 - 8x + 26)$ seen</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{4 \pm \sqrt{4^2 - 4 \times 13}}{2}$ or equivalent</td>
<td>Use of correct quadratic formula for their 3TQ or completes the square.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{4 \pm 6i}{2} = 2 \pm 3i$</td>
<td>oe</td>
<td>A1</td>
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<td><strong>Total</strong></td>
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<td>7</td>
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<td>Question Number</td>
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<tr>
<td>4(a)</td>
<td>$y = \frac{4}{x} = 4x^{-1} \Rightarrow \frac{dy}{dx} = -4x^{-2} = -\frac{4}{x^2}$</td>
<td>Use of the product rule. The sum of two terms including $\frac{dy}{dx}$, one of which is correct.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$xy = 4 \Rightarrow x \frac{dy}{dx} + y = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = -\frac{2}{t^2} \frac{1}{2}$</td>
<td>their $\frac{dy}{dt} \frac{1}{2}$</td>
<td></td>
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<tr>
<td></td>
<td>$\frac{dy}{dx} = -4x^{-2} \lor x \frac{dy}{dx} + y = 0 \lor \frac{dy}{dx} = -\frac{2}{t^2} \frac{1}{2}$</td>
<td>Correct derivative $-4x^{-2}, -\frac{y}{x}$ or $-\frac{1}{t^2}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(\text{So, } m_n = t^2)</td>
<td>Perpendicular gradient rule (m_n m_t = -1)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$y - \frac{2}{t} = t^2 (x - 2t)$</td>
<td>(y - \frac{2}{t} = \text{their } m_n (x - 2t)) or (y = mx + c) with their (m_n) and ((2t, \frac{2}{t})) in an attempt to find 'c'. Their gradient of the normal must be different from their gradient of the tangent and have come from calculus and should be a function of (t).</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$ty - t^2 x = 2 - 2t^4$ *</td>
<td></td>
<td>A1* cso</td>
</tr>
<tr>
<td>(b)</td>
<td>$t = -\frac{1}{2} \Rightarrow -\frac{1}{2}y - \left(-\frac{1}{2}\right)^3 x = 2 - 2\left(-\frac{1}{2}\right)^3$</td>
<td>Substitutes the given value of (t) into the normal</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$4y - x + 15 = 0$</td>
<td></td>
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<tr>
<td></td>
<td>$y = \frac{4}{x} \Rightarrow x^2 - 15x - 16 = 0 \lor$</td>
<td></td>
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<td></td>
<td>(\frac{2t, \frac{2}{t}}{t} \Rightarrow \frac{8}{t} - 2t + 15 = 0 \Rightarrow 2t^2 - 15t - 8 = 0 \lor$</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(x = \frac{4}{y} \Rightarrow 4y^2 + 15y - 4 = 0).</td>
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</table>
|                 | \((x+1)(x-16) = 0 \Rightarrow x = \text{or } \lor (2t+1)(t-8) = 0 \Rightarrow t = \text{or } \lor (4y-1)(y+4) = 0 \Rightarrow y = \text{ or }
\((P: x = -1, y = -4, Q: x = 16, y = \frac{1}{4}\) | Correct values for \(x\) and \(y\) | A1 |
<p>| | | | |
|                 | | | |
|                 | Total 9 | | |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
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<tbody>
<tr>
<td><strong>5(a)</strong></td>
<td>((r + 2)(r + 3) = r^2 + 5r + 6)</td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td>(\sum (r^2 + 5r + 6) = \frac{1}{6} n (n+1)(2n+1) + 5 \times \frac{1}{2} n(n+1), + 6n)</td>
<td>M1: Use of correct expressions for (\sum r^2) and (\sum r)</td>
<td>M1, B1ft</td>
<td></td>
</tr>
<tr>
<td>(= \frac{1}{3} n \left[ \frac{1}{2} (n+1)(2n+1) + \frac{15}{2} (n+1)+18 \right] )</td>
<td>B1ft: (\sum k = nk)</td>
<td></td>
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<tr>
<td>(= \frac{1}{3} n \left[ \frac{1}{2} n^2 + \frac{3}{2} n + \frac{1}{2} + \frac{15}{2} n + \frac{15}{2} + 18 \right] )</td>
<td>M1: Factors out (n) ignoring treatment of constant.</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td>(= \frac{1}{3} n \left[ n^2 + 9n + 26 \right] )</td>
<td>A1: Correct expression with (\frac{1}{3} n) or (\frac{1}{6} n) factored out, allow recovery.</td>
<td>A1*cs</td>
<td></td>
</tr>
<tr>
<td><strong>5(b)</strong></td>
<td>(\sum_{r=1}^{n} 3f(n) - f(n \text{ or } n+1)) and attempt to use part (a).</td>
<td>M1: f(3n) – f(n or n+1) and attempt to use part (a).</td>
<td>M1A1</td>
</tr>
<tr>
<td>(= n (9n^2 + 27n + 26) - \frac{1}{3} n (n^2 + 9n + 26))</td>
<td>A1: Equivalent correct expression</td>
<td></td>
<td></td>
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<tr>
<td>(= \frac{2}{3} n \left( \frac{27}{2} n^2 + 81 \right) n + 39 - \frac{1}{2} n^2 - \frac{9}{2} n - 13 )</td>
<td>Factors out (\frac{2}{3} n) dependent on previous M1</td>
<td>dM1</td>
<td></td>
</tr>
<tr>
<td>(= \frac{2}{3} n \left( 13n^2 + 36n + 26 \right) )</td>
<td>Accept correct expression.</td>
<td>A1</td>
<td></td>
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<tr>
<td>((a = 13, b = 36, c = 26))</td>
<td></td>
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<td><strong>Total</strong></td>
<td></td>
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<tr>
<td>6(a)</td>
<td>( y = 2a^2x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^2x^{-\frac{1}{2}} )</td>
<td>( \frac{1}{x^2} \rightarrow x^{\frac{1}{2}} )</td>
<td>( M1 )</td>
</tr>
<tr>
<td></td>
<td>( y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a )</td>
<td>( k) \frac{dy}{dx} = c )</td>
<td>( \frac{dy}{dx} \times \frac{1}{dx} ), Can be a function of ( p ) or ( t ).</td>
</tr>
<tr>
<td></td>
<td>or ( \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 2a - \frac{1}{2ap} )</td>
<td>Differentiation is accurate.</td>
<td>( A1 )</td>
</tr>
<tr>
<td></td>
<td>( \frac{dy}{dx} = a^2x^{-\frac{1}{2}} ) or ( 2y\frac{dy}{dx} = 4a ) or ( \frac{dy}{dx} = 2a - \frac{1}{2ap} )</td>
<td>Applies ( y - 2ap = \frac{1}{p}(x - ap^2) ) ( ) or ( y = (\text{their } m)x + c ) using ( x = ap^2 ) and ( y = 2ap ) in an attempt to find ( c ). Their ( m ) must be a function of ( p ) from calculus.</td>
<td>( M1 )</td>
</tr>
<tr>
<td></td>
<td>( py - x = ap^2 )</td>
<td>Correct completion to printed answer*</td>
<td>( A1 ) cso</td>
</tr>
<tr>
<td>(b)</td>
<td>( qy - x = aq^2 )</td>
<td></td>
<td>( B1 )</td>
</tr>
<tr>
<td>(c)</td>
<td>( qy - aq^2 = py - ap^2 )</td>
<td>Attempt to obtain an equation in one variable ( x ) or ( y )</td>
<td>( M1 )</td>
</tr>
<tr>
<td></td>
<td>( y(q - p) = aq^2 - ap^2 )</td>
<td>Attempt to isolate ( x ) or ( y )</td>
<td>( M1 )</td>
</tr>
<tr>
<td></td>
<td>( y = \frac{aq^2 - ap^2}{q - p} )</td>
<td></td>
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<tr>
<td></td>
<td>( y = a(p + q) ) or ( ap + aq )</td>
<td>A1: Either one correct simplified coordinate A1: Both correct simplified coordinates</td>
<td>( A1, A1 )</td>
</tr>
<tr>
<td></td>
<td>( x = apq )</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>( (R(apq, ap + aq)) )</td>
<td></td>
<td></td>
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<tr>
<td>(d)</td>
<td>( 'apq' = -a )</td>
<td>Their ( x ) coordinate of ( R = -a )</td>
<td>( M1 )</td>
</tr>
<tr>
<td></td>
<td>( pq = -1 )</td>
<td>Answer only: Scores 2/2 if ( x ) coordinate of ( R ) is ( apq ) otherwise 0/2.</td>
<td>( A1 )</td>
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<td></td>
<td>( (2) )</td>
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<td>Total 11</td>
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<td>Question Number</td>
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<tr>
<td></td>
<td>( z_1 = 2 + 3i, \quad z_2 = 3 + 2i )</td>
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(a) 
\[ z_1 + z_2 = 5 + 5i \Rightarrow |z_1 + z_2| = \sqrt{5^2 + 5^2} \]
Adds \( z_1 \) and \( z_2 \) and correct use of Pythagoras. i under square root award M0. M1
\[ \sqrt{50} (= 5\sqrt{2}) \]
A1 cao

(b) 
\[ \frac{z_1 z_3}{z_2} = \frac{(2 + 3i)(a + bi)}{3 + 2i} \]
Substitutes for \( z_1, z_2 \) and \( z_3 \) and multiplies by \( \frac{3 - 2i}{3 - 2i} \) M1
\[ (3 + 2i)(3 - 2i) = 13 \]
13 seen. B1
\[ \frac{z_1 z_3}{z_2} = \frac{(12a - 5b) + (5a + 12b)i}{13} \]
M1: Obtains a numerator with 2 real and 2 imaginary parts. dM1A1
A1: As stated or \( \frac{(12a - 5b)}{13} + \frac{(5a + 12b)}{13} i \) ONLY.

(c) 
\[ 12a - 5b = 17 \]
\[ 5a + 12b = -7 \]
Compares real and imaginary parts to obtain 2 equations which both involve \( a \) and \( b \). Condone sign errors only. M1
\[ 60a - 25b = 85 \]
\[ 60a + 144b = -84 \] \Rightarrow \( b = -1 \) Solves as far as \( a = \) or \( b = \) dM1
\[ a = 1, \quad b = -1 \]
Both correct A1
Correct answers with no working award 3/3.

(d) 
\[ \arg(w) = -\tan^{-1}\left(\frac{7}{17}\right) \]
Accept use of \( \pm \tan^{-1} \) or \( \pm \tan \) awrt \( 0.391 \) or \( 5.89 \) implies M1. M1
\[ \approx \text{awrt } 0.391 \text{ or awrt } 5.89 \]
A1

Total 11
<table>
<thead>
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<tbody>
<tr>
<td>8(a)</td>
<td>[ \mathbf{A}^2 = \begin{pmatrix} 6 &amp; -2 \ -4 &amp; 1 \end{pmatrix} \begin{pmatrix} 6 &amp; -2 \ -4 &amp; 1 \end{pmatrix} = \begin{pmatrix} 44 &amp; -14 \ -28 &amp; 9 \end{pmatrix} ]</td>
<td>M1: Attempt both ( \mathbf{A}^2 ) and ( 7\mathbf{A} + 2\mathbf{I} )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>[ 7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 42 &amp; -14 \ -28 &amp; 7 \end{pmatrix} + \begin{pmatrix} 2 &amp; 0 \ 0 &amp; 2 \end{pmatrix} = \begin{pmatrix} 44 &amp; -14 \ -28 &amp; 9 \end{pmatrix} ]</td>
<td>A1: Both matrices correct</td>
<td></td>
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<td>OR ( \mathbf{A}^2 - 7\mathbf{A} = \mathbf{A}(\mathbf{A} - 7\mathbf{I}) )</td>
<td>M1 for expression and attempt to substitute and multiply ((2\times2)(2\times2) = 2\times2)</td>
<td></td>
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<td></td>
<td>[ \mathbf{A}(\mathbf{A} - 7\mathbf{I}) = \begin{pmatrix} 6 &amp; -2 \ -4 &amp; 1 \end{pmatrix} \begin{pmatrix} -1 &amp; -2 \ -4 &amp; -6 \end{pmatrix} = \begin{pmatrix} 2 &amp; 0 \ 0 &amp; 2 \end{pmatrix} = 2\mathbf{I} ]</td>
<td>A1 cso</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>[ \mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I} \Rightarrow \mathbf{A} = 7\mathbf{I} + 2\mathbf{A}^{-1} ]</td>
<td>Require one correct line using accurate expressions involving ( \mathbf{A}^{-1} ) and identity matrix to be clearly stated as ( \mathbf{I} ).</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ \mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})^* ]</td>
<td>A1* cso</td>
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<td></td>
<td>Numerical approach award 0/2.</td>
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<tr>
<td>(c)</td>
<td>[ \mathbf{A}^{-1} = \frac{1}{2} \begin{pmatrix} -1 &amp; -2 \ -4 &amp; -6 \end{pmatrix} ]</td>
<td>Correct inverse matrix or equivalent</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>[ \frac{1}{2} \begin{pmatrix} -1 &amp; -2 \ -4 &amp; -6 \end{pmatrix} \begin{pmatrix} 2k + 8 \ -2k - 5 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -2k - 8 + 4k + 10 \ -8k - 32 + 12k + 30 \end{pmatrix} ]</td>
<td>Matrix multiplication involving their inverse and ( k ): ((2\times2)(2\times1) = 2\times1). N.B. ( \begin{pmatrix} 6 &amp; -2 \ -4 &amp; 1 \end{pmatrix} \begin{pmatrix} 2k + 8 \ -2k - 5 \end{pmatrix} ) is M0</td>
<td>M1</td>
</tr>
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<td></td>
<td>( \begin{pmatrix} k + 1 \ 2k - 1 \end{pmatrix} ) or ( (k + 1, 2k - 1) )</td>
<td>((k + 1)) first A1, ((2k - 1)) second A1</td>
<td>A1, A1</td>
</tr>
<tr>
<td>Or:</td>
<td>[ \begin{pmatrix} 6 &amp; -2 \ -4 &amp; 1 \end{pmatrix} \begin{pmatrix} x \ y \end{pmatrix} = \begin{pmatrix} 2k + 8 \ -2k - 5 \end{pmatrix} ]</td>
<td>Correct matrix equation.</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(6x - 2y = 2k + 8) (4x + y = -2k - 5 \Rightarrow x = \ldots ) or ( y = \ldots )</td>
<td>Multiply out and attempt to solve simultaneous equations for ( x ) or ( y ) in terms of ( k ).</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \begin{pmatrix} k + 1 \ 2k - 1 \end{pmatrix} ) or ( (k + 1, 2k - 1) )</td>
<td>((k + 1)) first A1, ((2k - 1)) second A1</td>
<td>A1, A1</td>
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<td>Total 8</td>
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</table>
(a) \( u_t = 8 \) given
\[ n = 1 \Rightarrow u_1 = 4^1 + 3(1) + 1 = 8 \quad (\therefore \text{true for } n = 1) \]
Assume true for \( n = k \) so that \( u_k = 4^k + 3k + 1 \)

\[
u_{k+1} = 4(4^k + 3k + 1) - 9k = 4^{k+1} + 12k + 4 - 9k = 4^{k+1} + 3k + 4\]

Correct completion to an expression in terms of \( k + 1 \)

If true for \( n = k \) then true for \( n = k + 1 \) and as true for \( n = 1 \) true for all \( n \)

(b) Condone use of \( n \) here.

\[
lhs = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \quad \text{Shows true for } m = 1 \quad \text{B1}
\]

\[
rhs = \begin{pmatrix} 2(1) + 1 & -4(1) \\ 1 & 1 - 2(1) \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}
\]

Assume \( \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} = k + 1 - 2k \)

\[
\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k + 1 & -4k \\ k & 1 - 2k \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k + 1 & -4k \\ k & 1 - 2k \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2k + 1 & -4k \\ k & 1 - 2k \end{pmatrix} \quad \text{M1 temp}
\]

\[
= \begin{pmatrix} 6k + 3 - 4k & -8k - 4 + 4k \\ 3k + 1 - 2k & -4k - 1 + 2k \end{pmatrix}
\]

Or equivalent 2x2 matrix.

\[
= \begin{pmatrix} 2k + 3 & -4k - 4 \\ k + 1 & -2k - 1 \end{pmatrix} \quad \text{B1 temp}
\]

Correct completion to a matrix in terms of \( k + 1 \)

If true for \( m = k \) then true for \( m = k + 1 \) and as true for \( m = 1 \) true for all \( m \)

Conclusion with all 4 underlined elements that can be seen anywhere in the solution; \( m \) defined incorrectly award A0.  

\[ \text{Total 10} \]
Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 2 (6668/01)
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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol \( \sqrt{\text{ }} \) will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **oe** – or equivalent (and appropriate)
   - **dep** – dependent
   - **indep** – independent
   - **dp**: decimal places
   - **sf**: significant figures
   - \( * \) The answer is printed on the paper
   - \( \square \) The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:
1. Factorisation
   \[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x =\]
   \[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x =\]

2. Formula
   Attempt to use correct formula (with values for \(a, b\) and \(c\)).

3. Completing the square
   Solving \(x^2 + bx + c = 0\): \[\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to } x = \ldots\]

Method marks for differentiation and integration:
1. Differentiation
   Power of at least one term decreased by 1. (\(x^n \rightarrow x^{n-1}\))

2. Integration
   Power of at least one term increased by 1. (\(x^n \rightarrow x^{n+1}\))

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
**Question Number** | **Scheme** | **Marks**
--- | --- | ---
1. (a) \[ \frac{2}{(2r+1)(2r+3)} = \frac{A}{2r+1} + \frac{B}{2r+3} = \frac{1}{2r+1} - \frac{1}{2r+3} \] (2) M1,A1

(b) \[ \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \ldots - \frac{1}{2n+1} - \frac{1}{2n+3} \]

\[ = \frac{1}{3} - \frac{1}{2n+3} = \frac{2n+3-3}{3(2n+3)} \] M1

\[ \sum_{r}^{n} \frac{3}{(2r+1)(2r+3)} = \frac{3}{2} \times \frac{2n}{3(2n+3)} = \frac{n}{2n+3} \] M1depA1 (3)

---

**Notes for Question 1**

(a) M1 for any valid attempt to obtain the PFs

A1 for \[ \frac{1}{2r+1} - \frac{1}{2r+3} \]

**NB** With no working shown award M1A1 if the correct PFs are written down, but M0A0 if either one is incorrect

(b) M1 for using their PFs to split each of the terms of the sum or of \[ \sum^{n} \frac{2}{(2r+1)(2r+3)} \] into 2 PFs.

At least 2 terms at the start and 1 at the end needed to show the diagonal cancellation resulting in two remaining terms.

M1dep for simplifying to a single fraction and multiplying it by the appropriate constant

A1cao for \[ \sum = \frac{n}{2n+3} \]

**NB:** If \( r \) is used instead of \( n \) (including for the answer), only M marks are available.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| **2** | **(a)** $z = 5 \sqrt{3} - 5i = r(\cos \theta + i \sin \theta)$  
  
  $r = \sqrt{(5^2 \times 3 + 5^2)} = 10$  
  
  **(b)** arg $z = \arctan\left(-\frac{5}{5\sqrt{3}}\right) = -\frac{\pi}{6}$ (or $-\frac{\pi}{6} \pm 2n\pi$)  
  
  **(c)** $\left|\frac{w}{z}\right| = \frac{2}{10} = \frac{1}{5}$ or 0.2  
  
  **(d)** arg $\left(\frac{w}{z}\right) = \frac{\pi}{4} - \left(-\frac{\pi}{6}\right) = \frac{5\pi}{12}$ (or $\frac{5\pi}{12} \pm 2n\pi$)  | **B1** (1)  
  
  **M1,A1** (2)  
  
  **B1** (1)  
  
  **M1,A1** (2)  |

[6]
Notes for Question 2

(a) 
B1 for \(|z| = 10\) no working needed

(b) 
M1 for \(\arg z = \arctan \left( \pm \frac{5}{5\sqrt{3}} \right)\), \(\tan(\arg z) = \pm \frac{5}{5\sqrt{3}}\), \(\arg z = \arctan \left( \pm \frac{5\sqrt{3}}{5} \right)\) or
\[
\tan(\arg z) = \pm \frac{5\sqrt{3}}{5} \quad \text{OR use their } |z| \text{ with sin or cos used correctly}
\]
A1 for \(-\frac{\pi}{6}\) \(\left( \text{or } -\frac{\pi}{6} \pm 2n\pi \right)\) (must be 4th quadrant)

(c) 
B1 for \(\left| \frac{w}{z} \right| = \frac{2}{10}\) or \(\frac{1}{5}\) or 0.2

(d) 
M1 for \(\arg \left( \frac{w}{z} \right) = \frac{\pi}{4} - \arg z\) using their \(\arg z\)
A1 for \(\frac{5\pi}{12}\) \(\left( \text{or } \frac{5\pi}{12} \pm 2n\pi \right)\)

Alternative for (d):
Find \(\frac{w}{z} = \left( \sqrt{6} - \sqrt{2} \right) + \left( \sqrt{6} + \sqrt{2} \right) i\)
\[
\tan\left( \arg \frac{w}{z} \right) = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} \quad \text{M1 from their } \frac{w}{z}
\]
\[
\arg \left( \frac{w}{z} \right) = \frac{5\pi}{12} \quad \text{A1 cao}
\]

Work for (c) and (d) may be seen together – give B and A marks only if modulus and argument are clearly identified
\[
\text{ie } \frac{1}{5} \left( \cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) \text{ alone scores B0M1A0}
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( (x = 0) \ \frac{d^3 y}{dx^3} = \sin 0 - 4 \times \frac{1}{2} = -2 )</td>
</tr>
<tr>
<td></td>
<td>( \frac{d^3 y}{dx^3} + 4 \frac{dy}{dx} - \cos x ( = 0 ) )</td>
</tr>
<tr>
<td></td>
<td>( (x = 0) \ \frac{d^3 y}{dx^3} = \cos 0 - 4 \times \frac{1}{8} = \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td>( y = y_0 + x \left( \frac{dy}{dx} \right)_0 + \frac{x^2}{2!} \left( \frac{d^2 y}{dx^2} \right)_0 + \frac{x^3}{3!} \left( \frac{d^3 y}{dx^3} \right)_0 + .... )</td>
</tr>
<tr>
<td></td>
<td>( y = \frac{1}{2} + x \times \frac{1}{8} + \frac{x^2}{2} \times (-2) + \frac{x^3}{6} \times \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td>( y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12} )</td>
</tr>
<tr>
<td></td>
<td><strong>Alt:</strong> ( y = \frac{1}{2} + \frac{x}{8} + ax^2 + bx^3 + ... )</td>
</tr>
<tr>
<td></td>
<td>( y'' = 2a + 6bx + ... )</td>
</tr>
<tr>
<td></td>
<td>( 2a + 6bx + ... = \sin x - \left( \frac{1}{2} + \frac{x}{8} + ax^2 + bx^3 ... \right) )</td>
</tr>
<tr>
<td></td>
<td>( 2a + 2 = 0 \quad a = -1 )</td>
</tr>
<tr>
<td></td>
<td>( 6b + \frac{1}{2} = 1 \quad b = \frac{1}{12} )</td>
</tr>
<tr>
<td></td>
<td>( y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12} )</td>
</tr>
<tr>
<td>Marks</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>M1 (2! or 2 and 3! or 6)</td>
</tr>
<tr>
<td></td>
<td>A1 cao [5]</td>
</tr>
<tr>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>A1 Correct differentiation and equation used</td>
</tr>
<tr>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>A1 cao</td>
</tr>
</tbody>
</table>
Notes for Question 3

B1 for \( \left( \frac{d^2 y}{dx^2} \right)_0 = -2 \) wherever seen

M1 for attempting the differentiation of the given equation. To obtain \( \frac{d^3 y}{dx^3} \pm k \frac{dy}{dx} \pm \cos x ( = 0 ) \) oe

A1 for substituting \( x = 0 \) to obtain \( \left( \frac{d^3 y}{dx^3} \right)_0 = \frac{1}{2} \)

M1 for using the expansion \([ y = f(x) ] = f(0) + xf'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{3!} f'''(0) \) with their values for \( \frac{d^3 y}{dx^3} \) and \( \frac{d^2 y}{dx^2} \). Factorial can be omitted in the \( x^2 \) term but must be shown explicitly in the \( x^3 \) term or implied by further working eg using 6.

A1cao for \( y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12} \) (Ignore any higher powers included) Exact decimals allowed. **Must include** \( y = \ldots \).

**Alternative:**

B1 for \( y = \frac{1}{2} + \frac{x}{8} + ax^2 + bx^3 + \ldots \)

M1 for differentiating this twice to get \( y'' = 2a + 6bx + \ldots \) (may not be completely correct)

A1 for correct differentiation and using the given equation and the expansion of \( \sin x \) to get \( 2a + 6bx + \ldots = \left( x - \frac{x^3}{3} + \ldots \right) - 4\left( \frac{1}{2} + \frac{x}{8} + \ldots \right) \)

M1 for equating coefficients to obtain a value for \( a \) or \( b \)

A1 cao for \( y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12} \) (Ignore any higher powers included)
4 (a) Assume true for \( n = k \): \( z^k = r^k \left( \cos k\theta + i \sin k\theta \right) \)

\[
n = k + 1: \quad z^{k+1} = (z^k \cdot z^1) = r^k \left( \cos k\theta + i \sin k\theta \right) \times r \left( \cos \theta + i \sin \theta \right)
\]

\[
= r^{k+1} \left( \cos k\theta \cos \theta - \sin k\theta \sin \theta + i \left( \sin k\theta \cos \theta + \cos k\theta \sin \theta \right) \right)
\]

\[
= r^{k+1} \left( \cos (k+1)\theta + i \sin (k+1)\theta \right)
\]

\[
∴ \text{if true for } n = k, \text{ also true for } n = k + 1
\]

\[
k = 1 \quad z^1 = r^1 \left( \cos \theta + i \sin \theta \right); \quad \text{True for } n = 1 \quad ∴ \text{true for all } n
\]

Alternative: See notes for use of \( r e^{i\theta} \) form

(b) \( w = 3 \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \)

\[
w^5 = 3^5 \left( \cos \frac{15\pi}{4} + i \sin \frac{15\pi}{4} \right)
\]

\[
w^5 = 243 \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i \right) \left[ \frac{243\sqrt{2}}{2} \right] \quad \text{o.e}
\]

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
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<tbody>
<tr>
<td>4</td>
<td>Assume true for ( n = k ): ( z^k = r^k \left( \cos k\theta + i \sin k\theta \right) )</td>
</tr>
<tr>
<td></td>
<td>( n = k + 1: \quad z^{k+1} = (z^k \cdot z^1) = r^k \left( \cos k\theta + i \sin k\theta \right) \times r \left( \cos \theta + i \sin \theta \right) )</td>
</tr>
<tr>
<td></td>
<td>( = r^{k+1} \left( \cos k\theta \cos \theta - \sin k\theta \sin \theta + i \left( \sin k\theta \cos \theta + \cos k\theta \sin \theta \right) \right) )</td>
</tr>
<tr>
<td></td>
<td>( = r^{k+1} \left( \cos (k+1)\theta + i \sin (k+1)\theta \right) )</td>
</tr>
<tr>
<td></td>
<td>∴ if true for ( n = k ), also true for ( n = k + 1 )</td>
</tr>
<tr>
<td></td>
<td>( k = 1 \quad z^1 = r^1 \left( \cos \theta + i \sin \theta \right); \quad \text{True for } n = 1 \quad ∴ \text{true for all } n )</td>
</tr>
<tr>
<td></td>
<td>Alternative: See notes for use of ( r e^{i\theta} ) form</td>
</tr>
<tr>
<td></td>
<td>( w = 3 \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) )</td>
</tr>
<tr>
<td></td>
<td>( w^5 = 3^5 \left( \cos \frac{15\pi}{4} + i \sin \frac{15\pi}{4} \right) )</td>
</tr>
<tr>
<td></td>
<td>( w^5 = 243 \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i \right) \left[ \frac{243\sqrt{2}}{2} \right] \quad \text{o.e} )</td>
</tr>
</tbody>
</table>

[7]
Notes for Question 4

(a) NB: Allow each mark if \( n, n+1 \) used instead of \( k, k+1 \)

M1 for using the result for \( n=k \) to write \( z^{k+1} = z^{k} \times z = r^{k} (\cos k\theta + i \sin k\theta) \times r (\cos \theta + i \sin \theta) \)

M1 for multiplying out and collecting real and imaginary parts, using \( i^2 = -1 \)

OR using sum of arguments and product of moduli to get \( r^{k+1} (\cos (k\theta + \theta) + i \sin (k\theta + \theta)) \)

M1dep for using the addition formulae to obtain single \( \cos \) and \( \sin \) terms

OR factorise the argument \( r^{k+1} (\cos (k + 1)\theta + i \sin (k + 1)\theta) \)

Dependent on the second M mark.

A1cso for \( r^{k+1} (\cos (k + 1)\theta + i \sin (k + 1)\theta) \) Only give this mark if all previous steps are fully correct.

A1cso All 5 underlined statements must be seen

Alternative: Using Euler’s form

\[
\begin{align*}
z &= r (\cos \theta + i \sin \theta) = re^{i\theta} \quad \text{M1 May not be seen explicitly} \\
z^{k+1} &= z^{k} \times z = (re^{i\theta})^{k} \times re^{i\theta} = r^{k} e^{ik\theta} \times re^{i\theta} \quad \text{M1} \\
&= r^{k+1} e^{i(k+1)\theta} \quad \text{M1dep on 2nd M mark} \\
&= r^{k+1} (\cos (k + 1)\theta + i \sin (k + 1)\theta) \quad \text{A1cso} \\
&= r^{k+1} (\cos \theta + i \sin \theta) \\
\end{align*}
\]

True for \( n = 1 \) : true for all \( n \) etc

A1cso All 5 underlined statements must be seen

(b) M1 for attempting to apply de Moivre to \( w \) or attempting to expand \( w^{5} \) and collecting real and imaginary parts, but no need to simplify these.

A1cao for \( 243 \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}i} \right) \left[ = \frac{243 \sqrt{2}}{2} - \frac{243 \sqrt{2}}{2}i \right] \) (oe eg \( 3^5 \) instead of 243)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| (a)             | \[\frac{dy}{dx} + 2\frac{y}{x} = 4x \]  
|                 | I F: \[ e^{\int \frac{2}{x} \, dx} = e^{2\ln x} = (x^2) \]  
|                 | \[x^2 \frac{dy}{dx} + 2xy = 4x^3 \]  
|                 | \[yx^2 = \int 4x^3 \, dx = x^4 (+c) \]  
|                 | \[y = x^2 + \frac{c}{x^2} \]  
|                 | M1     |       |
|                 | M1     |       |
|                 | M1dep  |       |
|                 | M1dep  |       |
|                 | A1cso  | (5)   |
| (b)             | \[x = 1, \ y = 5 \Rightarrow c = 4 \]  
|                 | \[y = x^2 + \frac{4}{x^2} \]  
|                 | A1ft   | (2)   |
| (c)             | \[\frac{dy}{dx} = 2x - \frac{8}{x^3} \]  
|                 | \[\frac{dy}{dx} = 0 \quad x^4 = 4, \ x = \pm\sqrt{2} \text{ or } \pm\sqrt{4} \]  
|                 | \[y = 2 + \frac{4}{2} = 4 \]  
|                 | A1cao  |       |
| **Alt:**        | Complete square on \[y = \ldots \] or use the original differential equation  
|                 | \[x = \pm\sqrt{2}, \quad y = 4 \]  
|                 | M1     |       |
|                 | A1, A1 |       |
|                 | B1 shape|       |
|                 | B1 turning points shown somewhere | (5) |
|                 | [12]   |       |
### Notes for Question 5

(a)  
M1 for dividing the given equation by $x$. May be implied by subsequent work.  
M1 for IF = $e^x = e^{2\ln x} = (x^2)$: The integral must be seen together with an attempt at integrating this.  
M1 dep for multiplying the equation $\frac{dy}{dx} + 2\frac{y}{x} = 4x$ by their IF dep on 2nd M mark  
M1 dep for attempting the integration of the resulting equation - constant not needed. Dep on 2nd and 3rd M marks  
A1 cso for $y = x^2 + \frac{c}{x^2}$, $\text{eg } xy = x^4 + c$  

**Alternative:** for first three marks: Multiply given equation by $x$ to get straight to the third line. All 3 M marks should be given.

(b)  
M1 for using $x = 1$, $y = 5$ in their expression for $y$ to obtain a value for $c$.  
A1 ft for $y = x^2 + \frac{4}{x^2}$ follow through their result from (a).

(c)  
M1 for differentiating their result from (b), equating to 0 and solving for $x$.  
A1 for $x = \pm \sqrt{2}$ (no follow through) or $\pm \sqrt{4}$ No extra real values allowed but ignore any imaginary roots shown.  
A1 cso for using the particular solution to obtain $y = 4$. No extra values allowed.  

**Alternatives for these 3 marks:**  
M1 for making $\frac{dy}{dx} = 0$ in the given differential equation to get $y = 2x^2$ and using this with their particular solution to obtain an equation in one variable  
**OR** complete the square on their particular solution to get $y = \left(x + \frac{2}{x}\right)^2 - 4$.  
A1 for $x = \pm \sqrt{2}$ (no follow through)  
A1 cso for $y = 4$ No extra values allowed.

B1 for the correct shape - must have two minimum points and two branches, both asymptotic to the $y$-axis  
B1 for a fully correct sketch with the coordinates of the minimum points shown somewhere on or beside the sketch. Decimals accepted here.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (a)</td>
<td>[ \begin{align*} 2x^2 + 6x - 5 &amp;= 5 - 2x \ 2x^2 + 8x - 10 &amp;= 0 \ x^2 + 4x - 5 &amp;= 0 \ (x + 5)(x - 1) &amp;= 0 \text{ or by formula} \ x &amp;= -5, x = 1 \ -2x^2 - 6x + 5 &amp;= 5 - 2x \ 2x^2 + 4x &amp;= 0 \ x &amp;= 0, x = -2 \end{align*} ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td><img src="image" alt="Graph" /></td>
<td>B1 line</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 quad curve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1ft (on x-coords from (a))</td>
</tr>
<tr>
<td>(c)</td>
<td>( x &lt; -5, \quad -2 &lt; x &lt; 0, \quad x &gt; 1 )</td>
<td>B1, B1, B1</td>
</tr>
<tr>
<td></td>
<td>Special case: Deduct the last B mark earned 1 if ( \leq ) or ( \geq ) used</td>
<td>[12]</td>
</tr>
</tbody>
</table>
## Notes for Question 6

### (a) NB: Marks for (a) can only be awarded for work shown in (a):

| M1 | for \(2x^2 + 6x - 5 = 5 - 2x\) |
| M1 | for obtaining a 3 term quadratic and attempting to solve by factorising, formula or completing the square |
| A1 | for \(x = -5, \; x = 1\) |
| M1 | for considering the part of the quadratic that needs to be reflected ie for \(-2x^2 - 6x + 5 = 5 - 2x\) oe |
| A1 | for a correct 2 term quadratic, terms in any order \(2x^2 + 4x = 0\) oe |
| A1 | for \(x = 0 \; x = -2\) |

**NB:** The question demands that algebra is used, so solutions which do not show how the roots have been obtained will score very few if any marks, depending on what is written on the page.

*Alternative:* Squaring both sides:

| M1 | Square both sides and simplify to a quartic expression |
| M1 | Take out the common factor \(x\) |
| A1 | \(x\), a correct linear factor and a correct quadratic factor |
| M1 | \(x\) and 3 linear factors |
| A1 | any two of the required values |
| A1 | all 4 values correct |

### (b)

| B1 | for a line drawn, with negative gradient, crossing the positive \(y\)-axis |
| B1 | for the quadratic curve, with part reflected and the correct shape. It should cross the \(y\)-axis at the same point as the line and be pointed where it meets the \(x\)-axis (ie not U-shaped like a turning point) |
| B1ft | for showing the \(x\) coordinates of the points where the line crosses the curve. They can be shown on the \(x\)-axis as in the MS (accept \(O\) for 0) or written alongside the points as long as it is clear the numbers are the \(x\) coordinates |
| B1ft | The line should cross the curve at all the crossing points found and no others for this mark to be given. |

### (c) NB: No follow through for these marks

| B1 | for any one of \(x < -5, \; -2 < x < 0, \; x > 1\) correct |
| B1 | for a second one of these correct |
| B1 | for the third one correct |

Special case: if \(\leq\) or \(\geq\) is used, deduct the last B mark earned.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 7 (a)           | \[
\frac{dv}{dx} = v + x \frac{dv}{dx} \\
\frac{d^2v}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x \frac{d^2v}{dx^2} \\
4x^2 \left( 2 \frac{dv}{dx} + x \frac{d^2v}{dx^2} \right) - 8x \left( v + x \frac{dv}{dx} \right) + \left( 8 + 4x^2 \right) \times xv = x^4 \\
4x^3 \frac{d^2v}{dx^2} + 4x^3v = x^4 \\
4 \frac{d^2v}{dx^2} + 4v = x 
\]
| M1A1            |       |       |
|                 | See end for an alternative for (a) |
|                 |       |       |
|                 | (b)   |       |
|                 | \[4\lambda^2 + 4 = 0\] |
|                 | \[\lambda^2 = -1 \text{ oe}\] |
|                 | \((v) = C \cos x + D \sin x \quad \text{or} \quad (v) = Ae^{ix} + Be^{-ix}\) |
|                 | A1    |       |
|                 | P.I: Try \(v = kx (+l)\) |
|                 | \[
\frac{dv}{dx} = k \quad \frac{d^2v}{dx^2} = 0 \\
4 \times 0 + 4 \left( kx (+l) \right) = x \\
k = \frac{1}{4} \quad (l = 0) \\
v = C \cos x + D \sin x + \frac{1}{4} x \quad \text{or} \quad v = Ae^{ix} + Be^{-ix} + \frac{1}{4} x 
\]
| M1dep           |       |       |
|                 |       |       |
|                 | (c)   |       |
|                 | \[y = x \left( C \cos x + D \sin x + \frac{1}{4} x \right) \quad \text{or} \quad y = x \left( Ae^{ix} + Be^{-ix} + \frac{1}{4} x \right)\) |
| B1ft            | (1)   |       |
**Question 7 continued**

<table>
<thead>
<tr>
<th>Alternative for (a):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v = \frac{y}{x} )</td>
</tr>
<tr>
<td>( \frac{dv}{dx} = \frac{dy}{dx} \times \frac{1}{x} - y \times \frac{1}{x^2} )</td>
</tr>
<tr>
<td>( \frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} \times \frac{1}{x} - \frac{dy}{dx} \times \frac{1}{x^2} + \frac{dy}{dx} \times \frac{1}{x^2} + 2y \times \frac{1}{x^3} )</td>
</tr>
<tr>
<td>( x^3 \frac{d^2v}{dx^2} = x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y )</td>
</tr>
<tr>
<td>( 4x^3 \frac{d^2v}{dx^2} + 4x^3v = 4x^2 \frac{d^2y}{dx^2} - 8x \frac{dy}{dx} + 8y + 4x^2y = x^4 )</td>
</tr>
<tr>
<td>( 4 \frac{d^2v}{dx^2} + 4v = x )</td>
</tr>
</tbody>
</table>
Notes for Question 7

(a)
M1 for attempting to differentiate \( y = xv \) to get \( \frac{dy}{dx} \) - product rule must be used

M1 for differentiating their \( \frac{dy}{dx} \) to obtain an expression for \( \frac{d^2y}{dx^2} \) - product rule must be used

A1 for \( \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} + x \frac{d^2y}{dx^2} \)

M1 for substituting their \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) and \( y = xv \) in the original equation to obtain a differential equation in \( v \) and \( x \)

M1 for collecting the terms to have at most a 4 term equation - 4 terms only if a previous error causes \( \frac{dy}{dx} \) to be included, otherwise 3 terms

A1 cao and cso for \( 4 \frac{d^2v}{dx^2} + 4v = x \) *

Alternative: (see end of mark scheme)
M1 for writing \( v = \frac{y}{x} \) and attempting to differentiate by quotient or product rule to get \( \frac{dv}{dx} \)

M1 for differentiating their \( \frac{dv}{dx} \) to obtain an expression for \( \frac{d^2v}{dx^2} \) - product or quotient rule must be used

A1 for \( \frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} \times \frac{1}{x} - \frac{dy}{dx} \times \frac{1}{x^2} - \frac{dy}{dx} \times \frac{1}{x^2} + 2y \times \frac{1}{x} \)

M1 for multiplying their \( \frac{d^2v}{dx^2} \) by \( x^3 \)

M1 for multiplying by 4 and adding \( 4x^3y \) to each side and equating to \( x^4 \) (as rhs is now identical to the original equation.

A1 cao and cso for \( 4 \frac{d^2v}{dx^2} + 4v = x \) *

(b)
M1 for forming the auxiliary equation and attempting to solve

A1 for \( \lambda^2 = -1 \) oe
A1 for the complementary function in either form. Award for a correct CF even if \( \lambda = i \) only is shown.
Notes for Question 7 continued

<table>
<thead>
<tr>
<th>M1</th>
<th>for trying one of $v = kx$, $k \neq 1$ or $v = kx + l$ and $v = mx^2 + kx + l$ as a PI and obtaining $\frac{dv}{dx}$ and $\frac{d^2v}{dx^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>for substituting their differentials in the equation $4 \frac{d^2v}{dx^2} + 4v = x$. Award M0 if the original equation is used. Dep on 2nd M mark of (b)</td>
</tr>
<tr>
<td>A1cao</td>
<td>for obtaining the correct result (either form) (c)</td>
</tr>
<tr>
<td>B1ft</td>
<td>for reversing the substitution to get $y = x \left( C \cos x + D \sin x + \frac{1}{4}x \right)$ or $y = x \left( A e^{ix} + B e^{-ix} + \frac{1}{4}x \right)$ follow through their answer to (b)</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
</tr>
<tr>
<td>8 (a)</td>
<td>((y =) r \sin \theta = a \sin 2\theta \sin \theta)</td>
</tr>
<tr>
<td></td>
<td>(\left( \frac{dy}{d\theta} \right) = a (2 \cos 2\theta \sin \theta + \sin 2\theta \cos \theta))</td>
</tr>
<tr>
<td></td>
<td>(\left( \frac{dy}{d\theta} \right) = 2a \sin \theta (\cos 2\theta + \cos^2 \theta))</td>
</tr>
<tr>
<td></td>
<td>At (P) (\frac{dy}{d\theta} = 0 \Rightarrow \sin \theta = 0 \ (n/a)) or (2\cos^2 \theta - 1 + \cos^2 \theta = 0)</td>
</tr>
<tr>
<td></td>
<td>(3\cos^2 \theta = 1)</td>
</tr>
<tr>
<td></td>
<td>(\cos \theta = \frac{1}{\sqrt{3}} \quad *)</td>
</tr>
<tr>
<td></td>
<td>(b) (r = a \sin 2\theta = 2a \sin \theta \cos \theta)</td>
</tr>
<tr>
<td></td>
<td>(r = 2a \sqrt{\left(1 - \frac{1}{3}\right) \sqrt{\frac{1}{3}}} = 2a \frac{\sqrt{2}}{3})</td>
</tr>
<tr>
<td></td>
<td>(c) (\text{Area} = \int_{0}^{\phi} \frac{1}{2} r^2 d\theta = \frac{1}{2} a^2 \int_{0}^{\phi} \sin^2 2\theta d\theta)</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{2} a^2 \int_{0}^{\phi} \frac{1}{2} (1 - \cos 4\theta) d\theta)</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{4} a^2 \left[ \theta - \frac{1}{4} \sin 4\theta \right]_0^\phi)</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{4} a^2 \left[ \phi - \frac{1}{4} \left(4 \sin \phi \cos \phi (2 \cos^2 \phi - 1)\right) \right])</td>
</tr>
<tr>
<td></td>
<td>(= \frac{1}{4} a^2 \left[ \arccos \left( \frac{1}{\sqrt{3}} \right) - \left( \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \times \left( \frac{2}{3} - 1 \right) \right) \right])</td>
</tr>
<tr>
<td></td>
<td>(\frac{1}{36} a^2 \left[ 9 \arccos \left( \frac{1}{\sqrt{3}} \right) + \sqrt{2} \right] \quad *)</td>
</tr>
</tbody>
</table>

[15]
Notes for Question 8

(a) 
M1 for obtaining the \( y \) coordinate \( y = r \sin \theta = a \sin 2\theta \sin \theta \)

M1dep for attempting the differentiation to obtain \( \frac{dy}{d\theta} \) Product rule and/or chain rule must be used; sin to become \( \pm \cos \) (cos to become \( \pm \sin \)). The 2 may be omitted. Dependent on the first M mark.

A1 for correct differentiation eg \( \frac{dy}{d\theta} = a \left( 2 \cos 2\theta \sin \theta + \sin 2\theta \cos \theta \right) \) or

M1 for using \( \sin 2\theta = 2 \sin \theta \cos \theta \) anywhere in their solution to (a)

M1 for setting \( \frac{dy}{d\theta} = 0 \) and getting a quadratic factor with no \( \sin^2 \theta \) included.

Alternative: Obtain a quadratic in \( \sin \theta \) or \( \tan \theta \) and complete to \( \cos \theta = \) later.

A1cao for \( \cos \theta = \frac{1}{\sqrt{3}} \) or \( \cos \phi = \frac{1}{\sqrt{3}} \) *

Question 8 (a) Variations you may see:

\[ y = \sin 2\theta \sin \theta \]
\[ y = \sin 2\theta \cos \theta \]
\[ y = 2 \sin \theta \cos \theta \]
\[ y = 2a \cos 2\theta \sin \theta \]

\[ \frac{dy}{d\theta} = a \left( 2 \cos 2\theta \sin \theta + \sin 2\theta \cos \theta \right) \]

\[ \frac{dy}{d\theta} = 2a \left( \sin 2\theta \cos \theta - \sin^2 \theta \right) \]

<table>
<thead>
<tr>
<th>( y = \sin 2\theta \sin \theta )</th>
<th>( y = 2 \sin \theta \cos \theta )</th>
<th>( y = 2a \cos 2\theta \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{d\theta} = a \left( 2 \cos 2\theta \sin \theta + \sin 2\theta \cos \theta \right) )</td>
<td>( \frac{dy}{d\theta} = 2a \left( \sin 2\theta \cos \theta - \sin^2 \theta \right) )</td>
<td>( \frac{dy}{d\theta} = 2a \left( -\sin \theta + 3 \sin \theta \cos^{2}\theta \right) )</td>
</tr>
</tbody>
</table>

At P: \( \frac{dy}{d\theta} = 0 \Rightarrow 2 \cos^2 \theta - \sin^2 \theta = 0 \) or:

\[ 2 \cos^2 \theta - 3 \sin^2 \theta = 0 \]

\[ 3 \cos^2 \theta - 1 = 0 \]

\[ 2 - 3 \sin^2 \theta = 0 \]

\[ \tan^2 \theta = 2 \]

\[ \cos^2 \theta = 1/3 \]

\[ \sin^2 \theta = 2/3 \]

\[ \tan \theta = \pm \sqrt{2} \Rightarrow \cos \theta = \pm \frac{1}{\sqrt{3}} \]

\[ \cos \theta = \pm \frac{1}{\sqrt{3}} \]

\[ \sin \theta = \pm \frac{\sqrt{2}}{3} \pm \frac{\sqrt{6}}{3} \Rightarrow \cos \theta = \pm \frac{1}{\sqrt{3}} \]

(b)

M1 for using \( \sin 2\theta = 2 \sin \theta \cos \theta \), \( \cos^2 \theta + \sin^2 \theta = 1 \) and \( \cos \phi = \frac{1}{\sqrt{3}} \) in \( r = a \sin 2\theta \) to obtain a numerical multiple of \( a \) for \( R \). Need not be simplified.

A1cao for \( R = 2a \frac{\sqrt{2}}{3} \)

Can be done on a calculator. Completely correct answer with no working scores 2/2; incorrect answer with no working scores 0/2
Notes for Question 8 continued

(c)

M1 for using the area formula \( \frac{1}{2} \int_0^\phi r^2 \, d\theta = \frac{1}{2} a^2 \int_0^\phi \sin^2 2\theta \, d\theta \) Limits not needed

M1 for preparing \( \int \sin^2 2\theta \, d\theta \) for integration by using \( \cos 2x = 1 - 2\sin^2 x \)

M1 for attempting the integration: \( \cos 4\theta \) to become \( \pm \sin 4\theta \) - the \( \frac{1}{4} \) may be missing but inclusion of 4 implies differentiation - and the constant to become \( k\theta \). Limits not needed.

A1 for \( = \frac{1}{4} a^2 \left[ \theta - \frac{1}{4} \sin 4\theta \right] \) Limits not needed

M1dep for changing \textbf{their} integrated function to an expression in \( \sin \theta \) and \( \cos \theta \) and substituting limits 0 and \( \phi \). Dependent on the second M mark of (c)

M1dep for a numerical multiple of \( a^2 \) for the area. Dependent on all previous M marks of (c)

A1cso for \( \frac{1}{36} a^2 \left[ 9 \arccos \left( \frac{1}{\sqrt{3}} \right) + \sqrt{2} \right] \) *

This is a given answer, so check carefully that it can be obtained from the previous step in their working.

\textbf{Also:} The final 3 marks can only be awarded if the working is \textbf{shown} i.e. \( \sin 4\theta \) cannot be obtained by calculator.
Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 3 (6669/01)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\sqrt{\phantom{-}}$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **oe** – or equivalent (and appropriate)
   - **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - *** The answer is printed on the paper
   - **□** The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

**Method mark for solving 3 term quadratic:**
1. Factorisation
   \[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \]
   \[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \]

2. Formula
   Attempt to use correct formula (with values for \(a\), \(b\) and \(c\)).

3. Completing the square
   Solving \(x^2 + bx + c = 0\):
   \[
   \left( x \pm \frac{b}{2} \right)^2 = q \pm c, \quad q \neq 0, \quad \text{leading to } x = ...
   \]

**Method marks for differentiation and integration:**
1. Differentiation
   Power of at least one term decreased by 1. (\(x^n \rightarrow x^{n-1}\))

2. Integration
   Power of at least one term increased by 1. (\(x^n \rightarrow x^{n+1}\))

**Use of a formula**
Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**
Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done “in your head”, detailed working would not be required.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a) &amp; (b)</td>
<td></td>
</tr>
</tbody>
</table>

Sight of both of these (can be implied by their work) (allow $\pm ae = \pm 13$ or $\pm ae = 13$ or $ae = \pm 13$)

<table>
<thead>
<tr>
<th>Mark (a) and (b) together</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ae = 13$ and $a^2(e^2 - 1) = 25$</td>
</tr>
</tbody>
</table>

Solves to obtain

$\frac{a^2}{...} \text{ or } a = ...$

Eliminates $e$ to reach

$\frac{a^2}{...} \text{ or } a = ...$

$a = 12$

Cao (not $\pm 12$) unless -12 is rejected

$e = 13/\text{“}12\text{”}$

Uses their $a$ to find $e$ or finds $e$ by eliminating $a$ (Ignore $\pm$ here) (Can be implied by a correct answer)

$x = (\pm) \frac{a}{e}, = \pm \frac{144}{13}$

M1: $(x = ) (\pm) \frac{a}{e}$

$\pm$ not needed for this mark nor is $x$

and even allow $y = (\pm) \frac{a}{e}$ here – just look for use of $\frac{a}{e}$ with numerical $a$ and $e$.

A1: $x = \pm \frac{144}{13}$ oe but must be an equation (Do not allow $x = \pm \frac{12}{13}$)

Total 6

If they use the eccentricity equation for the ellipse ($b^2 = a^2(1-e^2)$) allow the M’s
Question

2. (a)  

\[ k \arcsinh \left( \frac{2x}{3} \right) + c \quad \text{or} \quad k \ln \left[ p x + \sqrt{(p^2 x^2 + \frac{9}{4} p^2)} \right] + c \]  

M1

\[ \frac{1}{2} \arcsinh \left( \frac{2x}{3} \right) + c \quad \text{or} \quad \frac{1}{2} \ln \left[ p x + \sqrt{(p^2 x^2 + \frac{9}{4} p^2)} \right] + c \]  

A1

(2)

(b)  

So:  

\[ \frac{1}{2} \ln \left[ 6 + \sqrt{45} \right] - \frac{1}{2} \ln \left[ -6 + \sqrt{45} \right] = \frac{1}{2} \ln \left[ \frac{6 + \sqrt{45}}{-6 + \sqrt{45}} \right] = \frac{1}{2} \ln \left[ \frac{(6 + \sqrt{45})^2}{9} \right] \]  

M1

Uses correct limits and combines logs

= \ln[2 + \sqrt{5}] \quad \text{or} \quad \frac{1}{2} \ln[9 + 4\sqrt{5}] \quad \text{A1} \text{cso}

Note that the last 3 marks can be scored without the need to rationalise e.g.

\[ 2 \times \frac{1}{2} \ln [2x + \sqrt{4x^2 + 9}] \bigg|_0 = \ln(6 + \sqrt{45}) - \ln 3 = \ln \left( \frac{6 + \sqrt{45}}{3} \right) \]

M1: Uses the limits 0 and 3 and doubles
M1: Combines Logs
A1: \ln[2 + \sqrt{5}] \text{oe}

(3)

Alternative for (a)  

\[ x = \frac{3}{2} \sinh u \Rightarrow \int \frac{1}{\sqrt{9 \sinh^2 u + 9}} \cdot \frac{3}{2} \cosh u \ du = k \arcsinh \left( \frac{2x}{3} \right) + c \]  

M1

\[ \frac{1}{2} \arcsinh \left( \frac{2x}{3} \right) + c \]  

A1

Alternative for (b)  

\[ \left[ \frac{1}{2} \arcsinh \left( \frac{2x}{3} \right) \right]_3 = \frac{1}{2} \arcsinh 2 - \frac{1}{2} \arcsinh -2 \]

\[ \frac{1}{2} \ln(2 + \sqrt{5}) - \frac{1}{2} \ln(\sqrt{5} - 2) = \frac{1}{2} \ln \left( \frac{2 + \sqrt{5}}{\sqrt{5} - 2} \right) \]  

M1

Uses correct limits and combines logs

= \frac{1}{2} \ln \left( \frac{2\sqrt{5} + 4 + 2\sqrt{5}}{5 - 4} \right) \quad \text{M1}

Correct method to rationalise denominator (may be implied)
Method must be clear if answer does not follow their fraction

= \frac{1}{2} \ln[9 + 4\sqrt{5}] \quad \text{A1 cso}

Total 5
3. \[ \frac{dx}{d\theta} = 2\sinh 2\theta \quad \text{and} \quad \frac{dy}{d\theta} = 4\cosh \theta \]

Or equivalent correct derivatives

\[ A = (2\pi) \int 4\sinh \theta \sqrt{2\sinh \theta + 4\cosh \theta} \, d\theta \]

or

\[ A = (2\pi) \int 4\sinh \theta \sqrt{1 + (4\cosh \theta)^2 \cdot 2\sinh 2\theta} \, d\theta \]

Use of correct formula including replacing \( dx \) with \( 2\sinh 2\theta \, d\theta \) if chain rule used. Allow the omission of the \( 2\pi \) here.

\[ A = 32\pi \int \sinh \theta \cosh^2 \theta \, d\theta \]

\[ A = 32\pi \int (\sinh \theta + \sinh^3 \theta) \, d\theta \]

**Completely correct expression for \( A \) with the square root removed.** This mark may be recovered later if the \( 2\pi \) is introduced later.

\[ A = \frac{32\pi}{3} \left[ \cosh^3 \theta \right]_0^1 \]

M1: Valid attempt to integrate a correct expression or a multiple of a correct expression – dependent on the first M1

A1: Correct expression

\[ = \frac{32\pi}{3} \left[ \cosh^3 1 - 1 \right] \]

M1: Uses the limits 0 and 1 correctly. Dependent on both previous M’s

A1: Cao and cso (no errors seen)

(7)

**Example Alternative Integration for last 4 marks**

\[ \int \sinh \theta \cosh^2 \theta \, d\theta = \int \sinh \theta (1 + \sinh^2 \theta) \, d\theta = \int (\sinh \theta + \sinh^3 \theta) \, d\theta \]

\[ = \frac{1}{4} \cosh \theta + \frac{1}{12} \cosh 3\theta \]

**dM1:** \[ \int \sinh \theta \cosh^2 \theta \, d\theta = p \cosh \theta + q \cosh 3\theta \]

**A1:** \[ 32\pi \left[ \frac{1}{4} \cosh \theta + \frac{1}{12} \cosh 3\theta \right] \]

\[ A = 8\pi \left[ \cosh \theta + \frac{1}{3} \cosh 3\theta \right]_0^1 \]

\[ = 8\pi (\cosh 1 + \frac{1}{3} \cosh 3 - \cosh 0 - \frac{1}{3} \cosh 0) \]

\[ = \frac{32\pi}{3} \left[ \cosh^3 1 - 1 \right] \]

M1: Uses the limits 0 and 1 correctly. Dependent on both previous M’s

A1: Cao
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td><strong>Alternative Cartesian Approach</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 1 + \frac{y^2}{8}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Any correct Cartesian equation</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\frac{dx}{dy} = \frac{y}{4} \quad \text{or} \quad \frac{dy}{dx} = \frac{\sqrt{2}}{(x-1)^2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correct Derivative</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$A = \int 2\pi y \sqrt{1 + \left(\frac{y}{4}\right)^2}dy \quad \text{or} \quad A = \int 2\pi \sqrt{8(x-1)^2} \sqrt{1 + \left(\frac{2}{x-1}\right)}dx$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td><strong>Use of a correct formula</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A = 2\pi \times \frac{2}{3} \times 8 \left(1 + \frac{y^2}{16}\right)^{\frac{1}{2}} \quad \text{or} \quad A = \frac{4\pi \sqrt{8}}{3} (x+1)^{\frac{1}{2}}$</td>
<td>dM1 A1</td>
</tr>
<tr>
<td></td>
<td><strong>M1</strong>: Convincing attempt to integrate a relevant expression – dependent on the first M1 but allow the omission of $2\pi$</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>A1</strong>: Completely correct expression for $A$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A = 2\pi \times \frac{2}{3} \times 8 \left(1 + \sinh^2 1\right)^{\frac{1}{2}} - 2\pi \times \frac{2}{3} \times 8 \quad \text{or} \quad 2\pi \times \frac{2}{3} \times \sqrt{8} \quad 1 + \cosh 2\left(\frac{1}{2}\right) - \frac{32\pi}{3}$</td>
<td>ddM1</td>
</tr>
<tr>
<td></td>
<td><strong>Correct use of limits ($0 \rightarrow 4\sinh 1$ for $y$ or $1 \rightarrow \cosh 2$ for $x$)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use $1+\sinh^2 1 = \cosh^2 1$ to give $\frac{32\pi}{3} \left[\cosh^3 1 - 1\right]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use $\cosh 2 = 2\cosh^2 1 - 1$ to give $\frac{32\pi}{3} \left[\cosh^3 1 - 1\right]$</td>
<td>A1</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
</tbody>
</table>
| 4.              | \[
\frac{dy}{dx} = \frac{40}{\sqrt{(x^2 - 1)}} - 9
\] | M1: \[
\frac{dy}{dx} = \frac{p}{\sqrt{(x^2 - 1)}} - q
\] A1: Cao |
|                 | Put \[
\frac{dy}{dx} = 0 \text{ and obtain } x^2 = ...
\] (Allow sign errors only) | e.g. \[
\left(\frac{1681}{81}\right)
\] dM1 |
|                 | \[
x = \frac{41}{9}
\] | M1: Square root A1: \[
x = \frac{41}{9} \text{ or exact equivalent (not } \pm \frac{41}{9})
\] M1 A1 |
|                 | \[
y = 40\ln\left\{\left(\frac{41}{9}\right) + \sqrt{\left(\frac{41}{9}\right)^2 - 1}\right\} - "41"
\] Substitutes \[
x = \frac{41}{9}
\] into the curve and uses the logarithmic form of arcosh | M1 |
|                 | So \[
y = 80\ln 3 - 41
\] Cao | A1 |
<p>|                 | <strong>Total 7</strong> | |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 5. (a) (i) & (ii) | \[
\begin{pmatrix}
1 & 1 & a \\
2 & b & c \\
-1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
1
\end{pmatrix}
= \begin{pmatrix}
1+a \\
b+c \\
1
\end{pmatrix} = \lambda_1 \begin{pmatrix}
0 \\
1 \\
1
\end{pmatrix}, \text{ and so } a = -1, \lambda_1 = 1
\] | M1, A1, A1 |
|                | \[
\begin{pmatrix}
1 & 1 & a \\
2 & b & c \\
-1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
1
\end{pmatrix}
= \begin{pmatrix}
1-a \\
2-c \\
-2
\end{pmatrix} = \lambda_2 \begin{pmatrix}
1 \\
0 \\
-1
\end{pmatrix}, \text{ and so } c = 2, \lambda_2 = 2
\] | M1, A1, A1 |
|                | \[
\begin{pmatrix}
1 & 1 & a \\
2 & b & c \\
-1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1-b \\
1 \\
-1
\end{pmatrix}
= \begin{pmatrix}
1 \\
1 \\
-1
\end{pmatrix}
\] | M1A1 |
|                | \[
\begin{pmatrix}
1 & 1 & a \\
2 & b & c \\
-1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1-b \\
1 \\
1
\end{pmatrix}
= \begin{pmatrix}
1 \\
1 \\
-1
\end{pmatrix}
\] | M1 |
|                | \[
\begin{pmatrix}
1 & 2 & -1 \\
1 & 1 & 0 \\
0 & d & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
1 \\
d
\end{pmatrix}
\] | B1 |
|                | \[
\begin{pmatrix}
1 & 2 & -1 \\
1 & 1 & 0 \\
0 & d & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
1 \\
d
\end{pmatrix}
\] | B1 |
|                | \[
\begin{pmatrix}
1 & -2-d & 1 \\
-1 & 1 & -1 \\
d & -d & -1
\end{pmatrix}
\] | a correct first step |
|                | \[
\begin{pmatrix}
1 & -1 & d \\
-2-d & 1 & -d \\
1 & -1 & -1
\end{pmatrix}
\] | M1 |
|                | \[
\begin{pmatrix}
1 & -1 & d \\
-2-d & 1 & -d \\
1 & -1 & -1
\end{pmatrix}
\] | A1 |
|                | \[
\begin{pmatrix}
1 & -1 & d \\
-2-d & 1 & -d \\
1 & -1 & -1
\end{pmatrix}
\] | M1 A1 A1 |
| (b)(i) | \[
\det P = -d - 1
\] | B1 |
|        | Allow 1 - d - 2 or 1 - (2 + d) A correct (possibly un-simplified) determinant | |
| (ii)  | \[
\mathbf{P}^{-1} = \begin{pmatrix}
1 & 2 & -1 \\
1 & 1 & 0 \\
0 & d & 1
\end{pmatrix}
\begin{pmatrix}
1 & d+2 & 1 \\
1 & 1 & 1 \\
d & d & -1
\end{pmatrix}
\] | B1 |
|        | M1: Identifiable full attempt at inverse including reciprocal of determinant. Could be indicated by at least 6 correct elements. | |
|        | A1: Two rows or two columns correct (ignoring determinant) BUT M0A1A0 or M0A1A1 is not possible | |
|        | A1: Fully correct inverse | |
|        | \[
\frac{1}{-d - 1}
\] | M1 A1 A1 |
|        | \[
\begin{pmatrix}
1 & -1 & d \\
-2-d & 1 & -d \\
1 & -1 & -1
\end{pmatrix}
\] | |
<p>|        | Total 13 | |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ I_n = \int x^{n-1} \times x(16 - x^2)^{\frac{1}{2}} \, dx \]

M1: Obtains \( x(16 - x^2)^{\frac{1}{2}} \) prior to integration

A1: Correct underlined expression (can be implied by their integration)

\[ I_n = \left[-\frac{1}{4} x^{n-1} (16 - x^2)^{\frac{1}{2}} \right]_0^1 + \frac{n-1}{3} \int_0^1 x^{n-2} (16 - x^2)^{\frac{1}{2}} \, dx \]

dM1

**dM1: Parts in the correct direction (Ignore limits)**

\[ \therefore I_n = \frac{n-1}{3} \int x^{n-2} (16 - x^2)(16 - x^2)^{\frac{1}{2}} \, dx \]

i.e. \( I_n = \frac{16(n-1)}{3} I_{n-2} - \frac{n-1}{3} I_n \)

Manipulates to obtain at least one integral in terms of \( I_n \) or \( I_{n-2} \) on the rhs.

M1

\( I_n(1 + \frac{n-1}{3}) = \frac{16(n-1)}{3} I_{n-2} \)

Collects terms in \( I_n \) from both sides

M1

\( (n+2)I_n = 16(n-1)I_{n-2} \)

Printed answer with no errors

A1* cso

**Way 2**

\[ \int_0^1 x^n (16 - x^2)^{\frac{1}{2}} \, dx = \int_0^1 x^n \frac{(16 - x^2)}{(16 - x^2)^{\frac{1}{2}}} \, dx = \int_0^1 \frac{16x^n}{(16 - x^2)^{\frac{1}{2}}} \, dx - \int_0^1 \frac{x^n^{n-2}}{(16 - x^2)^{\frac{1}{2}}} \, dx \]

\[ = \int_0^1 16x^{n-1} \times x(16 - x^2)^{\frac{1}{2}} \, dx - \int_0^1 x^{n-1} \times x(16 - x^2)^{\frac{1}{2}} \, dx \]

M1A1

M1: Obtains \( x(16 - x^2)^{\frac{1}{2}} \) prior to integration

A1: Correct expressions

\[ = \left[-16x^{n-1}(16 - x^2)^{\frac{1}{2}} \right]_0^1 + 16(n-1) \int_0^1 x^{n-2} (16 - x^2)^{\frac{1}{2}} \, dx \]

\[ - \left[-x^{n-1}(16 - x^2)^{\frac{1}{2}} \right]_0^1 + (n+1) \int_0^1 x^n (16 - x^2)^{\frac{1}{2}} \, dx \]

dM1

**dM1: Parts in the correct direction on both (Ignore limits)**

\[ I_n = 16(n-1)I_{n-2} - (n+1)I_n \]

Manipulates to obtain at least one integral in terms of \( I_n \) or \( I_{n-2} \) on the rhs.

M1

\[ I_n(1 + n+1) = 16(n-1)I_{n-2} \]

Collects terms in \( I_n \) from both sides

M1

\( (n+2)I_n = 16(n-1)I_{n-2} \)

Printed answer with no errors

A1*

**Way 3**

\[ \int_0^1 x^n (16 - x^2)^{\frac{1}{2}} \, dx = \int_0^1 x \times x^{n-1} \frac{(16 - x^2)}{(16 - x^2)^{\frac{1}{2}}} \, dx \]

M1: Obtains \( x(16 - x^2)^{\frac{1}{2}} \) prior to integration

A1: Correct expression

\[ = \left[-x^{n-1}(16 - x^2)(16 - x^2)^{\frac{1}{2}} \right]_0^1 + \int_0^1 (16(n-1)x^{n-2} - (n+1)x^n)(16 - x^2)^{\frac{1}{2}} \, dx \]

dM1

**dM1: Parts in the correct direction (Ignore limits)**

\[ I_n = 16(n-1)I_{n-2} - (n+1)I_n \]

Manipulates to obtain at least one integral in terms of \( I_n \) or \( I_{n-2} \) on the rhs.

M1

\[ I_n(1 + n+1) = 16(n-1)I_{n-2} \]

Collects terms in \( I_n \) from both sides

M1

\( (n+2)I_n = 16(n-1)I_{n-2} \)

Printed answer with no errors

A1*
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ I_1 = \int_0^4 x \sqrt{(16 - x^2)} , dx = \left[ -\frac{1}{3} (16 - x^2)^{3/2} \right]_0^4 = \frac{64}{3} ]</td>
<td>M1: Correct integration to find ( I_1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1: ( \frac{64}{3} ) or equivalent (May be implied by a later work – they are not asked explicitly for ( I_1 ))</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( \frac{64}{3} ) must come from correct work</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Using ( x = 4 \sin \theta ):</td>
<td></td>
</tr>
<tr>
<td>[ I_1 = \int_0^{\pi} 4 \sin \theta \sqrt{(16 - 16 \sin^2 \theta)} \cos \theta , d\theta = \int_0^{\pi} 64 \sin \theta \cos^2 \theta , d\theta ]</td>
<td>M1: A complete substitution and attempt to substitute changed limits</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1: ( \frac{64}{3} ) or equivalent</td>
<td></td>
</tr>
<tr>
<td>( I_5 = \frac{64}{7} I_3, I_3 = \frac{32}{5} I_1 )</td>
<td>Applies to apply reduction formula twice. First M1 for ( I_5 ) in terms of ( I_3 ), second M1 for ( I_3 ) in terms of ( I_1 ) (Can be implied)</td>
<td>M1, M1</td>
</tr>
<tr>
<td>( I_5 = \frac{131072}{105} )</td>
<td>Any exact equivalent (Depends on all previous marks having been scored)</td>
<td>A1</td>
</tr>
</tbody>
</table>

Total 11
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>7(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{dx}{d\theta} = -a \sin \theta$ and $\frac{dy}{d\theta} = b \cos \theta$ so $\frac{dy}{dx} = \frac{b \cos \theta}{-a \sin \theta}$</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td>M1: Differentiates both $x$ and $y$ and divides correctly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1: Fully correct derivative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternative: $x^2 + \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \Rightarrow y' = -\frac{b^2}{a^2} \frac{x}{y} = -\frac{b^2 a \cos \theta}{a^2 b \sin \theta}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differentiates implicitly and substitutes for $x$ and $y$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1: $= \frac{b \cos \theta}{a \sin \theta}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal has gradient $\frac{a \sin \theta}{b \cos \theta}$ or $\frac{a^2 y}{b^2 x}$</td>
<td>Correct perpendicular gradient rule</td>
<td>M1</td>
</tr>
<tr>
<td>$(y - b \sin \theta) = \frac{a \sin \theta}{b \cos \theta} (x - a \cos \theta)$</td>
<td>Correct straight line method using a „changed“ gradient which is a function of $\theta$</td>
<td>M1</td>
</tr>
<tr>
<td>If $y = mx + c$ is used need to find $c$ for M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ax \sin \theta - by \cos \theta = (a^2 - b^2) \sin \theta \cos \theta$ *</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>Fully correct completion to printed answer</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Allow un-simplified</td>
<td>B1</td>
</tr>
<tr>
<td>$x = \frac{(a^2 - b^2) \cos \theta}{a}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = -\frac{(a^2 - b^2) \sin \theta}{b}$</td>
<td>Allow un-simplified</td>
<td>B1</td>
</tr>
<tr>
<td>$\left(= \frac{1}{2} \frac{(a^2 - b^2)^2 \cos \theta \sin \theta}{ab} \right) = \frac{1}{4} \frac{(a^2 - b^2)^2 \sin 2\theta}{ab}$</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>M1: Area of triangle is $\frac{1}{2} &quot;OA&quot; \times &quot;OB&quot;$ and uses double angle formula correctly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1: Correct expression for the area (must be positive)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Correct value for $\theta$ (may be implied by correct coordinates)</td>
<td>B1</td>
</tr>
<tr>
<td>Maximum area when $\sin 2\theta = 1$ so $\theta = \frac{\pi}{4}$ or 45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>So the point $P$ is at $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ oe</td>
<td>M1: Substitutes their value of $\theta$ where $0 &lt; \theta &lt; \frac{\pi}{2}$ or $0 &lt; \theta &lt; 90$ into their parametric coordinates</td>
<td>M1 A1</td>
</tr>
<tr>
<td>$\left(a \cos \frac{\pi}{4}, b \sin \frac{\pi}{4}\right)$ scores B1M1A0</td>
<td>A1: Correct exact coordinates</td>
<td></td>
</tr>
<tr>
<td>Mark part (c) independently</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total 12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Question 8(a)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attempt scalar product</td>
<td>M1</td>
</tr>
<tr>
<td>Use of correct formula</td>
<td>M1</td>
</tr>
<tr>
<td>Correct distance (Allow $29/\sqrt{29}$)</td>
<td>A1</td>
</tr>
</tbody>
</table>

### (a) Way 2

\[
r = (6i + 2j + 12k) + \lambda(3i - 4j + 2k)
\]

\[
\therefore 6 + 3\lambda, 3 + 2 - 4\lambda, -4 + 12 + 2\lambda = 5
\]

Substitutes the parametric coordinates of the line through $(6, 2, 12)$ perpendicular to the plane into the cartesian equation.

\[
\lambda = -1 \Rightarrow 3, 6, 10 \text{ or } -3i + 4j - 2k
\]

Solves for $\lambda$ to obtain the required point or vector. M1

### (a) Way 3

Parallel plane containing $(6, 2, 12)$ is

\[
r \cdot (3i - 4j + 2k) = 34
\]

\[
\Rightarrow r \cdot (3i - 4j + 2k) - \frac{34}{\sqrt{29}} = 0
\]

\[
\Rightarrow r \cdot (3i - 4j + 2k) - \frac{5}{\sqrt{29}} = 0
\]

\[
\frac{34}{\sqrt{29}} - \frac{5}{\sqrt{29}} = \frac{\sqrt{29}}{29}
\]

Correct distance A1

### (b)

For a cross product, if the method is unclear, 2 out of 3 components should be correct for M1

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| (cos$\theta$) = \[
\frac{(3i - 4j + 2k) \cdot (i + j - k)}{\sqrt{3^2 + 4^2 + 2^2} \cdot \sqrt{1^2 + 3^2 + 1^2}} \]

\[
= \frac{-11}{\sqrt{29} \sqrt{11}}
\]

Attempts scalar product of normal vectors including magnitudes | A1 |

Obtains angle using arccos (dependent on previous M1) dM1 A1

Do not isw and mark the final answer e.g. $90 - 52 = 38$ loses the A1

### (c)

\[
\begin{bmatrix}
i & j & k \\
1 & 3 & -1 \\
3 & -4 & 2
\end{bmatrix}
\]

M1: Attempt cross product of normal vectors M1A1

A1: Correct vector

\[
x = 0 : (0, \frac{5}{2}, \frac{15}{2}), \ y = 0 : (1, 0, 1), \ z = 0 : (\frac{15}{13}, \frac{-5}{13}, 0)
\]

M1A1

May use way 3 to find a point on the line

\[
r \times (-2i + 5j + 13k) = -5i - 15j + 5k
\]

A1: Correct equation M1A1
### Question Number
<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
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</thead>
<tbody>
<tr>
<td>(c) Way 2 “$x + 3y - z = 0$” and $3x - 4y + 2z = 5$ uses their cartesian form of and eliminate $x,$ $y$ or $z$ and substitutes back to obtain two of the variables in terms of the third</td>
<td>M1</td>
</tr>
<tr>
<td>$(x = 1 - \frac{3}{5} y$ and $z = 1 + \frac{11}{2} y)$ or $(y = \frac{5z - 5}{13}$ and $x = \frac{15 - 2z}{13}$) or $(y = \frac{5 - 5x}{2}$ and $z = \frac{15 - 13x}{2})$</td>
<td>A1</td>
</tr>
<tr>
<td>Cartesian Equations:</td>
<td></td>
</tr>
<tr>
<td>$x = \frac{y - 5}{2} = \frac{z - 15}{2}$ or $\frac{x - 1}{5} = \frac{y - 5}{13}$ or $\frac{x - 15}{13} = \frac{y + 5}{13} = z$</td>
<td></td>
</tr>
<tr>
<td>Points and Directions: Direction can be any multiple $(0, \frac{5}{2}, \frac{15}{2}), i - \frac{5}{2} j - \frac{13}{2}k$ or $(1, 0, 1), -\frac{2}{5} i + j + \frac{13}{5} k$ or $(\frac{15}{13}, -\frac{5}{13}, 0), -\frac{2}{5} i + \frac{5}{13} j + k$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>M1: Uses their Cartesian equations correctly to obtain a point and direction A1: Correct point and direction – it may not be clear which is which – i.e. look for the correct numbers either as points or vectors</td>
<td></td>
</tr>
<tr>
<td>Equation of line in required form: e.g. $r \times (-2i + 5j + 13k) = -5i - 15j + 5k$ Or Equivalent</td>
<td>M1 A1</td>
</tr>
<tr>
<td>Total 14</td>
<td></td>
</tr>
<tr>
<td>(e) Way 3</td>
<td></td>
</tr>
<tr>
<td>$\begin{pmatrix} 2\lambda + \mu \ \lambda - \mu \ 5\lambda - 2\mu \end{pmatrix} = \begin{pmatrix} 3 \ -4 \ 2 \end{pmatrix} \implies 12\lambda + 3\mu = 5$</td>
<td>M1: Substitutes parametric form of $\Pi_2$ into the vector equation of $\Pi_1$ M1A1</td>
</tr>
<tr>
<td>$\mu = \frac{5}{3}, \lambda = 0$ gives $\begin{pmatrix} 5 \ 3 \ -3/3 \end{pmatrix}$</td>
<td>A1: Correct equation</td>
</tr>
<tr>
<td>$\mu = 0, \lambda = \frac{5}{12}$ gives $\begin{pmatrix} 5/6 \ 12/12 \end{pmatrix}$</td>
<td>M1A1</td>
</tr>
<tr>
<td>Direction $\begin{pmatrix} -2 \ 5 \ 13 \end{pmatrix}$</td>
<td>A1: Correct coordinates and direction</td>
</tr>
<tr>
<td>Equation of line in required form: e.g. $r \times (-2i + 5j + 13k) = -5i - 15j + 5k$ Or Equivalent</td>
<td>M1A1</td>
</tr>
<tr>
<td>Do not allow ‘mixed’ methods – mark the best single attempt</td>
<td></td>
</tr>
<tr>
<td>NB for checking, a general point on the line will be of the form: $(1 - 2\lambda, \frac{5}{3} \lambda, 1 + 13\lambda)$</td>
<td></td>
</tr>
</tbody>
</table>
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \(\sqrt{\text{ }}\) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
General Rules for Marking Mechanics

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is accuracy error not method error.

- Omission of mass from a resolution is method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised ONCE per complete question.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) For $P$,</td>
<td>$-I = 3(1 - 4)$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$I = 9 \text{Ns}$</td>
<td>A1</td>
</tr>
<tr>
<td>(b) For $Q$,</td>
<td>$9 = m(1.5 - 3)$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$m = 2$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>OR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$12 - 3m = 3 + 1.5m$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$m = 2$</td>
<td>A1</td>
</tr>
</tbody>
</table>

### Notes for Question 1

**Q1(a)**

- M1 for attempt at Impulse = difference in momenta for particle $P$, (must be considering one particle i.e. have same mass in both terms) (M0 if g is included or if mass omitted).
- First A1 for $\pm 3(1 - 4)$
- Second A1 for 9 (Must be positive). Allow change of sign at end to obtain magnitude.

**N.B.** For M1 they may use CLM to find a value for $m$ first and then use it when considering the change in momentum of $Q$ to find the impulse.

**Q1(b)**

- **EITHER**
  - M1 for attempt at:
  - their Impulse from (a) = difference in momenta for particle $Q$, (must be considering one particle) (M0 if g is included or if mass omitted).
  - First A1 for $9 = m(1.5 - 3)$ oe.
  - Second A1 for $m = 2$.

- OR
  - M1 for attempt at CLM equation, with correct no. of terms, dimensionally correct. Allow consistent extra g’s and sign errors.
  - First A1 for a correct equation i.e. $12 - 3m = 3 + 1.5m$ oe.
  - Second A1 for $m = 2$.  

[6]
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>For system, $T - 950g - 50g = 1000 \times -2$</td>
<td>M1 A1 (3)</td>
</tr>
<tr>
<td></td>
<td>$T = 7800 \text{ N}$</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>For woman, $R - 50g = 50 \times -2$</td>
<td>M1 A1 (3)</td>
</tr>
<tr>
<td></td>
<td>$R = 390 \text{ N}$</td>
<td>A1</td>
</tr>
</tbody>
</table>

Notes for Question 2

Q2(a) (In both parts, use the mass to decide which part of the system is being considered and M marks can only be scored if an equation contains only forces acting on that part of the system)
M1 is for a complete method for finding $T$ i.e. for an equation in $T$ only, dimensionally correct, with the correct number of terms.
First A1 for a correct equation.
Second A1 for 7800 (N).

Q2(b) M1 is for a complete method for finding $R$ i.e. for an equation in $R$ only, dimensionally correct, with the correct number of terms.
First A1 for a correct equation.
Second A1 for 390 (N).
N.B. Equation for lift only is: $T - 950g - R = 950 \times (-2)$
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>$T \cos \alpha - F = 2g \cos 60^\circ$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$T \sin \alpha + R = 2g \cos 30^\circ$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$F = \frac{1}{3} R$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>eliminating $F$ and $R$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$T = g(1 + \frac{1}{\sqrt{3}})$, 1.6g (or better), 15.5, 15 (N)</td>
<td>DM1 A1</td>
</tr>
</tbody>
</table>

Notes for Question 3

First M1 for resolving parallel to the plane with correct no. of terms and both $T$ and $2g$ terms resolved. 
First A1 for a correct equation. (use of $\alpha$ instead of $30^\circ$ or $60^\circ$ or vice versa is an A error not M error; similarly if they use sin(3/5) or cos(4/5) when resolving, this can score M1A0) 
Second M1 for resolving perpendicular to the plane with correct no. of terms and both $T$ and $2g$ terms resolved. 
Second A1 for a correct equation (use of $\alpha$ instead of $30^\circ$ or $60^\circ$ or vice versa is an A error not M error; similarly if they use sin(3/5) or cos(4/5) when resolving, this can score M1A0) 
B1 for $F = 1/3 R$ seen or implied. 
Third M1, dependent on first two M marks and appropriate angles used when resolving in both equations, for eliminating $F$ and $R$. 
Fourth M1 dependent on third M1, for solving for $T$. 
Third A1 for 15(N) or 15.5 (N). 
N.B. The first two M marks can be for two resolutions in any directions. 
Use of $\tan \frac{\alpha}{3} = 4/3$ leads to an answer of 17.83…and can score max 7/8.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. (a)</td>
<td>$240 = \frac{1}{5}(u + 34)10$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$u = 14$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td>4. (b)</td>
<td>$34 = 14 + 10a \implies a = 2$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$120 = 14t + \frac{1}{2} \times 2 \times t^2$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$t^2 + 14t - 120 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solving, $t = -20$ or $6$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$t = 6$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>OR</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>$34 = 14 + 10a \implies a = 2$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$v^2 = 14^2 + 2 \times 2 \times 120 \implies v = 26$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>AND $26 = 14 + 2t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t = 6$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9)</td>
</tr>
</tbody>
</table>

**Notes for Question 4**

**Q4(a)**
- First M1 for a complete method to produce an equation in $u$ only.
- First A1 for a correct equation. ($u^2 - 48u + 476 = 0$ oe is possible).
- Second A1 for $u = 14$.

**Q4(b)**
- **EITHER**
  - First M1 for an equation in $a$ only. (M0 if $v = 34$ when $s = 120$ is used)
  - First A1 for $a = 2$. (This may have been found in part (a))
  - Second M1 for a 3-term quadratic equation in $t$ only, allow sign errors (must have found a value of $a$. (M0 if $v = 34$ when $s = 120$ is used)
  - Second A1 for a correct equation.
  - Third M1 dependent on previous M1 for solving for $t$.
  - Third A1 for $t = 6$

- **OR**
  - First M1 for an equation in $a$ only.
  - First A1 for $a = 2$. (This may have been found in part (a))
  - Second M1 for a complete method to obtain an equation in $t$ only, allow sign errors. (must have found a value of $a$)
  - Second A1 for a correct equation.
  - Third M1 dependent on previous M1 for solving for $t$.
  - Third A1 for $t = 6$
<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>Speed</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Figures</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>[(120 + T)22] = 2145</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>T = 75</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td>(c)</td>
<td>[(t + t - 30)22] = 990</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>t = 60</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Answer = 60 - 10 = 50</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td>(d)</td>
<td>990 = 0.5a50^2</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>a = 0.79, 0.792, 99/125 oe</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
</tbody>
</table>

Notes for Question 5

Q5(a)
First B1 for a trapezium starting at the origin and ending on the t-axis. Second B1 for the figures marked (allow missing 0 and a delineator oe for T) (allow if they have used T = 75 correctly on their graph)

Q5(b)
First M1 for producing an equation in their T only by equating the area of the trapezium to 2145, with the correct no. of terms. If using a single trapezium, we need to see evidence of using ½ the sum of the two parallel sides or if using triangle(s), need to see ½ base x height. Second A1 cao for a correct equation in T (This is not f.t. on their T) Third A1 for T = 75. N.B. Use of a single suvat equation for the whole motion of the car e.g. \( s = t(t+u)/2 \) is M0

Q5(c)
First M1 for producing an equation in t only (they may use \( t - 30 \) oe as their variable) by equating the area of the trapezium to 990, with the correct no. of terms. If using a trapezium, we need to see evidence of using ½ the sum of the two parallel sides or if using triangle(s), need to see ½ base x height.
First A1 for a correct equation. Second A1 for \( t = 60 \) (Allow 30 + 30). Third A1 for answer of 50. N.B. Use of a single suvat equation for the whole motion of the car e.g. \( s = t(t+u)/2 \) is M0. Use of the motion of the motorcycle is M0 (insufficient information). Use of \( v = 22 \) for the motorcycle is M0.

Q5(d)
First M1 for an equation in a only. First A1 for \( a = 0.79, 0.792, 99/125 \) oe N.B. Use of \( v = 22 \) for the motorcycle is M0.
### Question 6.

#### (a)

- \[ M(P), \quad 50g \times 2 = Mg \times (x - 2) \]
- \[ M(Q), \quad 50g \times 3 = Mg \times (12 - x) \]

- (i) \[ M = 25 \text{ (kg)} \]
- (ii) \[ x = 6 \text{ (m)} \]

#### (b)

- \[ (\uparrow) R + R = 25g + 50g \]
- \[ M(A), \quad 2R + 12R = 25g \times 6 + 50g \times AX \]
- \[ AX = 7.5 \text{ (m)} \]

### Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="" alt="Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>

### Marks

- \[ M1 \text{ A1} \]
- \[ DM1 \text{ A1} \]
- \[ (8) \]
- \[ M1 \text{ A1 ft} \]
- \[ DM1 \text{ A1} \]
- \[ (6) \]
- \[ [14] \]
### Notes for Question 6

| Q6(a) | First M1 for moments about $P$ equation with usual rules (or moments about a different point AND vertical resolution and $R$ then eliminated) (M0 if non-zero reaction at $Q$)  
|       | Second M1 for moments about $Q$ equation with usual rules (or moments about a different point AND vertical resolution) (M0 if non-zero reaction at $P$)  
|       | Second A1 for a correct equation in $M$ and same unknown.  
|       | Third M1, dependent on first and second M marks, for solving for $M$  
|       | Third A1 for 25 (kg)  
|       | Fourth M1, dependent on first and second M marks, for solving for $x$  
|       | Fourth A1 for 6 (m)  
|       | N.B. No marks available if rod is assumed to be uniform but can score max 5/6 in part (b), provided they have found values for $M$ and $x$ to f.t. on.  
|       | If they have just invented values for $M$ and $x$ in part (a), they can score the M marks in part (b) but not the A marks. |

| Q6(b) | First M1 for vertical resolution or a moments equation, with usual rules.  
|       | First A1 1 ft on their $M$ and $x$ from part (a), for a correct equation. (must have equal reactions in vertical resolution to earn this mark)  
|       | Second M1 for a moments equation with usual rules.  
|       | Second A1 1 ft on their $M$ and $x$ from part (a), for a correct equation in $R$ and same unknown length.  
|       | Third M1, dependent on first and second M marks, for solving for $AX$ (not their unknown length) with $AX \leq 15$  
|       | Third A1 for $AX = 7.5$ (m)  
|       | N.B. If a single equation is used (see below), equating the sum of the moments of the child and the weight about $P$ to the sum of the moments of the child and the weight about $Q$, this can score M2 A2 1 ft on their $M$ and $x$ from part (a), provided the equation is in one unknown. Any method error, loses both M marks.  
<p>|       | e.g. 25g.4 + 50g(x – 2) = 25g.6 + 50g(12 – x) oe. |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>$t = 0$ gives $v = i - 3j$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>speed $= \sqrt{t^2 + (\mathbf{-3})^2}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \sqrt{10} = 3.2$ or better</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td>(b)</td>
<td>$t = 2$ gives $v = (-3i + 3j)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Bearing is $315^\circ$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td>(c)(i)</td>
<td>$1 - 2t = 0 \Rightarrow t = 0.5$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>$-(3t - 3) = -3(1 - 2t)$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Solving for $t$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$t = 2/3, 0.67$ or better</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6)</td>
</tr>
</tbody>
</table>

**Notes for Question 7**

**Q7(a)**
- B1 for $i - 3j$.
- M1 for $\sqrt{\text{sum of squares of cpt.s}}$.
- A1 for $\sqrt{10}, 3.2$ or better.

**Q7(b)**
- M1 for clear attempt to sub $t = 2$ into given expression.
- A1 for $315^\circ$.

**Q7(c)**
- (i) First M1 for $1 - 2t = 0$.
  - First A1 for $t = 0.5$.
  - N.B. If they offer two solutions, by equating both the $i$ and $j$ components to zero, give M0.
- (ii) First M1 for $\frac{1 - 2t}{3t - 3} = \pm \frac{-1}{-3}$ o.e. (Must be an equation in $t$ only)  
  - First A1 for a correct equation (the $+$ sign) 
  - Second M1, dependent on first M1, for solving for $t$.
  - Second A1 for $2/3, 0.67$ or better.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. (a)</td>
<td>For $A$, $T = 2ma$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>For $B$, $3mg - T = 3ma$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$3mg = 5ma$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$\frac{3g}{5} = a$ (5.9 or 5.88 m s$^2$)</td>
<td>A1</td>
</tr>
<tr>
<td>8. (b)</td>
<td>$T = \frac{6mg}{5}; 12m ; 11.8m$</td>
<td>B1</td>
</tr>
<tr>
<td>8. (c)</td>
<td>$F = \sqrt{T^2 + T^2}$</td>
<td>M1 A1 ft</td>
</tr>
<tr>
<td></td>
<td>$F = \frac{6mg\sqrt{2}}{5}; 1.7mg$ (or better); 16.6$m$; 17$m$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Direction clearly marked on a diagram, with an arrow, and 45$^\circ$ (oe) marked</td>
<td>B1</td>
</tr>
</tbody>
</table>

**Notes for Question 8**

**Q8(a)**
- B1 for $T = 2ma$
- First M1 for resolving vertically (up or down) for $B$, with correct no. of terms. (allow omission of $m$, provided 3 is there)
- First A1 for a correct equation.
- Second M1, dependent on first M1, for eliminating $T$, to give an equation in $a$ only.
- Second A1 for 0.6g, 5.88 or 5.9.
- N.B. ‘Whole system’ equation: $3mg = 5ma$ earns first 4 marks but any error loses all 4.

**Q8(b)**
- B1 for $\frac{6mg}{5}$, 11.8$m$, 12$m$

**Q8(c)**
- M1 $\sqrt{(T^2 + T^2)}$ or $\frac{T}{\sin 45^\circ}$ or $\frac{T}{\cos 45^\circ}$ or $2T\cos 45^\circ$ or $2T\sin 45^\circ$ (allow if $m$ omitted)
- (M0 for $T\sin 45^\circ$)
- First A1 ft on their $T$.
- Second A1 cao for $\frac{6mg\sqrt{2}}{5}$ oe, 1.7mg (or better), 16.6$m$, 17$m$
- B1 for the direction clearly shown on a diagram with an arrow and 45$^\circ$ marked.
Mark Scheme (Results)

Summer 2013

GCE Mechanics 2 (6678/01)
Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- **bod** – benefit of doubt
- **ft** – follow through
- The symbol ✓ will be used for correct ft
- **cao** – correct answer only
- **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
- **isw** – ignore subsequent working
- **awrt** – answers which round to
- **SC**: special case
- **oe** – or equivalent (and appropriate)
- **dep** – dependent
- **indep** – independent
- **dp** decimal places
- **sf** significant figures
- **\*** The answer is printed on the paper
- **\[** The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme
General Rules for Marking Mechanics

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is accuracy error not method error.

- Omission of mass from a resolution is method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of \( g = 9.8 \) should be given to 2 or 3 SF.

- Use of \( g = 9.81 \) should be penalised once per (complete) question.

- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised ONCE per complete question.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Use of $I = mv - mu$</td>
<td>M1</td>
<td>Must be subtracting. Condone subtraction in the wrong order</td>
</tr>
<tr>
<td></td>
<td>$2v = (3i + 6j) + 2(i - 4j)$</td>
<td>A1</td>
<td>Correct unsimplified equation ($= 5i - 2j$)</td>
</tr>
<tr>
<td></td>
<td>$v = 2.5i - j$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Speed = $\sqrt{2.5^2 + 1^2} = \sqrt{7.25} = 2.69$ (m s$^{-1}$)</td>
<td>M1</td>
<td>Use of correct Pythagoras with their $v$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>Exact form or 2s.f. or better. Watch out for fortuitous answers from $2.5i + j$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[5]</td>
<td></td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
<td>Notes</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>2a</strong></td>
<td>Work done = $15\mu R = 15 \times 0.4 \times 3 \cos 20^\circ$</td>
<td>M1</td>
<td>$F_{\text{max}} = \mu \times 3g \cos 20^\circ$ (11.05). $R$ must be resolved but condone trig confusion.</td>
</tr>
<tr>
<td></td>
<td>$= 18g \cos 20 = 166 \text{ (J)}$</td>
<td>M1</td>
<td>$15\times$ their $F_{\text{max}}$. Independent M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>$15 \times F_{\text{max}}$ + ..... is M0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>or 170 (J)</td>
</tr>
<tr>
<td><strong>2b</strong></td>
<td>Energy: WD against $F + \text{GPE} + \text{final KE} = \text{initial KE}$</td>
<td>M1/A2ft</td>
<td>Must include all four correct terms (including resolving). Condone sign errors and trig confusion. Any sign errors in the KE terms count as a single error. Follow their WD</td>
</tr>
<tr>
<td></td>
<td>their WD $+ 3g \sin 20^\circ \times 15 + \frac{1}{2} 3v^2 = \frac{1}{2} 3 \times 20^2$</td>
<td>A1</td>
<td>-1ee Follow their WD</td>
</tr>
<tr>
<td></td>
<td>$v = 13.7 \text{ (m s}^{-1})$</td>
<td></td>
<td>or 14</td>
</tr>
<tr>
<td><strong>Or 2b</strong></td>
<td>$3a = - 0.4 \times 3g \cos 20 + 3g \sin 20$ and use of $v^2 = u^2 + 2as$</td>
<td>M1</td>
<td>Complete method. Their $F_{\text{max}}$ component of weight</td>
</tr>
<tr>
<td></td>
<td>$v^2 = 20^2 + 2 \times a \times 15 (=188.93\ldots)$</td>
<td>A1ft</td>
<td>A correct equation with their $F_{\text{max}}$. Allow for $a = +7.03\ldots$ acting down the slope</td>
</tr>
<tr>
<td></td>
<td>$v = 13.7 \text{ (m s}^{-1})$</td>
<td>A1</td>
<td>$a = -7.035\ldots$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Correct equation for their $a$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>or 14 (m s$^{-1}$)</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
<td>Notes</td>
</tr>
<tr>
<td>----------------</td>
<td>--------</td>
<td>-------</td>
<td>-------</td>
</tr>
</tbody>
</table>
| 3a | \( v = 0 = 2t^2 - 14t + 20 \)  
\[ = 2t^2 - 2t - 5 \]
\[ t = 2 \text{ or } t = 5 \] | M1  
M1  
A1 | [3] Set \( v = 0 \)  
Solve for \( t \) |

There are many different approaches to part (b). The allocation of the two M marks is

M1: A method to find the time when the velocity is a minimum  
M1: Evaluate the speed at that time

e.g. b  
\[ t = 0, \ v = 20 \text{ (m s}^{-1}) \]
\[ a = 4t - 14 = 0 \]  
\[ t = \frac{7}{2}, \ v = 2 \times \frac{3}{2} \times \frac{-3}{2} = \frac{-9}{2} \]  
Max speed = 20 ms\(^{-1}\)  

\[ \text{B1} \]

\[ \text{M1} \]

\[ \text{M1A1} \]

Must see ±4.5  
Clearly stated & correct conclusion.  
Depends on the two M marks. From correct solution only.

balt1  
\[ t = 0, \ v = 20 \text{ (m s}^{-1}) \]
Sketch with symmetry about their \( t = 3.5 \)  
\( v(\text{their 3.5}) \)  
\(-4.5\)  
Max speed = 20 ms\(^{-1}\)  

\[ \text{B1} \]

\[ \text{M1} \]

\[ \text{M1} \]

\[ \text{M1} \]

\[ \text{A1} \]

\[ \text{A1} \]

[5]  
Evaluate \( v \) at min.  
Correct work  
Clearly stated & correct conclusion.  
Depends on the two M marks. From correct solution only.

b alt 2  
\[ t = 0, \ v = 20 \text{ (m s}^{-1}) \]
Justification of minimum or tabulate sufficient values to confirm location  
Evaluate \( v \) at min.  
Correct work  
Correct conclusion. Depends on the two M marks  

\[ \text{B1} \]

\[ \text{M1} \]

\[ \text{M1} \]

\[ \text{A1} \]

\[ \text{A1} \]

[5]  
Clearly stated & from correct solution only.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
</table>
| **b alt 3**     | $t = 0$, $v = 20$ (m s$^{-1}$)  
Complete the square as far as $\left(t - \frac{7}{2}\right)^2$  
$2\left(t - \frac{7}{2}\right)^2 - \frac{9}{2}$  
Max speed = 20 ms$^{-1}$ | B1 | Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only. |
| **c**           | $\int 2t^2 - 14t + 20 \ dt = \frac{2}{3} t^3 - 7t^2 + 20t + C$  
Distance $= \left[ \frac{2}{3} t^3 - 7t^2 + 20t \right]_0^2 - \left[ \frac{2}{3} t^3 - 7t^2 + 20t \right]_0^4$  
$= 2 \left[ \frac{2}{3} t^3 - 7t^2 + 20t \right]_0^4 - \left[ \frac{2}{3} t^3 - 7t^2 + 20t \right]_0^2$  
$= 2 \left[ \frac{16}{3} - 7 \times 4 + 40 \right] - \left[ \frac{2 \times 64}{3} - 7 \times 16 + 80 \right] = 24$ (m) | M1 | Integration. Need to see majority of powers going up  
A1 | All correct. Condone C missing  
M1 | Correct method to find the distance, for their 2  
A1 | Correct unsimplified  
|                 |       |       | [5] |


### Question 4a

For a valid division into basic elements:

- e.g. pair of rhombuses

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AOCB$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>$OCDE$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>whole</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td>1/2</td>
<td>$\bar{y}$</td>
<td></td>
</tr>
</tbody>
</table>

\[
2\bar{y} = 1 \times \frac{1}{2} + 1 \times \frac{1}{2} \\
\bar{y} = 0.5 \text{ (m)}
\]

### a alt 2

Rhombus + two triangles

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AOB$</td>
<td>B1</td>
</tr>
<tr>
<td>$OBCD$</td>
<td>B1</td>
</tr>
<tr>
<td>$DOE$</td>
<td></td>
</tr>
<tr>
<td>whole</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\bar{y}$</td>
</tr>
</tbody>
</table>

\[
4\bar{y} = 2 \times 1 \\
\bar{y} = 0.5 \text{ (m)}
\]
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a alt 3</strong></td>
<td>Hexagon $AOEF$ whole</td>
<td>B1 B1</td>
<td>Hexagon – rhombus</td>
</tr>
<tr>
<td></td>
<td>$4\bar{y} = 0 - 2 \times 1$</td>
<td>M1A1 A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\bar{y} = 0.5$ (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>a alt 4</strong></td>
<td>$h =$ height of each triangle $= \sqrt{3}$</td>
<td></td>
<td>4 triangles</td>
</tr>
<tr>
<td></td>
<td>Distances of c of m from horizontal through $O$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>$q$</td>
<td>$r$</td>
<td>$s$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{2}{3}h \cos 30$</td>
<td>$\frac{2}{3}h \cos 30$</td>
<td>0</td>
</tr>
<tr>
<td>$4\bar{y} = 2 \times 1 \times \frac{2\sqrt{3}}{3} \cos 30 \left( = \frac{4\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = 2 \right)$</td>
<td>M1A1 A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{y} = 0.5$ (m)</td>
<td></td>
<td></td>
<td>[5]</td>
</tr>
</tbody>
</table>
In 4(b) the first two marks are
M1: Identify a triangle, with one angle correct, and attempt to find the lengths of two sides
A1ft: 2 sides correct, follow their answer to (a)
DM1: Work sufficient to be able to go on to find the required angle. Dependent on the preceding M1
A1ft: follow their answer to (a)
DM1: Find the required angle. Dependent on the preceding M1
A1 Correct answer
.... for example ......

2\cos30 = \sqrt{3} \quad \text{, "0.5"+2\sin30 = 1.5}
\tan\theta = \frac{\text{their 1.5}}{\text{their } \sqrt{3}}
\text{Required angle} = \theta - 30 = \tan^{-1}\frac{1.5}{\sqrt{3}} - 30 = 40.89... - 30 = 10.9^\circ

M1A1ft Their 0.5 & their \sqrt{3}
DM1 Use of tan in a right angled triangle.
Accept the reciprocal
A1ft Correct for their angle. Ft their 0.5
DM1 Correct strategy to find required angle
e.g. "\theta"–30°
or 90°–30°–"\theta"
A1 [6] Accept 11°, 10.9° or better
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4balt</td>
<td><img src="image" alt="Diagram" /></td>
<td>M1A1ft</td>
<td>Their 0.5</td>
</tr>
<tr>
<td></td>
<td>SAS in a relevant triangle</td>
<td>DM1</td>
<td>Correct cosine rule.</td>
</tr>
<tr>
<td></td>
<td>$d^2 = 2^2 + 0.5^2 - 2 \times 2 \times 0.5 \cos 120 = 5.25$</td>
<td>A1ft</td>
<td>Correct equation. Their 0.5</td>
</tr>
<tr>
<td></td>
<td>$\sin \theta = \frac{\sin 120}{0.5} = \frac{\sin 120}{\sqrt{5.25}}$</td>
<td>DM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\theta = 10.9^\circ$</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>

[6]
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5a</strong></td>
<td><img src="image" alt="Diagram" /></td>
<td></td>
<td>Moments about A. Requires all three terms and terms of correct structure (force x distance). Condone consistent trig confusion</td>
</tr>
<tr>
<td></td>
<td>( bF = a \cos \theta mg + 2a \cos \theta mg = 3a \cos \theta mg )</td>
<td>M1</td>
<td>-1 each error</td>
</tr>
<tr>
<td></td>
<td>( F = \frac{3amg \cos \theta}{b} ) <em>Answer given</em></td>
<td>A2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Resolve horizontally. Condone trig confusion</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td><strong>5b</strong></td>
<td></td>
<td></td>
<td>Resolve vertically. Condone sign error and trig confusion</td>
</tr>
<tr>
<td></td>
<td>( H = F \sin \theta = \frac{3amg \cos \theta \sin \theta}{b} )</td>
<td>M1</td>
<td>RHS correct. Or equivalent.</td>
</tr>
<tr>
<td></td>
<td>( 2mg = \pm V + F \cos \theta )</td>
<td>A1</td>
<td>Correct equation</td>
</tr>
<tr>
<td></td>
<td>( \pm V = 2mg - \frac{3amg \cos \theta}{b} \times \cos \theta \left( = 2mg - \frac{3amg \cos^2 \theta}{b} \right) )</td>
<td>A1</td>
<td>RHS correct. Or equivalent</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
<td>Notes</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>5c</strong></td>
<td>2mg - ( \frac{3amg \cos^2 \theta}{b} ) = \tan \theta &lt;br&gt; ( \frac{3amg \cos \theta \sin \theta}{b} ) = \sin \theta &lt;br&gt; ( \frac{3a \cos \theta \sin \theta}{3a \cos \theta \sin \theta} = \frac{\sin \theta}{\cos \theta} ) &lt;br&gt; ( 2b - 3a \cos^2 \theta = 3a \sin^2 \theta \Rightarrow 2b = 3a, \quad \frac{a}{b} = \frac{2}{3} )</td>
<td>M1, A1, DM1</td>
<td>Use of tan, either way up. ( V, H, F ) substituted. Correct for their components in ( \theta ) only. Simplify to obtain the ratio of ( a ) and ( b ), or equivalent</td>
</tr>
<tr>
<td><strong>5c alt 2</strong></td>
<td>The centre of mass of the combined rod + particle is ( \frac{3}{2} a ) from ( A )</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 forces in equilibrium must be concurrent ( \Rightarrow b = \frac{3}{2} a ) &lt;br&gt; ( \frac{a}{b} = \frac{2}{3} )</td>
<td>M1, A1</td>
<td>Not on the spec, but you might see it.</td>
</tr>
<tr>
<td><strong>alt c 3</strong></td>
<td>( R ) acts along the rod, so resolve forces perpendicular to the rod. &lt;br&gt; ( F = mg \cos \theta + mg \cos \theta ) &lt;br&gt; ( 2mg \cos \theta = \frac{3amg \cos \theta}{b} )</td>
<td>M1, A1, DM1</td>
<td>Resolve and substitute for ( F ) Eliminate ( \theta )</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
<td>Notes</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>alt c 4</strong></td>
<td>$R$ acts along the rod. Take moments about $C$ $mg \cos \theta \ 2a - b = mg \cos \theta \ b - a$ $2a - b = b - a, \quad \Rightarrow \frac{a}{b} = \frac{2}{3}$</td>
<td>M1 A1 DM1A1</td>
<td>Moments about $B$ gives $2a - b \ F = amg \cos \theta$ and substitute for $F$</td>
</tr>
<tr>
<td><strong>c alt 5</strong></td>
<td>Resultant parallel to the rod $\Rightarrow R = 2mg \sin \theta$ And $V^2 + H^2 = R^2$ $2mg \sin \theta = \left( \frac{3amg \cos \theta \sin \theta}{b} \right)^2 + \left( 2mg - \frac{3amg \cos^2 \theta}{b} \right)^2$ Eliminate $\theta$ $\Rightarrow \frac{a}{b} = \frac{2}{3}$</td>
<td>M1 A1 DM1 A1</td>
<td>Substitute for $V, H$ and $R$ in terms of $\theta$</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
<td>Notes</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
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<td>-------</td>
</tr>
</tbody>
</table>
| **6a** | Conservation of energy:  
\[
\frac{1}{2}mu^2 + mg \times 8 = \frac{1}{2} m \ 2u^2 \\
mu^2 + 16mg = 4mu^2 \\
16mg = 3mu^2, \quad u = \sqrt{\frac{16g}{3}} \\
u = 7.2 | M1 | Energy equation must contain the correct terms, but condone sign error. |

Correct unsimplified | A2 -1ee |

Solve for \( u \) | DM1 |

Accept 7.23. Accept \( \sqrt{\frac{16g}{3}} \) | A1 [5] |

**6b** | Vertical distance:  
\[-8 = u \sin \theta \times 2 - \frac{g}{2} \times 4 \]
\[
\sin \theta = \frac{2g - 8}{2u} = 0.802... \\
\theta = 53.3^\circ | M1 | Condone sign errors or trig error. \( u \) must be resolved. |

Correct equation for their \( u \). | A2 -1ee |

or 53° | A1 [4] |

**6c** | Min speed at max height, i.e. \( u \cos \theta \)
\[
= 4.3 \ (\text{m} \cdot \text{s}^{-1}) | M1 | Condone consistent trig confusion with part (b) |

or 4.32 (\text{ms}^{-1}) | A1 [2] |
<table>
<thead>
<tr>
<th>Question Number</th>
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<th>Notes</th>
</tr>
</thead>
</table>
| **7a** | CLM: $2mu = 2mv + 3mw$  
Impact: $w - v = eu$  
Subst $v = w - eu$: $2u = 2w - eu + 3w = 5w - 2eu$  
$w = \frac{2}{5} (1 + e) u$  
*Answer Given* | M1  
A1  
M1  
A1  
DM1  
A1 | All three terms required, but condone sign errors  
Condone sign error, but must be subtracting and $e$ must be used correctly.  
Penalise inconsistent signs here.  
Solve for $w$. Requires the two preceding M marks |
| **7b** | $w = \frac{7u}{10}$  
CLM: $3mw = 3mx + 4my$ and Impact: $y - x = \frac{3w}{4}$  
Subst: $3w = 3x + 4\left(x + \frac{3w}{4}\right)$  
$x = 0$,  
y = $\frac{3}{4} w = \frac{21}{40} u$ | B1  
M1A1  
DM1  
A1  
A1 | Seen, or implied by correct speeds.  
Both needed  
Solve for $x$ or $y$. Dependent on the preceding M mark  
$0.525u$, |
| **7c** | $v = -\frac{u}{20}$  
Speed of separation = $\frac{u}{20} + \frac{21u}{40} = \frac{23u}{40}$ | B1  
M1  
A1 | Correct velocity of $P$  
Correct use of their values and substitute for $e$.  
Check directions carefully  
$0.575u$ |
Mark Scheme (Results)

Summer 2013

GCE Mechanics 3 (6679/01)
Edexcel and BTEC Qualifications

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General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ✕ The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme
**General Rules for Marking Mechanics**

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is accuracy error not method error.
- Omission of mass from a resolution is method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td>[6]</td>
</tr>
</tbody>
</table>

B1 for resolving vertically and using $F = \mu R$ to obtain $F = \mu mg$. This may not be seen explicitly, but give B1 when seen used in an equation.

M1 for attempting to change revs per minute to rad s$^{-1}$, must see $\left(2\pi\right)$. (Can use 60 or $60^2$)

A1 for $\frac{20}{60} \times 2\pi$ (rad s$^{-1}$) oe

M1 for NL2 horizontally along the radius - acceleration in either form for this mark, $F$ or $\mu mg$ or $\mu m$ all allowed. $r$ to be 0.4 now or later. This is not dependent on the previous M mark.

A1ft for $\mu mg = m \times 0.4 \times \left(\frac{2}{3} \pi\right)^2$ follow through on their $\omega$

A1cso for $\mu = 0.18$ or 0.179, must be 2 or 3 sf.

NB: Use of $\leq$ is allowed, provided used correctly, until the final statement, which must be $\mu =.....$
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2 (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 2t + \frac{1}{2} = 0.5 \frac{dv}{dt} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( \int (4t + 1) , dt = \int dv )</td>
<td>M1 dep c not needed</td>
<td></td>
</tr>
<tr>
<td>( 2t^2 + t = v + c )</td>
<td>A1 inc the value for c (3)</td>
<td></td>
</tr>
<tr>
<td>( t = 0 \ v = 0 \ c = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v = 2t^2 + t \text{ m s}^{-1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{dx}{dt} = 2t^2 + t )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( x = \frac{2}{3} t^3 + \frac{1}{2} t^2 + k )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( t = 0 \ x = 0 \ k = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = \frac{2}{3} t^3 + \frac{1}{2} t^2 )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( v = 6 \ 6 = 2t^2 + t \ 2t^2 + t - 6 = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (2t - 3)(t + 2) = 0 \ t = \frac{3}{2} )</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>( x = \frac{2}{3} \times \left( \frac{3}{2} \right)^3 + \frac{1}{2} \left( \frac{3}{2} \right)^2 )</td>
<td>M1 dep</td>
<td></td>
</tr>
<tr>
<td>( x = \frac{27}{8} \text{ (oe 3.4, 3.375, 3.38) m} )</td>
<td>A1 cso (6)</td>
<td></td>
</tr>
</tbody>
</table>
**Notes for Question 2**

| (a) | M1 for NL2 with acceleration in the form $\frac{dv}{dt}$, seen explicitly or implied by the integration mass can be 0.5 or $m$
|     | M1dep for integrating with respect to $t$ - constant not needed
|     | A1cso for **showing** that $c = 0$ and giving the final result $v = 2t^2 + t$ Must see $t = 0, v = 0$ as a minimum

*By definite integration:*
| M1 | as above
| M1dep | for integrating, ignore limits
|     | A1 for substituting the limits 0 and $v$ and 0 and $t$ and obtaining $v = 2t^2 + t$
| (b) | M1 for integrating their $v$ with respect to $t$ constant not needed
|     | A1 for **showing** that $k = 0$ If no constant shown this mark is lost.
|     | M1 for setting $v = 6$ using their answer from (a) **and** attempting to solve the resulting quadratic equation, any valid method. If solved by calculator, **both** solutions must be shown.
|     | A1 for $t = \frac{3}{2}$ negative solution need not be shown with an algebraic solution
| M1dep | for using **their** (positive) value for $t$ to obtain $x = ...$. If two positive values were obtained, then allow M1 for substituting either value. Dependent on the first M1 of (b) but not the second.
|     | A1cso for $x = \frac{27}{8}$ (oe eg 3.375, 3.38) (All marks for (b) must have been awarded)

*By definite integration:*
| M1 | for integrating their $v$ with respect to $t$ limits not needed
|     | A1 for correct integration with lower limits 0.
| M1 | for setting $v = 6$ using their answer from (a) **and** attempting to solve the resulting quadratic equation, any valid method. If solved by calculator, **both** solutions must be shown.
|     | A1 for $t = \frac{3}{2}$ negative solution need not be shown with an algebraic solution
| M1dep | for substituting **their** limits into **their** integrated $v$ (sub should be shown). Dependent on the first M1 of (b) but not the second
|     | A1cso for $x = \frac{27}{8}$ (oe eg 3.375, 3.38)
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>3 (i)</strong></td>
<td>For $Q$ ( T = 2mg )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>For $P$ ( T \cos \theta = mg )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \cos \theta = \frac{1}{2} ) ( \theta = 60^\circ ) *</td>
<td>A1cso</td>
</tr>
<tr>
<td><strong>3 (ii)</strong></td>
<td>For $P$ ( \rightarrow T \sin \theta = mr\omega^2 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( 2mg \sin \theta = m \times 5l \sin \theta \times \omega^2 )</td>
<td>M1depA1</td>
</tr>
<tr>
<td></td>
<td>( \omega^2 = \frac{2g}{5l} ) ( \omega = \sqrt{\frac{2g}{5l}} ) *</td>
<td>A1cso</td>
</tr>
</tbody>
</table>
Notes for Question 3

In this question, award marks as though the question is not divided into two parts - ie give marks for equations wherever seen.

(i) 

B1 for using \( Q \) (no need to state \( Q \) being used) to state that \( T = 2mg \) or \( T_Q = 2mg \) with \( T_p = T_Q \) seen or implied later.

M1 for attempting to resolve vertically for \( P \) \( T \) must be resolved but sin/cos interchange or omission of \( g \) are accuracy errors.

\[ mg + 2mg = T + T \cos \theta \] gets M0

A1cso for combining the two equations to obtain \( \theta = 60^\circ \)  

NB: This is a "show" question, so if no expression is seen for \( T \) and just \( 2mg \cos \theta = mg \) shown, award 0/3 as this equation could have been produced from the required result, so insufficient working.

(ii) 

M1 for attempting NL2 for \( P \) along the radius. The mass used must be \( m \) if the particle is not stated to be \( P \); a mass of \( 2m \) would imply use of \( Q \). \( T \) must be resolved. Acceleration can be in either form.

A1 for \( T \sin \theta = mr \omega^2 \) or \( T \frac{\sqrt{3}}{2} = mr \omega^2 \)

M1 dep for eliminating \( T \) between the two equations for \( P \) and substituting for \( r \) in terms of \( l \) and \( \theta \) dependent on the second but not the first M mark.

A1 for \( 2mg \sin \theta = m \times 5l \sin \theta \times \omega^2 \) or \( \frac{T \sin \theta}{T \cos \theta} = \tan \theta = 5l \sin \theta \left( \frac{\omega^2}{g} \right) \) \( \theta \) or \( 60^\circ \)

A1cso for re-arranging to obtain \( \omega = \sqrt{\frac{2g}{5l}} \)  

Ensure the square root is correctly placed

Alternatives: Some candidates "cancel" the \( \sin \theta \) without ever showing it.

M1A1 for \( T = m \times 5l \omega^2 \)
M1A1 for \( 2mg = 5ml \omega^2 \)
A1cso as above

Vector Triangle method: Triangle must be seen

- \( T = 2mg \)
- \( \cos \theta = \frac{mg}{2mg} \)
- \( \theta = 60^\circ \)
- Correct triangle
- \( \sin \theta = \frac{5ml \sin \theta \omega^2}{2mg} \)
- \( \omega = ... \)
A1cso (as above)
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>4 (a)</td>
<td>$T = \frac{\lambda x}{l}$</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$20 = \frac{\lambda \times 0.3}{1.2}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 80 \text{ N}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Initial EPE = $\frac{\lambda x^2}{2l} = \frac{80 \times 0.3^2}{2.4} = 3 \text{ J}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$80 \times 0.3^2 - 0.4 \times 2g \times 0.3 = \frac{1}{2} \times 2v^2$</td>
<td>M1A1ft</td>
</tr>
<tr>
<td></td>
<td>$v^2 = 0.648$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v = 0.8 \text{ or } 0.805 \text{ m s}^{-1}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Comes to rest $0.4 \times 2g \times y = 3$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{3}{0.4 \times 2 \times 9.8} = 0.38 \text{ or } 0.383 \text{ m}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Alternatives: Energy from string going slack to rest:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2} \times 2 \times 0.648 = 0.4 \times 2g \times x$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 0.8265\ldots$</td>
<td>M1 Complete method</td>
</tr>
<tr>
<td></td>
<td>$y = 0.3 + 0.08265\ldots = 0.38 \text{ or } 0.383$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>NL2 to obtain the accel when string is slack $\left(-\frac{2g}{5}\right)$ and $v^2 = u^2 + 2as$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0 = 0.648 + 2 \times \left(-\frac{2g}{5}\right)s$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$BC = \frac{0.648 \times 5}{4g} + 0.3 = 0.38 \text{ or } 0.383$</td>
<td>M1A1</td>
</tr>
<tr>
<td>Notes for Question 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(a)</strong> M1 for attempting Hooke's Law, formula must be correct, either explicitly or by correct substitution.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1 for ( 20 = \frac{\lambda \times 0.3}{1.2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1 for obtaining ( \lambda = 80 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1 for the initial EPE ( \frac{''\lambda'' \times 0.3^2}{2.4} ) ( (= 3 \text{ J}) ) their value for ( \lambda ) allowed. May only be seen in the equation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 for a work-energy equation with one EPE term, one KE term and work done against friction (Award if second EPE/KE terms included provided these become 0). The EPE must be dimensionally correct, but need not be fully correct (eg denominator 1.2 instead of 2.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1ft for a completely correct equation follow through their EPE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1 cao for ( v = 0.80 ) or ( 0.805 ) must be 2 or 3 sf</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>NB:</strong> This is damped harmonic motion (due to friction) so all SHM attempts lose the last 4 marks.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong> M1 for any complete method leading to a value for either ( BC ). If the distance travelled after the string becomes slack is found the work must be completed by adding 0.3 Their EPE found in (a) used in energy methods.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MS</strong> method is energy from ( B ) to ( C ) ie work done against friction = loss of EPE.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>OR</strong> Energy from point where the string becomes slack to ( C ) ie work done against friction = loss of KE and completed for the required distance</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>OR</strong> NL2 to obtain the acceleration ( \left( -\frac{2g}{5} \right) ) while the string is slack and ( v^2 = u^2 + 2as ) to find the distance and completed for the required distance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1cso for ( BC = 0.38 ) or ( 0.383 ) (m) must be 2 or 3 sf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>5(a)</strong></td>
<td>[ V = \pi \int_{-1}^{1} x^4 , dx = \pi \left( \left[ \frac{1}{5} (x+1)^5 \right]_{-1}^{1} \right) = \frac{1}{5} \pi \left[ 3^5 - 1 \right] = \frac{242 \pi}{5} ]</td>
<td>M1, A1</td>
</tr>
<tr>
<td></td>
<td>[ \int_{-1}^{1} x^2 , dx = \pi \int_{-1}^{1} x , dx = \pi \left( \left[ \frac{x^3}{3} \right]<em>{-1}^{1} \right) = \frac{2 \pi^2}{3} - \pi \left( \left[ \frac{1}{3} (x+1)^3 \right]</em>{-1}^{1} \right) = \frac{2 \pi^2}{3} - \pi \left( \frac{242 \pi}{30} \right) ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td><strong>ALT:</strong> by expanding [ V = \pi \int_{-1}^{1} \left( x^3 + 4x^2 + 6x + 4x^2 + x \right) , dx = \pi \left( \left[ \frac{x^4}{5} + \frac{4x^3}{5} + \frac{6x^2}{5} + \frac{4x}{5} + \frac{1}{2} x^2 \right]_{-1}^{1} \right) = \frac{2 \pi^2}{3} - \pi \left( \frac{242 \pi}{30} \right) ]</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td><strong>OR by subst:</strong> [ V = \pi \int_{0}^{1} \left( u - 1 \right) u^{4} , du = \pi \left( \left[ \frac{u^6 - u^5}{6} \right] \right) = \pi \left( \frac{3^5 - \frac{3^5}{5} \left( \frac{1}{6} - \frac{1}{5} \right)}{6} \right) ]</td>
<td>M1A1M1</td>
</tr>
<tr>
<td><strong>5(b)</strong></td>
<td>[ \bar{x} = \frac{\frac{2 \pi^2}{3} - \frac{242 \pi}{30}}{\frac{242 \pi}{5}} = \frac{\frac{2 \pi^2}{3} - \frac{242 \pi}{30}}{\frac{242 \pi}{5}} ]</td>
<td>M1, A1 (8)</td>
</tr>
<tr>
<td></td>
<td>hemisphere [ S ] [ T ] Mass ratio [ 10 \times \frac{2 \pi}{3} \times 1 = \frac{242 \pi}{5} ] [ \left( \frac{20}{3} + \frac{242}{5} \right) \pi = \frac{826}{15} \pi ] [ B1 \text{ft on } S ] Dist from [ A ] [ 2 + \frac{3 \times 1}{8} = \frac{19}{8} ] [ 0.493 ] [ \bar{x} ] [ B1 \text{ft on } S ] [ \frac{20}{3} \times \frac{19}{8} + \frac{242}{5} \times 0.493 = \left( \frac{20}{3} + \frac{242}{5} \right) \bar{x} ]</td>
<td>M1A1ft</td>
</tr>
<tr>
<td></td>
<td>[ \bar{x} = 0.7208... \text{ cm (awrt 0.72)} ]</td>
<td>A1 (5) [13]</td>
</tr>
</tbody>
</table>
Notes for Question 5

**NB:** Some candidates will omit \( \pi \) throughout (as they know it cancels). In such cases award all marks if earned. If \( \pi \) is omitted from one integration only but then appears in the result of that integration at the last stage or is then omitted from the second integration, all marks can be gained. But if omitted from one integration, including the last stage, and included with the other mark strictly according to the MS.

**a)**

M1 for using \( V = \int_{0}^{2} \pi y^2 \, dx = \pi \int_{0}^{2} (x+1)^4 \, dx \) - limits not needed and attempting the integration by inspection or expansion (algebra **must** be seen)

A1 for correct integration - limits not needed

M1 for substituting the correct limits into their integrated function - no need to simplify

M1 for attempting to integrate \( \int_{0}^{2} \pi y^2 \, dx = \pi \int_{0}^{2} (x+1)^4 \, dx \) - limits not needed - by parts. This mark can be awarded once the integral has been expressed as the difference of an appropriate integrated function and an integral

A1 for correct, complete integration

\[ \left[ \frac{x(x+1)^5}{5} \right]_{0}^{2} - \pi \left[ \frac{(x+1)^6}{30} \right]_{0}^{2} \text{ or } \left[ \frac{2 \times 3^3 \pi}{3} - \pi \right] \left[ \frac{(x+1)^6}{30} \right]_{0}^{2} \]

Limits not needed

M1 for substituting the correct limits into their integrated function - no need to simplify

---

**Alternative methods for** \( \int_{0}^{2} \pi y^2 \, dx = \pi \int_{0}^{2} (x+1)^4 \, dx \)

M1 for expanding and integrating or making a suitable substitution and attempting the integration - limits not needed

A1 for correct integration - limits not needed

M1 for substituting the correct limits into their integrated function - no need to simplify

---

M1 for using \( \bar{x} = \frac{\int \pi y^2 \, dx}{\int \pi y^2 \, dy} \) Their integrals need not be correct.

A1cao for \( \bar{x} = 1.5068... \) Accept 1.5, 1.51 or better or \( \frac{547}{363} \)

**b)**

B1ft for correct mass ratio, follow through their volume for \( S \) need \( \pi \) now

B1ft for correct distances, follow through their distance for \( S \), but remember it must be 2 - answer from (a) if working from \( A \). Distances from the common face are \( -\frac{3}{8} \), ans from \( (a), \bar{x} \) Distances from other end are \( \frac{5}{8} \), 1 + ans from \( (a), \bar{x} \)

M1 for a dimensionally correct moments equation

A1ft for a fully correct moments equation, follow through their distances and mass ratio

A1cao for 0.7208...Accept 0.72 or better (Exact is \( \frac{1191}{1652} \))
### Question Number 6(a)

\[ \frac{24e}{1.5} = \frac{18(1.5 - e)}{0.75} \]

\[ 16e = 36 - 24e \]

\[ e = 0.9 \]

\[ AO = 2.4 \text{ m}^* \]

### Question Number 6(b)

\[ \frac{18(0.6 - x)}{0.75} - \frac{24(0.9 + x)}{1.5} = mx \text{ or } 0.8\ddot{x} \]

\[ 14.4 - 24x - 14.4 - 16x = mx \text{ or } 0.8\ddot{x} \]

\[ \ddot{x} = -\frac{40x}{0.8 \text{ or } m} \quad (= -50x) \quad \therefore \text{SHM} \]

### Question Number 6(c)

\[ \ddot{x} = -50x \Rightarrow \omega = \sqrt{50} \text{ or } 5\sqrt{2} \]

\[ \text{max. speed} = \sqrt{2} \Rightarrow a \times 5\sqrt{2} = \sqrt{2} \]

\[ a = \frac{1}{5} \]

\[ -0.1 = 0.2 \cos\left(5\sqrt{2}t\right) \]

\[ t = \frac{1}{5\sqrt{2}} \cos^{-1}\left(-\frac{1}{2}\right) \]

\[ t = \frac{1}{5\sqrt{2}} \times \frac{2\pi}{3} = \frac{\pi\sqrt{2}}{15} \text{ or } 0.296s (0.2961...) \quad \text{Accept 0.30, or better} \]

---

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td></td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>[ \frac{24e}{1.5} = \frac{18(1.5 - e)}{0.75} ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ 16e = 36 - 24e ]</td>
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<tr>
<td></td>
<td>[ e = 0.9 ]</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[ AO = 2.4 \text{ m}^* ]</td>
<td>A1ft</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td>M1A1A1</td>
</tr>
<tr>
<td></td>
<td>[ \frac{18(0.6 - x)}{0.75} - \frac{24(0.9 + x)}{1.5} = mx \text{ or } 0.8\ddot{x} ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ 14.4 - 24x - 14.4 - 16x = mx \text{ or } 0.8\ddot{x} ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \ddot{x} = -\frac{40x}{0.8 \text{ or } m} \quad (= -50x) \quad \therefore \text{SHM} ]</td>
<td>(5)</td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>[ \ddot{x} = -50x \Rightarrow \omega = \sqrt{50} \text{ or } 5\sqrt{2} ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \text{max. speed} = \sqrt{2} \Rightarrow a \times 5\sqrt{2} = \sqrt{2} ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ a = \frac{1}{5} ]</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[ -0.1 = 0.2 \cos\left(5\sqrt{2}t\right) ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ t = \frac{1}{5\sqrt{2}} \cos^{-1}\left(-\frac{1}{2}\right) ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ t = \frac{1}{5\sqrt{2}} \times \frac{2\pi}{3} = \frac{\pi\sqrt{2}}{15} \text{ or } 0.296s (0.2961...) \quad \text{Accept 0.30, or better} ]</td>
<td>A1(5)</td>
</tr>
</tbody>
</table>

[14]
Notes for Question 6

(a)
M1 for using Hooke's Law for each string, equating the two tensions and solving to find the extension in either string. The extensions should add to 1.5. The formula for Hooke's law must be correct, either shown explicitly in its general form or implicitly by the substitution.

A1 for a correct equation

A1 for  \( e = 0.9 \)

A1cso for 2.4 (m) *

Alternative: Find the ratio of the two extensions and divide 1.5 m in that ratio.
M1 complete method A1 correct ratio A1 extension in AO

A1 2.4 (m)

(b)
M1 for an equation of motion for \( P \). There must be a difference of two tensions. The acceleration can be \( a \) or \( \ddot{x} \) here and \( x \) should be measured from the equilibrium position (O) unless a suitable substitution is made later. Mass can be \( m \) or 0.8

A1, A1 for \( \frac{18(0.6 - x)}{0.75} - \frac{24(0.9 + x)}{1.5} = m\ddot{x} \) or 0.8\( \ddot{x} \) or \( a \) instead of \( \dddot{x} \). Give A1A1 if the equation is completely correct and A1 if only one error. Note that if the difference of the tensions is the wrong way round, this is one error.

M1dep for simplifying to \( \ddot{x} = f(x) \) Must be \( \ddot{x} \) now.

A1 for \( \ddot{x} = -\frac{40x}{0.8} \) or \( m \) (\( = -50x \)) and the conclusion (ie \( \therefore \) SHM)

(c)
B1 for \( \omega = \sqrt{50} \) or \( 5\sqrt{2} \) need not be shown explicitly

M1 for using max speed \( = \omega a \) with their \( \omega \)

A1 for \( a = \frac{1}{5} \)

M1 for using \( x = a \cos \omega t \) with their \( \omega \) and \( a \) and \( x = \pm(0.3 - a) \) or \( x = a \sin \omega t \) provided the work is completed by adding a quarter of their period is added to the time to complete the method.

A1cao for \( t = \frac{\pi\sqrt{2}}{15} \) or 0.296s (0.2961...) Accept 0.30 or better
7

(a) \[ T - 5mg \cos \theta = \frac{5mv^2}{a} \]

\[ \frac{1}{2} \times 5mv^2 - \frac{1}{2} \times 5m \times \frac{9ag}{5} = 5 mga \cos \theta \]

\[ 5mv^2 = 10 mga \cos \theta + 9 mga \]

\[ T = 5 mg \cos \theta + 10 mga \cos \theta + 9 mga \]

\[ T = 3 mg (5 \cos \theta + 3) \]  

(b) \[ T = 0 \] \[ \cos \theta = -\frac{3}{5} \]

\[ v^2 = \frac{9ag}{5} - \frac{6ag}{5} = \frac{3ag}{5} \]

\[ v = \sqrt{\frac{3ag}{5}} \]

(c) horiz comp of vel at B = \[ \frac{3ag}{5} \times \frac{3}{5} \]

vert comp = \[ \frac{3ag}{5} \times \frac{4}{5} \]

(i) \[ x = -\frac{4a}{5} + \frac{3}{5} \sqrt{\frac{3ag}{5}}t \]

\[ y - \frac{3a}{5} = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2 \]

(ii) \[ y = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2 + \frac{3a}{5} \]

[16]
Notes for Question 7

(a)
M1 for attempting NL2 along the radius when the string makes an angle $\theta$ with the downward vertical. The acceleration can be in either form, the weight must be resolved and $T$ must be included (not resolved). Sin/cos interchange or omission of $g$ are accuracy errors as is omission of $5$ in one or both terms. Radius can be $a$ or $r$.

A1 for a correct equation $T - 5mg \cos \theta = \frac{5mv^2}{a}$ Acceleration must be in the $\frac{v^2}{r}$ form now.

M1 for a conservation of energy equation from the horizontal to the same point. There must be a difference of 2 KE terms and a loss of PE term (which may be indicated by a difference of 2 PE terms). The initial KE can be $\frac{1}{2} \times \text{mass} \times \left(\frac{9ag}{3}\right)^2$ or $\frac{1}{2} \times \text{mass} \times u^2$ for this mark. Omission of $g$ and sin/cos interchange are accuracy errors. Mass can be $m$ or $5m$ here or just "mass". Use of $v^2 = u^2 + 2as$ gets M0

A1 for a fully correct equation $\frac{1}{2} \times (5m)v^2 - \frac{1}{2} \times (5m) \times \frac{9ag}{5} = (5m)ga \cos \theta$

M1dep for eliminating $v^2$ between the 2 equations. Dependent on both previous M marks.

A1cso for $T = 3mg(5\cos \theta + 3)$ *

(b)
B1 for obtaining $\cos \theta = -\frac{3}{5}$

M1 for using their value for $\cos \theta$ - must be numerical - in the energy equation to get $v^2 = \ldots$ (no need to simplify) Accept with $5m$ or $m$.

OR making $T = 0$ and $\cos \theta = -\frac{3}{5}$ (their value) in $T - 5mg \cos \theta = \frac{5mv^2}{a}$

A1cao for $v = \sqrt{\frac{3ag}{5}}$ oe Check square root is applied correctly.

(c)
M1 for resolving their $v$ to get the horizontal component of the speed at $B$. May not be seen explicitly, but seen in their attempt at $x$.

M1 for resolving their $v$ to get the vertical component of the speed at $B$

Both of these M marks can be given if sin and cos are interchanged or numerical substitutions not made.

M1dep for attempting to obtain $x$ by using the distance from $B$ to the $y$-axis with the horizontal distance travelled (found using their horizontal component, so dependent on the first M1 of (c))

A1cso for $x = \frac{4a}{5} + \frac{3}{5} \sqrt{\frac{3ag}{5}}$
M1dep for attempting to obtain \( y \) by using \( s = ut + \frac{1}{2}at^2 \) with their vertical component and using the initial vertical distance above the \( x \)-axis. Dependent on the second M mark of (c)

A1ft for \( y = \frac{3a}{5} = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2 \) Follow through their initial vertical component

A1cao for \( y = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2 + \frac{3a}{5} \)
Mark Scheme (Results)

Summer 2013

GCE Mechanics 4 (6680/01)
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Summer 2013
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \( \sqrt{ } \) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme
General Rules for Marking Mechanics

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is accuracy error not method error.

- Omission of mass from a resolution is method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.

- Use of $g = 9.81$ should be penalised once per (complete) question.

- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised ONCE per complete question.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.
<table>
<thead>
<tr>
<th>Question Number</th>
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<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>I(a)</td>
<td>Equation of motion: ( \frac{1}{2} g - \frac{3}{2} v = \frac{1}{2} \frac{dv}{dt} )</td>
<td>M1 Differential equation. All 3 terms required but condone sign errors</td>
</tr>
<tr>
<td></td>
<td>NB: these two marks are available in (b) if not scored in (a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \int 1 dt = \int \frac{1}{9.8 - 3v} dv ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ t + (C) = -\frac{1}{3} \ln(9.8 - 3v) ]</td>
<td>A1=A1</td>
</tr>
<tr>
<td></td>
<td>[ t = 0, v = 0 \Rightarrow C = -\frac{1}{3} \ln 9.8 ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ t = -\frac{1}{3} \ln \left( \frac{9.8 - 3v}{9.8} \right) ]</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[ 3v = 9.8(1 - e^{-3t}) ] <em>Given Answer</em></td>
<td>A1 (8) Watch out. cwo</td>
</tr>
<tr>
<td>(a) alt</td>
<td>Equation of motion: ( \frac{1}{2} g - \frac{3}{2} v = \frac{1}{2} \frac{dv}{dt} )</td>
<td>M1 All 3 terms required but condone sign errors</td>
</tr>
<tr>
<td></td>
<td>[ e^{3t} \frac{dv}{dt} + 3e^{3t} v = ge^{3t}, \frac{d}{dt}(ve^{3t}) = ge^{3t} ]</td>
<td>A1=A1</td>
</tr>
<tr>
<td></td>
<td>[ ve^{3t} = \frac{1}{3} ge^{3t} (+c) ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ t = 0, v = 0 \Rightarrow 0 = \frac{1}{3} g + C ]</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[ \Rightarrow ve^{3t} = \frac{1}{3} g \left( e^{3t} - 1 \right), 3v = 9.8(1 - e^{-3t}) ]</td>
<td>A1 Correct equation in any equivalent form</td>
</tr>
</tbody>
</table>

*Given form cwo*
<table>
<thead>
<tr>
<th>Question Number</th>
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</table>
| **1(b)**       | \( \frac{dx}{dt} = \frac{9.8}{3} \left(1 - e^{-3t}\right) \Rightarrow x = \frac{9.8}{3} \left(t + \frac{1}{3} e^{-3t}\right) + C \) | M1 Integrate the given \( v \) to find \( x \)  
A1 \( C \) not needed  
M1 Use the initial conditions to evaluate \( C \) or use limits correctly in a definite integral  
A1 \( 5.45, \frac{g}{9} \left(5 + e^{-t}\right) \) or equivalent |
| **(b) alt**    | \( g - 3v = v \frac{dv}{dx} \) | |
|                | \( \int 1dx = \int \frac{v}{g - 3v} dv = \int \left( -\frac{1}{3} + \frac{g}{3(g - 3v)} \right) dv \) | M1 Separate the variables and rearrange the RHS |
|                | \( x = -\frac{v}{3} - \frac{g}{9} \ln(g - 3v) + C \) | A1 +\( C \) not needed |
|                | \( x = 0, v = 0 \Rightarrow C = \frac{g}{9} \ln g \) and  
\( t = 2, v = \frac{g}{3} \left(1 - e^{-6}\right) (= 3.258, \ldots) \) | M1 Use the initial conditions to find \( C \) & find the value of \( v \) when \( t = 2 \)  
A1 |
|                | \( x = \frac{g}{9} \left(1 - e^{-6}\right) - \frac{g}{9} \ln(e^{-6}) = 5.4 \) | A1 |

(5)  
(13)
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<th>Question Number</th>
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<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(a)</td>
<td>Shortest time $50 \div \frac{10}{9} = 45$ (s)</td>
<td>M1,A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Drifts $\frac{2}{3} \times 45$, $= 30$ (m)</td>
<td>M1 \ A1 \  $\frac{2}{3}$ their time</td>
</tr>
<tr>
<td>(c)</td>
<td>Trig or pythag to find velocity of swimmer in direction $AB$</td>
<td>M1 \ A1 \ 0.88 or better</td>
</tr>
</tbody>
</table>

Diagram:

- $A$:
  - $\frac{10}{9}$
  - $\frac{8}{9}$

- $B$:
  - $\frac{2}{3}$

$50 \div \frac{8}{9}$, $= 56.25$ (s) | DM1,A1 Dependent on the previous M 56 or better | (8)
0.6u or uc \alpha

1.2u or 2u \cos \alpha

\[2m \times 1.2u - 3m \times 0.6u = 3ma + 2mb\]

\[(3a + 2b = 0.6u)\]

\[\frac{e(1.2u + 0.6u)}{a - b}\]

\[(a - b = 0.3u)\]

\[a = 0.24u \text{ or } b = -0.06u\]

\[(1.2u - (-0.06u)) \times 2m = 2.52mu\]

or \[(0.24u - (-0.6u)) \times 3m = 2.52mu\]
<table>
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<tr>
<th>Question Number</th>
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</table>
| 4(a)            | PE of ring = $-mgx$  
                 PE of particle = $-3mg(L - \sqrt{x^2 + d^2})$  
                 $\Rightarrow V = 3mg\sqrt{x^2 + d^2} - mgx + \text{constant. AG}$ | B1  
                 Taking the level of the peg as zero PE  
                 M1  
                 A1  
                 A1  
                 (4) Watch out |
| (b)             | $\frac{dV}{dx} = \frac{3mg.2x}{2\sqrt{x^2 + d^2}} - mg$  
                 $\frac{dV}{dx} = 0 \Rightarrow 3x = \sqrt{x^2 + d^2}, 9x^2 = x^2 + d^2, 8x^2 = d^2$  
                 $x = \frac{d}{8} = \left(\frac{\sqrt{2}d}{4}\right)$ | M1  
                 Set $\frac{dV}{dx} = 0$ and solve for $x$  
                 A1 0.354$d$ of better |
| (c)             | $\frac{d^2V}{dx^2} = 3mg\left(\frac{\sqrt{x^2 + d^2}.1 - x}{x^2 + d^2} - \frac{2x}{2\sqrt{x^2 + d^2}}\right) = \frac{2x}{x^2 + d^2}$  
                 $3mg\left(\frac{\sqrt{9x^2}.1 - x}{9x^2}\right) = \frac{3mgd^2}{(x^2 + d^2)^{\frac{3}{2}}} > 0$ | M1  
                 Product or quotient rule  
                 $\frac{d^2V}{dx^2} = \frac{3mg}{\sqrt{x^2 + d^2}} - \frac{3mgx}{2}.2x\left(x^2 + d^2\right)^{\frac{3}{2}}$  
                 A1  
                 OR $= 3mg\left(\frac{3x - x}{9x^2}\right) > 0$ Correct unsimplified.  
                 $\frac{16\sqrt{2}mg}{9d}, 2.5 \frac{mg}{d}, \frac{d^2V}{d\theta^2} = \frac{9mgd}{\sqrt{8}}$ |
|                 | Stable | A1ft  
                 Correct conclusion for their expression (10) |
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
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</tr>
</thead>
<tbody>
<tr>
<td>5(a)</td>
<td>Minimum $V = 12 \cos 50^\circ$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\approx 7.71$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Use of triangle with right angle between $v_c$ and $v_s$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Condone sin/cos confusion.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correct unsimplified trig expression</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.71 only</td>
<td></td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>5(b)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Vector triangle for relative velocities when \( V = 14 \) km h\(^{-1}\) could have relative velocity due S. Could show both possibilities.

Select the vector triangle with the relative velocity due N.

- \( \sin \theta = \frac{\sin 40}{\frac{12}{14}} \)
- Bearing 033°

Use of sine rule or equivalent to find \( \theta \)

Final answer. Accept 33.4°
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>6(a)</td>
<td><img src="image" alt="Diagram" /></td>
<td>M1 Diagram or clear explanation using distances&lt;br&gt;A1 Watch out for fudges.</td>
</tr>
<tr>
<td></td>
<td>$a + Ut = y + (a + x)$&lt;br&gt;$Ut = x + y$ <em>Answer Given</em></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$T = \frac{9ma \times x}{a} = 9mx$&lt;br&gt;$T - 6m\ddot{y} = m\dddot{y}$&lt;br&gt;$9mx - 6m(U - \dot{x}) = -m\ddot{x}$&lt;br&gt;$\dddot{x} + 6\dddot{x} + 9x = 6U$</td>
<td>B1&lt;br&gt;M1 Equation of motion of $P$. Requires all 3 terms in terms of $x$ and/or $y$&lt;br&gt;A2 Expressed in terms of $x$. -1 each error&lt;br&gt;A1 Answer given. Watch out for fudges</td>
</tr>
<tr>
<td>(c)</td>
<td>$t = 0, x = 0, \dot{x} = U$&lt;br&gt;$0 = AU + \frac{2U}{3}, A = -\frac{2}{3}$&lt;br&gt;$\dddot{x} = BUe^{-3t} - 3(A + Bt)Ue^{-3t}$&lt;br&gt;$U = BU - 3AU, B = 3A + 1 = -1$</td>
<td>M1 Use initial conditions to find $A$&lt;br&gt;A1&lt;br&gt;M1 Differentiate&lt;br&gt;A1&lt;br&gt;A1 Or equivalent</td>
</tr>
<tr>
<td>(d)</td>
<td>$\dddot{y} = U - \dot{x} = U - \left(-Ue^{-3t} + 2Ue^{-3t} + 3Ute^{-3t}\right)$&lt;br&gt;$= U \left(1 - e^{-3t} - 3te^{-3t}\right)$</td>
<td>M1&lt;br&gt;A1&lt;br&gt;A1 Or equivalent</td>
</tr>
<tr>
<td>Question Number</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>7(a)</td>
<td>State that impulse acts perpendicular to the wall and demonstrate that ((2\mathbf{i} + \mathbf{j}).(-\mathbf{i} + 2\mathbf{j}) = 0)</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>Impulse momentum equation: (m(\mathbf{v} - \mathbf{u}) = m[(a-b)\mathbf{i} + a\mathbf{j}] = \lambda(-\mathbf{i} + 2\mathbf{j})) [\Rightarrow a = -2(a-b), 3a = 2b] [OR] [Taking scalar products of velocities with ((2\mathbf{i} + \mathbf{j}))]: (\begin{pmatrix} b \ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \ 1 \end{pmatrix} = 2b) and (\begin{pmatrix} a \ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \ 1 \end{pmatrix} = 3a) No change parallel to the wall so (2b = 3a).</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Impact equation: (a = eb) [\Rightarrow e = \frac{2}{3}]</td>
<td>A1</td>
</tr>
</tbody>
</table>

- **B1**: Requires scalar product or gradient diagram.
- **M1**: Requires all terms present and of the correct structure -1 each error
- **A2**: Requires all terms present and of the correct structure
- **A1**: Requires all terms present and of the correct structure
\[ b \cos \theta = a \sqrt{2} \cos (45 - \theta) \]

\[ b \cos \theta = a \cos \theta + a \sin \theta, \quad 2b - 2a = a \]

\[ 2b = 3a \]

Use of \( \tan \theta = \frac{1}{2} \)

\[ a \sqrt{2} \sin (45 - \theta) = eb \sin \theta \]

\[ a \cos \theta = (a + eb) \sin \theta, \quad 2a = a + eb \]

\[ e = \frac{2}{3} \]

(c) Fraction of KE lost

\[ \frac{b^2 - 2a^2}{b^2} = \frac{1 - 2 \times \frac{4}{9}}{1} = \frac{1}{9} \]

M1 Parallel to the wall. Condone trig confusion?

A2 -1 each error. Both angles in same variable?

A1

B1 When seen in (b). Implied by 26.6 or 18.4

M1 Perpendicular to the wall. Condone consistent trig confusion?

A1

\[ e = \sqrt{\frac{10a^2}{b^2} - 4} \]

A1 0.67 or better

\[(12)\]
Mark Scheme (Results)

Summer 2013

GCE Mechanics 5 (6681/01)
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General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{\text{ }}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\star$ The answer is printed on the paper
   - $\square$ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
### General Rules for Marking Mechanics

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
  - Omission or extra g in a resolution is accuracy error not method error.
  - Omission of mass from a resolution is method error.
  - Omission of a length from a moments equation is a method error.
  - Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of \( g = 9.8 \) should be given to 2 or 3 SF.
- Use of \( g = 9.81 \) should be penalised once per (complete) question.
- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised ONCE per complete question.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.
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<tbody>
<tr>
<td>1.</td>
<td>G.S. is $r = Ae^{2t}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$t = 0 : A \cdot j = 0 \Rightarrow A = pi + r k$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$(pi + rk) \times j = i + k$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$-ri + pk = i + k \Rightarrow r = -1; p = 1$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$r = (i - k)e^{2t}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[7]</td>
</tr>
</tbody>
</table>

**Notes for Question 1**

B1 for $r = Ae^{2t}$ oe.  
First M1 for use of initial conditions $t = 0, r.j = 0$.  
(M0 if no explicit r expression to sub into)  
First A1 for $A = pi + rk$ (or $q = 0$)  
Second M1 for attempt at cross-product $r \times j$ when $t = 0$  
(M0 if no explicit r expression to sub into)  
Second A1 for $(-r i + p k)$  
Third M1 for using the second condition to find values for $p$ and $r$.  
Third A1 for a correct answer.  
N.B. All marks available apart from the final A1 if unsound work seen e.g. logs of vectors, provided that logs are removed at the start to give an explicit expression for $r$ which can be evaluated in order to find the value of the constant.
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</thead>
<tbody>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>$I_G = \frac{1}{3}ma^2 + \frac{1}{4}ma^2 = \frac{2}{3}ma^2$ (perp axes)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$I_G = \frac{2}{3}ma^2 + m(a\sqrt{2})^2 = \frac{8}{3}ma^2$ i.e. $k^2 = \frac{8}{3}a^2$ **</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>$-mga\sqrt{2}\sin \theta = \frac{8a}{3}m\alpha^2\dot{\phi}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$\dot{\phi} = -\frac{3g\sqrt{2}}{8a} \theta$, for small $\theta$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$T = 2\pi \sqrt{\frac{8a}{3g\sqrt{2}}} \phantom{\frac{3}{3}}$</td>
<td>M1 A1</td>
</tr>
</tbody>
</table>

**Notes for Question 2**

2(a) First M1 for use of perpendicular axes rule with appropriate no. of terms
First A1 for correct expression for $I_G$ (or from formulae sheet)
Second M1 for use of parallel axes rule to obtain $I_B$
Second A1 for PRINTED ANSWER.

Alternative, using result(s) on formulae sheet:
First M1A1 $I_{AB} = I_{BC} = 4/3ma^2$ (from formulae sheet)
Second M1 for use of perpendicular axes rule with appropriate no. of terms
$I_B = I_{AB} + I_{BC} = 8/3ma^2$

2(b) First M1 for moments equation (dim correct and $mg$ resolved)
First A1 for correct equation
Second M1 dependent for use of $\sin \theta = \theta$ and SHM equation
Third M1 for use of $T = 2\pi/\omega$ (only if proper SHM eqn with -)
Third A1 for answer.
<table>
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<tr>
<th>Question Number</th>
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</thead>
<tbody>
<tr>
<td>3.</td>
<td>( \frac{dm}{dt} = c \Rightarrow m = m_0 + ct )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>((m + \delta m)(v + \delta v) - mv = (mg - mk\nu)\delta t)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( m\delta v + v\delta m = (mg - mk\nu)\delta t )</td>
<td>M1 A2</td>
</tr>
<tr>
<td></td>
<td>( \frac{dv}{dt} + kv + \frac{m}{\delta t} \frac{dm}{dt} = g )</td>
<td>DM1 A1</td>
</tr>
<tr>
<td></td>
<td>( \frac{dv}{dt} + v\left(k + \frac{c}{m_0 + ct}\right) = g )</td>
<td></td>
</tr>
</tbody>
</table>

### Notes for Question 3

First M1 for \( \frac{dm}{dt} = c \) and integrating  
First A1 for \( m = m_0 + ct \)  
Second M1 for impulse-momentum equation (correct number of terms, excluding any \( \delta m\delta v \) or \( \delta m\delta t \) terms)  
Second and third A1: (-1 each error)  
OR:  
Second M1 for \( mg - mk\nu = \frac{d}{dt}(mv) \)  
Second and third A1 for \( mg - mk\nu = v\frac{dm}{dt} + m\frac{dv}{dt} \) (-1 each error)  
Third M1, dependent on second M1, for sub. for \( m \)  
Third A1 for PRINTED ANSWER
<table>
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<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (a)</td>
<td>(3i - 2j + k) + (-2i + j - k) + F_3 = 0</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>F_3 = (-i + j) (N)</td>
<td>A1</td>
</tr>
<tr>
<td>4 (b)</td>
<td>(-2i + 3j) (\times) (3i - 2j + k) + (3i + 2k) (\times) (-2i + j - k) + (\mathbf{r}) (\times) (-i + j) = 0</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(3i + 2j - 5k) + (-2i - j + 3k) + (-zi - zj + x+y)(k) = 0</td>
<td>A2,1,0</td>
</tr>
<tr>
<td></td>
<td>(1-z=0, \quad 1-z=0, \quad -2+x+y=0)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(\mathbf{r} = (2i + k) + t(\mathbf{-i + j})) is a solution</td>
<td>A1</td>
</tr>
<tr>
<td><strong>OR</strong> Use Concurrency Principle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 (c)</td>
<td>F_1 + F_2 + F_4 = (3i + j + k) (\Rightarrow) F_4 = (2i + 2j + k)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(\mathbf{r}_1 \times F_1 + \mathbf{r}_2 \times F_2 + (i-2j+3k) \times (2i+2j+k) = (i+j+k) \times (3i+j+k))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(3i + 2j - 5k) + (-2i - j + 3k) + (-8i + 5j + 6k) = (2j - 2k) + (\mathbf{G})</td>
<td>A2,1,0 ft on a</td>
</tr>
<tr>
<td></td>
<td>(\mathbf{G} = (-7i + 4j + 6k))</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(</td>
<td>\mathbf{G}</td>
</tr>
</tbody>
</table>

**Notes for Question 4**

4(a) M1 for \(\sum \mathbf{F}_i = 0\) 
A1 for \((-i + j)\)

In (b) condone consistent use of \(\mathbf{F} \times \mathbf{r}\). 
First M1 for \(\text{M}(O)\) or \(\text{M}(P_1)\) or \(\text{M}(P_2)\) (must be using correct forces)
First A1 and Second A1 -each product. 
Second M1 for changing equation to \(\mathbf{r} = \mathbf{a} + \lambda \mathbf{F}_3\) 
Third A1 for any correct equation.

4(b) **OR:** Use Concurrency Principle 
First M1 for \(\mathbf{r}_1 + s\mathbf{F}_1 = \mathbf{r}_2 + t\mathbf{F}_2\) 
First A1 for \(s = 1\) or \(t = 1\) 
Second A1 for \(\mathbf{i} + \mathbf{j} + \mathbf{k}\) 
Second M1 for changing equation to \(\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}\) 
Third A1 for any correct equation.

4(c) First M1 \(\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_4 = (3i + j + k)\) 
First A1 for \(\mathbf{F}_4 = (2i + 2j + k)\) 
Second M1 for 
\((\mathbf{r}_1 \times \mathbf{F}_1) + (\mathbf{r}_2 \times \mathbf{F}_2) + (i-2j+3k) \times (\text{their} \mathbf{F}_4) = (i+j+k) \times (3i+j+k)\) 
Second A1ft and Third A1ft for correct equation with products evaluated \((-1ee)\) 
Fourth A1 for \(\mathbf{G} = (-7i + 4j + 6k)\) 
Third M1 for \(|\mathbf{G}| = \sqrt{(-7)^2 + 4^2 + 6^2}\) 
Fifth A1 for \(\sqrt{101}\) oe (2 or more SF)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>Taking strips parallel to $BC$:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$l_x = \frac{2ax}{h}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$\delta m = \frac{2ax}{h} \cdot \frac{M , \delta x}{ah} = \frac{2Mx \delta x}{h^2}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$\delta I = \frac{1}{3} \delta m \left(\frac{ax}{h}\right)^2 + \delta m , x^2$</td>
<td>M1 A1 A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{2M}{3h^4} (a^2 + 3h^2) x^4 , \delta x$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$I = \frac{2M}{3h^4} (a^2 + 3h^2) \int_0^h x^4 , dx$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{2M}{3h^4} (a^2 + 3h^2) \left[ \frac{x^5}{5} \right]_0^h$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{M}{6} (a^2 + 3h^2)$ **</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Notes for Question 5**

- First M1 for attempt to find length of strip in terms of $x$, $h$ and $a$, using similar triangles or equivalent (must be dim correct)
- First A1 for a correct expression
- Second M1 for attempt to find mass of strip in terms of $x$, $h$, $M$ and $\delta x$ (must be dim correct) (this mark is not available if $\rho$ is not found)
- Second A1 for a correct expression.
- Third M1 for use of parallel axes rule on strip
- Third A1 for $\frac{1}{12} \delta ml^2$ term
- Fourth A1 for $\delta mx^2$ term
- Fifth A1 for a correct expression in $a$, $x$, $h$, $M$ and $\delta x$
- Fourth M1 dependent on Third M1, for integrating their $\delta I$ (which must have a $\delta x$)
- Sixth A1 for PRINTED ANSWER
- N.B. Third M1 They may use perpendicular axes on whole triangle, with same working.
- (N.B. if $\rho$ is used but never eliminated, can score max M1 A1 M0 A0 M1 A1 A1 A0 M1 A0)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>( 4mg - T_i = 4mf )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( T_2 - mg = mf )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>((T_1 - T_2)a - 2mga = \frac{1}{2} 2ma^2, \frac{f}{a})</td>
<td>M1 A3</td>
</tr>
<tr>
<td></td>
<td>( 4mg - mg - 2mg = 6mf \Rightarrow f = \frac{g}{6} )</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>\text{ang accln} = \frac{g}{6a}</td>
<td>A1</td>
</tr>
</tbody>
</table>

\( (10) \)

**Notes for Question 6**

- First M1 for equation of motion for \( 4m \)
- First A1 for correct equation using either \( f \) or \( \alpha \)
- Second M1 for equation of motion for \( m \)
- Second A1 for correct equation using either \( f \) or \( \alpha \)
- Third M1 for equation of motion for pulley
- A3 for a correct equation using either \( f \) or \( \alpha \)
- Fourth M1, dependent on previous three M’s, for producing an equation in \( \alpha \) and \( g \) only.
- Sixth A1 for answer
- S.C. M4A5 ‘whole system’ equation:
  \( (4mg - mg - 2mg)a = (4ma^2 + ma^2 + ma^2)\alpha \)
### Question 7

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### (a)

\[ I_A = \frac{1}{2} mr^2 + mr^2 = \frac{3}{2} mr^2 \]

\[ \rightarrow X = mr \ddot{\theta} = 0 \]

\[ \downarrow mg - Y = mr \ddot{\theta} \]

\[ M(A) \]  

\[ mgr = \frac{3}{2} mr^2 \ddot{\theta} \]

\[ Y = \frac{1}{2} mg \]

#### (b)

\[ \frac{1}{2} \frac{3}{2} mr^2 \omega^2 = mgr \]

\[ I, 2r = \frac{3}{2} mr^2 \omega \]

\[ I = \frac{m}{2} \sqrt{3gr} \]

### Notes for Question 7

#### 7(a)

- First M1 for use of parallel axes rule
- First A1 for correct expression
- Second M1 for resolving horizontally (usual rules)
- Second A1 for a correct equation
- Third A1 for 0
- Third M1 for resolving vertically (usual rules)
- Fourth A1 for a correct equation
- Fourth M1 for moments about \( A \) (usual rules)
- Fifth A1 for a correct equation
- Fifth M1, dependent on previous two M marks, for solving for \( Y \).
- A1 for answer

#### 7(b)

- First M1 for energy equation
- First A1 for a correct equation
- Second M1 for angular impulse-momentum equation
- Second A1 for a correct equation
- Third M1, dependent on previous M’s, for solving for \( I \)
- Third A1 for the answer
Mark Scheme (Results)

Summer 2013

GCE Statistics 1 (6683/01)
Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \(\sqrt{ }\) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - [] The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a)</td>
<td>( (S_{bh}) = 64980 - \frac{7150 \times 110}{9} = -22408.9 \ldots )</td>
<td>(-22,400) M1 A1</td>
</tr>
<tr>
<td></td>
<td>( (S_{hh}) = 7171500 - \frac{7150^2}{9} = 1491222.2 \ldots )</td>
<td>1,490,000 A1</td>
</tr>
<tr>
<td></td>
<td>( r = \frac{-22408.9}{\sqrt{1491222 \times 371.56}} = -0.95200068\ldots ) awrt – 0.952</td>
<td>M1A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Yes as ( r ) is close to (-1) (if (-1 &lt; r &lt; -0.5)) or Yes as ( r ) is close to 1 (if 1 &gt; ( r ) &gt; 0.5) [If (-0.5 \leq r \leq 0.5) allow “no since ( r ) is close to 0”] [If (</td>
<td>r</td>
</tr>
<tr>
<td>(c)</td>
<td>( b = \frac{-22408.9}{1491222.2} = -0.015027\ldots ) (allow (-\frac{25}{1625})) awrt – 0.015</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(d)</td>
<td>( a = \frac{110}{9} - &quot;their, b&quot; \times \frac{7150}{9} = (12.2 - \frac{-0.015 \times 794.4}{9}) = 24.1604\ldots ) so ( t = 24.2 - 0.015b )</td>
<td>M1, A1</td>
</tr>
<tr>
<td></td>
<td>0.015 is the drop in temp, (in °C), for every 1(m) increase in height above sea level.</td>
<td>B1</td>
</tr>
<tr>
<td>(f)</td>
<td>Change = (“24.2 – 0.015” ( \times 500)) – (“24.2 – 0.015” ( \times 1000)) or ( 500 \times &quot;0.015&quot; ) ( = \pm 7.5 ) (awrt ± 7.5) (only if a value &lt; 100)</td>
<td>M1 A1ft</td>
</tr>
</tbody>
</table>

Notes

(a) M1 for at least one correct expression (condone transcription error)

1\(^{st}\) A1 for \( S_{bh} \) = awrt 1\,490\,000 or \( S_{bh} \) = awrt –22\,400 (Condone \( S_{ax} \) or \( S_{xy} \) =... or even \( S_{xy} \)...)

2\(^{nd}\) A1 for \( S_{bh} = -22\,400 \) and \( S_{hh} = 1\,490\,000 \) only. [This mark is assessing correct rounding]

(Assume no labels but mis-labelling \( S_{bh} \) as \( S_{hh} \) etc loses the final A1)

(b) M1 for attempt at correct formula. Allow minor transcription errors of 2 or 3 digits.

Must have their \( S_{bh}, S_{hh} \) and given \( S_{t} \) (3sf or better) in the correct places. Condone missing “—”

Award M1A0 for awrt –0.95 with no expression seen. M0 for \( \frac{64980}{\sqrt{7171500 \times 7.864}} \)

(c) B1ft must comment on supporting and state: high/strong/clear (negative or positive) correlation “points lie close to a straight line” is B0 since there is no evidence of this.

(d) 1\(^{st}\) M1 for a correct expression for \( b \). Follow through their \( S_{bh} \) & \( S_{bh} \). Condone missing “—”

1\(^{st}\) A1 for awrt –0.015 or allow exact fraction from rounded values.

2\(^{nd}\) M1 for a correct method for \( a \). Follow through their value of \( b \)

2\(^{nd}\) A1 for a correct equation for \( t \) and \( h \) with \( a = \) awrt 24.2 and \( b = \) awrt –0.015. No fractions

(e) B1 Must mention \( h \) (or height) and \( t \) (or temperature) and their (1 sf) value of \( b \) in a correct comment

(f) M1 for a correct expression seen based on their equation. Allow transcription error of 1 digit. If answer is \( 500 \times \) their \( b \) to 2sf and < 100 (M1A1), If answer is \( 500 \times \) their \( b \) to 2sf and > 100 (M1A0)
<table>
<thead>
<tr>
<th>Question</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2. (a)</td>
<td>25 (allow any x where $24 &lt; x &lt; 26$)</td>
</tr>
</tbody>
</table>
| (b)      | $Q_2$ (or median or $m$) = 51  
           | $IQR = 63 - 46 = 17$ (or $Q_3 - Q_1 = 17$) |
| (c)      | Outliers given by $46 - 1.5 \times 17 = 20.5$ or $63 + 1.5 \times 17 = 88.5$  
           | Outliers limits are **20.5 and 88.5** |
| (d)      | **Medians:** Median for females lower than males  
           | **IQR:** IQR for females smaller than males. Allow “lower/higher” but not “wider”  
           | **Range:** Range of females is less than males  
           | **Skewness:** Male and female marks are both positively skew  
           | Ignore other statements about average, spread, mean, st. Dev, variation, outliers etc |

<table>
<thead>
<tr>
<th>Notes</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Mark (b) and (c) together BUT must see clear statement that median (or $m$ or $Q_2$) = 51 and IQR = 17</td>
<td></td>
</tr>
</tbody>
</table>
| (b)   | M1 for 2 quartiles (at least one correct) and attempt to find the difference. Must see their 63 – their 46  
       | A1 for 17 only. [Answer only of IQR= 17 scores M1A1] |
| (c)   | **A fully correct box-plot (either version) with no supporting work scores 5/5. Otherwise:**  
       | 1st M1 for correct attempt to calc’ at least one limit for outliers, ft their quartiles or IQR  
       | or award for sight of 20.5 or 88.5  
       | 1st A1 for identifying both limits of 20.5 and 88.5  
       | 2nd M1 for a box with an upper and a lower whisker(s) with at least 2 correct values (or correct ft)  
       | (condone no median marked) (condone 2 upper or 2 lower whiskers)  
       | 2nd A1ft for their 20.5 or 26, 46, 51, 63 and 85 or their 88.5 in appropriate places and readable off their scale. Follow through their 20.5 and their 88.5 only, other values need to be correct  
       | If there are 2 upper or 2 lower whiskers A0  
       | B1 for only 2 outliers appropriately marked at 14 and 90 Do not award if whiskers go beyond these values.  
       | **Apply ± 0.5 square accuracy for diagram**  
       | A box plot not on the graph paper can only score the 1st M1A1 |
| (d)   | 1st B1ft for one correct comment comparing median, IQR, range or skewness  
       | 2nd B1ft for a second correct comment comparing median, IQR, range or skewness  
       | Do not allow contradictory statements |
Correct answers only score full marks for each part

If a probability is not in [0, 1] award M0

(a) M1 for denominator of 200 and attempt to add 2 + 8 or 35 + 75 or 30 + 50
   A1 for 0.55 or exact equivalent fraction e.g. $\frac{11}{20}$

(b) M1 for a fully correct expression (e.g. $1 - 0.01$)
   A1 for 0.99 or an exact equivalent fraction

(c) M1 for a correct ratio or a correct formula and at least one correct prob (i.e. a correct num or denom). BUT award M0 if num is $\frac{67}{80}$ or $\frac{80}{200}$ or if num>denom
   A1 for 0.375 or $\frac{3}{8}$ or any exact equivalent.

(d) M1 for a box and the 3 regions $F$, $C$ and $H$ labelled or implied and single set $B$ labelled. There should be no intersections between $F$, $C$ and $H$ unless marked by zeros. They may have 3 circles for $F$, $C$ and $B$ with $H = F \cap C'$ etc. Condone lack of zero in the given diagram.
   $F$ 1st B1 for the 9 and 1 or 0.045 and 0.005 (o.e.) in the correct regions
   $H$ 2nd B1 for the 77 and 33 or 0.385 and 0.165 (o.e.) in the correct regions
   $C$ 3rd B1 for the 64 and 16 or 0.32 and 0.08 (o.e.) in the correct regions
   May have $B$ in 3 bits that are disconnected.

(e) M1 for a numerator made up of their 1 + their 16 + their 33 and a denom of 200 and num < 200
   Also allow sum of their probabilities (provided sum < 1)
   A1 for 0.25 or any exact equivalent
### Question 4 (a)

\[ \sum f t = 4837.5 \quad \text{(allow 4838 or 4840)} \]

Mean \( \frac{\text{"4837.5"}}{200} = 24.1875 \)

awrt 24.2 or \( \frac{387}{16} \)

\[ \sigma = \sqrt{\frac{134281.25}{200} - \left( \frac{4837.5}{200} \right)^2} \]

= 9.293 .......

(accept \( s = 9.32 \))

awrt 9.29

B1 M1 A1

### Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
</table>
| 4. (a)   | \[ \sum f t = 4837.5 \quad \text{(allow 4838 or 4840)} \]  
Mean \( \frac{\text{"4837.5"}}{200} = 24.1875 \)  
awrt 24.2 or \( \frac{387}{16} \)  
\[ \sigma = \sqrt{\frac{134281.25}{200} - \left( \frac{4837.5}{200} \right)^2} \]  
= 9.293 .......

(accept \( s = 9.32 \))  
awrt 9.29 |
| (b) \[ Q_2 = [20.5] + \frac{(100/100.5 - 62)}{88} \times 5 = 22.659... \]  
awrt 22.7 |
| (c) \[ Q_1 = 10.5 + \frac{(50/50.25)}{62} \times 10[= 18.56] \quad (*) \]  
\( (n + 1 \text{ gives } 18.604...) \) |
| (d) \[ Q_3 = 25.5 \quad \text{(Use of } n + 1 \text{ gives } 25.734...) \]  
IQR = 6.9 \( \quad \text{(Use of } n + 1 \text{ gives } 7.1) \) |
| (e) The data is skewed  \( \text{condone “negative skew”} \) |
| (f) Mean decreases and st. dev. remains the same. \( \text{[Must mention mean and st. dev.] (from(a))} \)  
The median and quartiles would decrease. \( \text{[Must refer to median and at least } Q_1. \text{] (b)(c))} \)  
The IQR would remain unchanged \( \text{(from (d))} \) |

### Notes

Correct answers only score full marks in each part except (c)

(a) B1 for 4837.5 or 4838 or 4840 seen.

If no \( \sum f t \) seen (or attempt at \( \sum f t \) seen), B1 can be implied by a correct mean of awrt 24.2

1\text{st} M1 for attempt at their \( \sum f t \) allow 1sf so \( \sum f = \text{awrt } 200 \) and \( \sum f t = \text{awrt } 5000 \).

Or award M1 for a clear attempt at mean where at least 4 correct products of \( \sum f t \) are seen

2\text{nd} M1 for correct expression including square root root seen. Follow through their mean.

Allow a transcription error in 134281.25 but not an incorrect re-calculation.

(b) M1 for a correct fraction \( \times 5 \). Ignore end point but must be +.

Allow use of \( (n + 1) \) giving 100.5…

(c) B1cs0 for a fully correct expression including end point. NB Answer is given.

Allow use of \( (n + 1) \) giving 50.25…but use of 50.5 scores B0

(d) 1\text{st} B1 for 25.5 (or awrt 25.7 using \( n + 1 \))

2\text{nd} B1ft for their \( Q_3 \) – their \( Q_1 \) (or 18.6) (provided > 0) Accept awrt 2sf. Correct ans. only scores 2/2

(e) B1 Must mention that the data is skewed or not symmetrical. Do not award for “outliers”

(f) 1\text{st} B1 for one correct comment from the above. May refer to parts (a), (b), (c) or (d)

2\text{nd} B1 for two correct comments from the above

3\text{rd} B1 for all 3 correct comments from the above
Question | Scheme | Marks
--- | --- | ---
5. **(a)**  
3a + 2b = 0.7  
\[a + 2a + 3a + 4b + 5b + 1.8 = 4.2\]  
5b = 1  
b = 0.2  
a = 0.1  
Attempt to solve | M1 | M1
| M1 | cao | B1
| B1 | cao | (5)

**Note:** Probabilities outside [0, 1] should be awarded M0

**ALT** for stating b and a.

1st M1 for an attempt at a linear equation in a and b based on sum of probs. = 1  
2nd M1 for an attempt at a second linear equation in a and b based on E(X) = 4.2 Allow one slip.  
3rd M1 for an attempt to solve their 2 linear equations based on sum of probs and E(X). Must reduce to a linear equation in one variable. 1st B1 for b and 2nd B1 for a. Answers only score B1B1 only  
The 3rd M1 may be implied if M2 is scored and both correct answers are given.

**(b)**  
\[E(X^2) = 1 \times 0.1 + 2^2 \times 0.1 + 3^2 \times 0.1 + 4^2 \times 0.2 + 5^2 \times 0.2 + 6^2 \times 0.3 = 20.4\] \((*)\)  
[Var(X) = ] 20.4 – 4.2² \([= 2.76]\)  
Var(5 – 3X) = 9 Var(X)  
\[= 24.84\] or \[24.8\] (allow \[\frac{641}{25}\]) | cao | B1

**Note:** Condone use of \(X(x)\) instead of \(Y(y)\)  
Ignore incorrect or no label if table fully correct

**(c)**  
[Var(X) = ] 20.4 – 4.2² \([= 2.76]\)  
Var(5 – 3X) = 9 Var(X)  
\[= 24.84\] or \[24.8\] (allow \[\frac{641}{25}\]) | cao | A1

**Note:** Ignore incorrect or no label if table fully correct

**Note:** Probabilities outside [0, 1] should be awarded M0

**ALT** for stating b and a.

1st M1 for an attempt at a linear equation in a and b based on sum of probs. = 1  
2nd M1 for an attempt at a second linear equation in a and b based on E(X) = 4.2 Allow one slip.  
3rd M1 for an attempt to solve their 2 linear equations based on sum of probs and E(X). Must reduce to a linear equation in one variable. 1st B1 for b and 2nd B1 for a. Answers only score B1B1 only  
The 3rd M1 may be implied if M2 is scored and both correct answers are given.

**(e)**  
P\((Y = 1) = 0.1\)  
e.g. P\((Y = 2) = F(2) – F(1) = 0.1\)  
\[\begin{array}{|c|c|c|c|c|c|} 
\hline
\text{y} & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{P(Y = y)} & 0.1 & 0.1 & 0.4 & 0.2 & 0.2 \\
\hline
\end{array}\]  
Condone use of \(X(x)\) instead of \(Y(y)\)  
Ignore incorrect or no label if table fully correct | cao | M1, A1

**Note:** Ignore incorrect or no label if table fully correct

**Notes**

1st M1 for a correct expression for Var(\(\lambda\)). Must see \(-4.2^2\)  
2nd M1 for \((-3)^2\) Var(\(\lambda\)) or better, no need for a value. Accept \(-3^2\) if it clearly is used as +9 later.

**Note:** Probabilities outside [0, 1] should be awarded M0

**ALT** for stating b and a.

1st M1 for showing that sum of probs. = 1  
2nd M1 for showing that E(X) = 4.2  
3rd M1 for an overall comment “(therefore) a = …and b = …” No comment loses this mark.

**Note:** Probabilities outside [0, 1] should be awarded M0

**ALT** for stating b and a.

1st M1 for an attempt at a linear equation in a and b based on sum of probs. = 1  
2nd M1 for an attempt at a second linear equation in a and b based on E(X) = 4.2 Allow one slip.  
3rd M1 for an attempt to solve their 2 linear equations based on sum of probs and E(X). Must reduce to a linear equation in one variable. 1st B1 for b and 2nd B1 for a. Answers only score B1B1 only  
The 3rd M1 may be implied if M2 is scored and both correct answers are given.

**Note:** Probabilities outside [0, 1] should be awarded M0

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| 6. (a)  | [Let \( X \) be the amount of beans in a tin. \( P(X < 200) = 0.1 \)]<br>\[
\frac{200 - \mu}{7.8} = -1.2816<br>\mu = 209.996.\ldots\text{awrt 210}
\] | M1 B1 |
|         | [ calc gives 1.28155156\ldots] | A1 |
| (b)     | \( P(X > 225) = P\left( \frac{Z > \frac{225 - "210"}{7.8}}{7.8} \right) \)
|         | \[
= P\left( Z > 1.92 \right) \text{ or } 1 - P(Z < 1.92)\text{ (allow 1.93)}
\]
|         | \[
= 1 - 0.9726 = 0.0274 \text{ (or better)}\text{ [calc gives 0.0272037\ldots]}\text{ = awrt 2.7\% allow 0.027}
\] | A1 |
| (c)     | [Let \( Y \) be the new amount of beans in a tin]<br>\[
\frac{210 - 205}{\sigma} = 2.3263 \text{ or } \frac{200 - 205}{\sigma} = -2.3263 \text{ [calc gives 2.3263478\ldots]}\]
|         | \[
\sigma = \frac{5}{2.3263} = 2.15 \text{ (2.14933\ldots)}
\] | M1 B1 |

### Notes

Condone poor handling of notation if answers are correct but A marks must have correct working.

(a) M1 for an attempt to standardise (allow \( \pm \)) with 200 and 7.8 and set \( = \pm \) any \( z \) value (\( |z| > 1 \))

B1 for \( z = \pm 1.2816 \) (or better used as a \( z \))[May be implied by 209.996(102\ldots) or better seen]

A1 for awrt 210 (can be scored for using 1.28 but then they get M1B0A1)

If answer is awrt 210 and 209.996… or better seen then award M1B1A1

\( z = 1.28 \) gives 209.984 and \( z = 1.282 \) gives 209.996 and both score M1B0A1

If answer is awrt 210 or awrt 209.996 then award M1B0A1 (unless of course \( z = 1.2816 \) is seen)

(b) M1 for attempting to standardise with 225, their mean and 7.8. Allow \( \pm \)

1\(^{st} \) A1 for \( Z > \) awrt 1.92/3. Allow a diagram but must have 1.92/3 and correct area indicated.

Must have the \( Z \) so \( P(X > 225) \) with or without a diagram is not sufficient.

Award for \( 1 - 0.9726 \) or \( 1 - 0.9732 \)

2\(^{nd} \) A1 for 2.7 \% or better (calculator gives 2.72\ldots) Allow awrt 0.027. Correct ans scores 3/3

(c) 1\(^{st} \) M1 for an attempt to standardise with 200 or 210, 205 and \( \sigma \) and set \( = \pm \) any \( z \) value (\( |z| > 2 \))

B1 for \( z = 2.3263 \) (or better) and compatible signs.

If B0 in (a) for using a value in \([1.28, 1.29)\) but not using 1.2816: allow awrt 2.33 here

2\(^{nd} \) dM1 Dependent on the first M1 for correctly rearranging to make \( \sigma = \ldots \text{ May be implied e.g. } \frac{\sigma}{\sqrt{2}} = 2.32 \rightarrow \sigma = 2.16 \text{ (M1A0) BUT must have } \sigma > 0 \)

A1 for awrt 2.15. Must follow from correct working but a range of possible \( z \) values will do.

NB 2.320 < \( z \) < 2.331 will give an answer of awrt 2.15
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper
   - $\square$ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>(5,5,5) or (1,5,5) or (2,5,5)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(5,5,5) (5,5,1) (5,1,5) (1,5,5) (5,5,2) (5,2,5) (2,5,5) or (5,5,5) and (5,5,1) (× 3) and (5,5,2) (× 3)</td>
<td></td>
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<tr>
<td></td>
<td></td>
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<tr>
<td>1(b)</td>
<td>(5,5,5) ( \left( \frac{3}{10} \right)^3 \cdot \frac{27}{1000} = 0.027 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(5,5,1) ( 3 \times \frac{1}{2} \times \left( \frac{3}{10} \right)^2 = \frac{135}{1000} \text{ or } \frac{27}{200} = 0.135 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(5,5,2) ( 3 \times \frac{1}{5} \times \left( \frac{3}{10} \right)^2 = \frac{54}{1000} = \frac{27}{500} = 0.054 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P(M = 5) = \left( \frac{3}{10} \right)^3 + 3 \times \frac{1}{2} \times \left( \frac{3}{10} \right)^2 + 3 \times \frac{1}{5} \times \left( \frac{3}{10} \right)^2 = \frac{27}{125} = 0.216 ) oe</td>
<td>A1A1</td>
</tr>
<tr>
<td>1(c)</td>
<td>( P(M = 1) = (0.5)^3 + 3(0.5)^2(0.2) + 3(0.5)^2(0.3) ) ( = 0.5 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( P(M = 2) = \left( \frac{1}{3} \right)^3 + 3 \times \left( \frac{1}{3} \right)^2 \times \frac{1}{2} + 3 \times \left( \frac{1}{5} \right)^2 \times \frac{3}{10} + 6 \times \frac{1}{2} \times \frac{1}{5} \times \frac{3}{10} ) ( = 0.284 ) or ( \frac{71}{250} ) oe</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( m )</td>
<td>( P(M = m) )</td>
</tr>
</tbody>
</table>
| Notes           | **1(a)** for two of the given triples, any order  
1st B1 for all 7 cases. no incorrect extras  
**1(b)** \( \left( \frac{3}{10} \right)^3 \text{ or } 0.027 \) oe. This can be a single term in a summation  
B1 \( \left( \frac{3}{10} \right)^3 \text{ or } 0.027 \) oe  
M1 either "3" \( \times \frac{1}{2} \times \left( \frac{3}{10} \right)^2 \) or "3" \( \times \frac{1}{5} \times \left( \frac{3}{10} \right)^2 \) oe. May omit the 3 \( \times \) or have another positive integer in place of the 3. These may be seen as a single term in a summation  
A1 \( \left( \frac{3}{10} \right)^3 + 3 \times \frac{1}{2} \times \left( \frac{3}{10} \right)^2 + 3 \times \frac{1}{5} \times \left( \frac{3}{10} \right)^2 \) oe  
A1 \( 0.216 \) oe  
**1(c)**  
1st M1 correct calculation for \( P(M = 1) \) or \( P(M = 2) \), working must be shown and **not** implied by a correct answer.  
1st A1 either \( P(M = 1) \) or \( P(M = 2) \) correct  
2nd M1 correct calculation for both \( P(M = 1) \) and \( P(M = 2) \), or their probabilities adding up to 1, but do not allow probabilities of 0.5, 0.2 and 0.3  
2nd A1 both \( P(M = 1) \) and \( P(M = 2) \) correct  
3rd A1 depend on both M marks awarded. All three values written down with their correct probabilities. They must be in part (c) but they do not need to be in a table.  
NB A fully correct table with no working will get M0 A0 M1 A1 A0.
<p>| | | | |</p>
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>2(a)</td>
<td>$P(X = 1) = 0.25e^{-0.25} = 0.1947$ awrt 0.195</td>
<td>M1A1</td>
<td>(2)</td>
</tr>
</tbody>
</table>
| 2(b) | $X \sim \text{Po}(1.5)$  
$P(X > 2) = 1 - P(X \leq 2)$  
$= 1 - 0.8088$  
$= 0.1912$ awrt 0.191 | B1  
M1  
A1 | (3) |
| 2(c) | $[\lambda = 300 \times 0.25 = 75]$  
$X \sim \text{N}(75,75)$  
$P(X < 90) = P(X \leq \frac{89.5 - 75}{\sqrt{75}})$  
$= P(Z \leq 1.6743..)$  
$= awrt 0.953$ or 0.952 | B1  
B1  
M1M1  
A1 | (5) |

### Notes

2(a) M1 0.25e^{-0.25} o.e  
2(b) B1 stating or using Po(1.5)  
M1 stating or using $1 - P(X \leq 2)$  
2(c)  
$1^{st}$ B1 for normal approximation and correct mean  
$2^{nd}$ B1 Var $(X) = 75$ or sd $= \sqrt{75}$ or awrt 8.66 (may be given if correct in standardisation formula)  
$1^{st}$ M1 using either 89.5 or 88.5  
$2^{nd}$ M1 Standardising using their mean and their sd, using [89.5, 88.5 or 89] and for finding correct area  
NB use of Poisson gives an answer of 0.9498 and gains no marks
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| **3(a)**        | $X \sim \text{Po}(7)$  
$P(X > 10) = 1 - P(X \leq 10)$  
$= 1 - 0.9015$  
$= 0.0985$ awrt 0.0985 | B1 M1 A1 |
| **3(b)**        | $P(X > d) < 0.05$ Or $P(X \geq d) < 0.05$  
$P(X \leq d) > 0.95$  
$P(X \leq 11) = 0.9467$  
$P(X \leq 12) = 0.9730$  
Least number of games $= 12$  
Least number of games $13$ | M1 |
| **3(c)**        | $H_0: \lambda = 1, \mu = 28$  
$H_1: \lambda > 1, \mu > 28$  
$Y \sim \text{Po}(28)$ approximated by $\text{N}(28,28)$  
$P(Y \geq 36) = P(Z \geq \frac{36.5 - 28}{\sqrt{28}})$  
$= P(Z \geq 1.42)$  
$= 0.0778$ or $1.42 < 1.6449$ CR $X \geq 37.2$  
$0.0778 > 0.05$ so do not reject $H_0$ not significant. Not in CR  
There is no evidence that the average rate of sales per day has increased. | B1 B1 M1M1 A1 M1 M1 A1cs |

**Notes**

3(a) B1 stating or using Po(7)  
M1 stating or using $1 - P(X \leq 10)$

3(b) M1 using or writing $P(X > d) < 0.05$ or $P(X < d) > 0.95$ (condone $\geq$ instead of $>$ and $\leq$ instead of $<$) May be implied by correct answer. Different letters may be used.  
$1^{st}$ A1 $P(X \leq 12) / P(X < 13) = awrt 0.973$ or $P(X \leq 11) / P(X < 12) = awrt 0.947$  
May be implied by a correct answer  
$2^{nd}$ A1 12 or 13  
**NB** An answer of 12/13 on its own with no working gains M1A1A1

3(c) $1^{st}$ B1 both hypotheses correct using $\lambda$ or $\mu$, and 1 or 28  
$2^{nd}$ B1 for writing or using a normal approximation with correct mean and Var (may be given if sd correct in standardisation formula)  
$1^{st}$ M1 for use of a continuity correction 35.5 or 36.5 or $x \pm 0.5$  
$2^{nd}$ M1 Standardising using their mean and their sd. If they have not written down a mean and sd then these need to be correct here to award the mark. They must use $[35.5, 36.5, 36, x \pm 0.5]$ For CR must have $= awrt 1.64$ or $1.65$  
$1^{st}$ A1 awrt 0.0778 or 0.9222 or the statement $1.42 < awrt 1.65/1.64$ or CR $X \geq 37.2 / X > 37.2$  
$3^{rd}$ M1 a correct conclusion for their probability. May be implied by a correct contextual conclusion. **NB** Non contextual contradicting statements gets M0  
$2^{nd}$ A1 a correct contextual conclusion for their hypotheses and a fully correct solution with no errors seen. Need the words “rate/average number”, “sales” and “increased” oe  
**NB** If found $P(X = 36)$ they can get B1B10M0A0M0A0

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</tr>
<tr>
<td><strong>4(a)</strong></td>
<td>( \text{E}(X) = \frac{5b}{2} )</td>
<td>B1</td>
</tr>
</tbody>
</table>
| **4(b)** | \[
\begin{align*}
\text{Var}(X) & = \text{E}(X^2) - (\text{E}(X))^2 \\
& = \int_{a}^{b} \frac{4b x^2}{3b} dx - \left(\frac{5b}{2}\right)^2 \\
& = \left[ \frac{x^3}{3b} \right]_{a}^{b} - \frac{25b^2}{4} \\
& = \frac{63b^3}{9b} - \frac{25b^2}{4} \\
& = \frac{3b^2}{4} \\
& = \frac{3b^2}{4} \\
\end{align*}
\] | M1 | M1d | (3) |
| **4(c)** | \[
\begin{align*}
\text{Var}(3 - 2X) & = 4\text{Var}(X) \\
& = 3b^2 \\
\end{align*}
\] | M1 | A1 | (2) |
| **4(d)** | \[
F(x) = \begin{cases} 
0 & x < 1 \\
\frac{x-1}{3} & 1 \leq x \leq 4 \\
1 & x > 4 
\end{cases}
\] | B1B1 | (2) |
| **4(e)** | \[
\frac{x-1}{3} = 0.5 \text{ so } x = 2.5
\] | B1 | (1) |
| **Alt 4(b)** | \[
\text{Var}(X) = \int_{a}^{b} \left(\frac{x-a}{b-a}\right)^2 dx \\
& = \int_{a}^{b} \frac{4b}{12b} x^2 - 10bx + 25b^2 dx \\
& = \left[ \frac{x^3}{3} - 10bx^2 + 25b^2 x \right]_{a}^{b} \\
& = \frac{9b^3}{12b} \\
& = \frac{3b^2}{4} \\
\] | M1 | M1 | A1cso(3) |

**Notes**

4(b) NB remember the answer is given (AG) so they must show their working

1st M1 for using \( \int \frac{x^2}{3b} dx \) - (their (a))\(^2\) limits not needed and condone missing dx. NB need not use the letter x but if they use b instead do not award if they cancel down to \( \frac{b}{3} \)

Check they have subtracted (their(a))\(^2\)

2nd M1 dependent on previous M being awarded. For some correct integration \( x^n \rightarrow x^{n+1} \) and correct limits substituted at some point. condone 4b\(^3\) instead of (4b)\(^3\)

A1 for correct solution with no incorrect working seen.

4(c) M1 for writing or using 4Var(X)

4(d) 1st B1 top and bottom line. Allow use of \( \leq \) instead of \(<\) and \( \geq \) instead of \( >\)

2nd B1 middle row. Allow use of \( <\) instead of \( \leq \)

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<tr>
<td><strong>5(a)</strong></td>
<td>( F(1) = \frac{4}{10} + a + b = 0 )</td>
<td>M1</td>
</tr>
</tbody>
</table>
\( a = \frac{-3}{5} \) or \( b = \frac{1}{5} \)

F(2) = 1, 2 + 2a + b = 1
Solving gives \( a = -\frac{3}{5}, b = \frac{1}{5} \)

**Alt**

F(2) - F(1) = 1, 2 + 2a + b - \( \frac{4}{10} \) - a - b = 1
\( a = -\frac{3}{5} \)

F(2) =1 or F(1) = 0

\[ 2 - \frac{6}{5} + b = 1 \text{ or } \frac{4}{10} - \frac{3}{5} + b = 0 \]
\[ b = \frac{1}{5} \]

5(b) Differentiating cdf gives \( f(x) = \frac{3}{10}x^2 + \frac{6}{10}x + a, \quad 1 \leq x \leq 2 \)

\[ = \frac{3}{10}(x^2 + 2x - 2) \]

5(c) \( E(X) = \int_1^2 \frac{3}{10}(x^3 + 2x^2 - 2x)dx \)

\[ = \frac{3}{10} \left[ \frac{1}{4}x^4 + \frac{2}{3}x^3 - x^2 \right]_1 \]
\[ = \frac{13}{8} \]

5(d) F(1.425) = 0.24355, F(1.435) = 0.25227

0.25 lies between F(1.425) and F(1.435) hence result.

<table>
<thead>
<tr>
<th>Notes</th>
<th>Total 12 marks</th>
</tr>
</thead>
</table>
| 5(a)  | 1st M1 using F(1) = 0. Clear attempt to form a linear equation for \( a \) and \( b \)
|       | 1st A1 either \( a = -0.6 \) or \( b = 0.2 \) Previous M must be awarded
|       | 2nd M1 using F(2) = 1. Clear attempt to form a second linear equation for \( a \) and \( b \)
|       | 2nd A1 if 1st A1 awarded then both \( a \) and \( b \) must be correct otherwise award if
|       | either \( a = -0.6 \) or \( b = 0.2 \)
|       | alt 1st M1 F(2) - F(1) = 1. Leading to a value for \( a \): 1st A1 \( a = -0.6 \)
|       | 2nd M1 using F(2) = 1 or F(1) = 0. Leading to a value for \( b \): 2nd A1 \( b = 0.2 \)
|       | NB correct values for \( a \) and \( b \) with no working scores no marks.
| 5(b)  | B1 They must differentiate and then factorise. cso
| 5(c)  | 1st M1 for clear attempt to use \( xf(x) \) with an intention of integrating (Integral sign
|       | enough) Ignore limits. Must substitute in \( f(x) \) or “their \( f(x) \)”. 
|       | 2nd M1 dependent on previous M being awarded for some correct integration… at least
|       | one correct term with the correct coefficient.
|       | 1st A1 for fully correct (possibly unsimplified) integration. Ignore limits
|       | 2nd A1 Accept 1.63 and 1.625 or some other exact equivalent
| 5(d)  | M1 expression showing substitution of 1.425 or 1.435 into \( F(x) \) [or into \( F(x) - 0.25 \]
|       | [or putting their \( F(x) = 0.25 \) and attempting to solve leading to \( x = \ldots \)] May be implied by
|       | either pair of the correct answers as given below for the 1st A1
|       | 1st A1 awrt 0.244 and awrt 0.252 [or awrt -0.00645 and awrt 0.00227] [or \( x = \) awrt 1.432]
|       | 2nd A1 0.25 lies between F(1.425) and F(1.435) [or change in sign therefore root
|       | between] [or “1.432” lies between 1.425 and 1.435 therefore root
|       | between]. Statement must be true for their method

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
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</thead>
<tbody>
<tr>
<td>6(a)</td>
<td>( X \sim B(20,0.25) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( P(X \geq 10) = 1 - 0.9861 = 0.0139 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( P(X \leq 1) = 0.0243 )</td>
<td>A1</td>
</tr>
<tr>
<td>(0 ≤)X ≤ 1 ∪ 10 ≤ X(≤ 20)</td>
<td>A1A1</td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>------</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6(b)</th>
<th>A1A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₀: p = 0.25</td>
<td>B1</td>
</tr>
<tr>
<td>H₁: p &lt; 0.25</td>
<td>M1A1</td>
</tr>
<tr>
<td>X~B(20,0.25)</td>
<td>M1d</td>
</tr>
<tr>
<td>P(X ≤ 3) = 0.2252 or CR X ≤ 1</td>
<td>A1cso</td>
</tr>
<tr>
<td>Insufficient evidence to reject H₀, Accept H₀, Not significant. 3 does not lie in the Critical region. No evidence that the changes to the process have reduced the percentage of defective articles (oe)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
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<tr>
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<td></td>
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<td></td>
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</tbody>
</table>

<p>| 6(b)  | B1 both hypotheses with p |
|       | 1ˢᵗ M1 using B(20, 0.25) and finding P(X ≤ 3) or P(X ≥ 4) may be implied by a correct CR |
|       | 1ˢᵗ A1 0.2252 (allow 0.7748) if not using CR or CR X ≤ 1 or X &lt; 2 |
|       | 2ⁿᵈ M1 dependent on previous M being awarded. A correct statement (do not allow if there are contradicting non contextual statements) |
|       | A1cso Conclusion must contain the words changes/new process oe, reduced oe number/percentage oe, and defective articles/defectives. There must be no incorrect working seen. |</p>
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7(a)</strong></td>
<td>Distribution ( X \sim B(n, 0.1) )</td>
<td>B1</td>
</tr>
</tbody>
</table>
| **7(b)** | \( Y \sim B(10, 0.1) \)  
\( P(Y \geq 4) = 1 - P(Y \leq 3) \)  
\( = 1 - 0.9872 \)  
\( = 0.0128 \) | B1 | (1) |
| **7(c)** | \( 0.9^n < 0.05 \) \( n \geq 28.4 \)  
\( n = 29 \)  
alternative  
\( B(28, 0.1): P(0) = 0.0523 \)  
\( B(29, 0.1): P(0) = 0.0471 \)  
\( n = 29 \) | M1 | (3) |
| **7(d)** | \( C \sim Po(5) \)  
\( P(C > 10) = 1 - P(C \leq 10) \)  
\( = 1 - 0.9863 \)  
\( = 0.0137 \) | B1 | (3) |

**Notes**

- **7(a)** B1 for “binomial” or B(…)
- **7(b)** B1 writing or using B(10,0.1)  
  M1 writing or using \( 1 - P(Y \leq 3) \)  
  A1 awrt 0.0128
- **7(c)** M1 \((0.9)^n < 0.05 \), oe, or \((0.9)^n > 0.05\), oe, or seeing 0.0523 or seeing 0.0471  
  1\(^{st}\) A1 \([P(0)] = 0.0471\) or getting awrt 28.4 May be implied by correct answer.  
  2\(^{nd}\) A1 cao \( n = 29 \) should not come from incorrect working.  
  NB An answer of 29 on its own with no working gains M1A1A1
- **7(d)** B1 writing or using Po(5)  
  M1 writing or using \( 1 - P(C \leq 10) \)  
  A1 awrt 0.0137

Total marks 10
Mark Scheme (Results)

Summer 2013

GCE Statistics 3 (6691/01)
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Summer 2013
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes:
   - bod – benefit of doubt
   - ft – follow through
   - the symbol \(\sqrt{\text{ }}\) will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - \(\star\) The answer is printed on the paper
   - \(\square\) The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate’s response may differ from the final mark scheme.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>Cholesterol Level</td>
<td>High</td>
</tr>
<tr>
<td>High</td>
<td>7.6</td>
<td>12.4</td>
</tr>
<tr>
<td>Low</td>
<td>30.4</td>
<td>49.6</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>62</td>
</tr>
</tbody>
</table>

| H₀ | Cholesterol level is independent of intake of saturated fats (no association) |
| H₁ | Cholesterol level is not independent of intake of saturated fats (association) |

<table>
<thead>
<tr>
<th>O</th>
<th>E</th>
<th>(O - E)²/E</th>
<th>O²/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>7.6</td>
<td>2.547… or ( \frac{242}{95} ) or 18.947… or ( \frac{360}{19} )</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>12.4</td>
<td>1.56129… or ( \frac{242}{75} ) or 5.161… or ( \frac{160}{17} )</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>30.4</td>
<td>0.6368… or ( \frac{121}{90} ) or 22.236… or ( \frac{845}{38} )</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>49.6</td>
<td>0.3903… or ( \frac{121}{176} ) or 58.790… or ( \frac{3645}{62} )</td>
<td></td>
</tr>
</tbody>
</table>

\[ \sum \frac{(O - E)^2}{E} = 5.1358234… \text{ or } \frac{1.2^2}{7.6} + \frac{8^2}{12.4} + \frac{26^2}{30.4} + \frac{54^2}{49.6} - 100 = 5.14 \text{ (awrt 5.14)} \]

\[ \chi^2(0.05) = 3.841 \]

5.14 > 3.841 so sufficient evidence to reject H₀ [Condone “accept H₁”]

Association between cholesterol level and saturated fat intake

**Notes**

**Minimum working** use part marks: \( E_i \) (2), Hyp (1), 5.14 (3), 3.841 (2), Conclusion (2)

1st M1 for some use of \( \frac{\text{Row Total} \times \text{Col. Total}}{\text{Grand Total}} \). May be implied by correct \( E_i \)

1st A1 for all expected frequencies correct. Allow M1A0 for \( E_i \) rounded to integers

1st B1 for both hypotheses. Must mention “cholesterol” and “fats” at least once Use of “relationship” or “correlation” or “connection” is B0

2nd dM1 for at least 2 correct terms (as in 3rd or 4th column) or correct expressions with their \( E_i \) Dependent on 1st M1 Accept 2sf accuracy for the M mark

2nd A1 for all correct terms. May be implied by a correct ans.(2 dp or better) Allow truncation eg 2.54… 3rd A1 for awrt 5.14

2nd B1 for correct degrees of freedom (may be implied by a cv of 3.841)

3rd M1 for a correct statement linking their test statistic and their cv(cv could be 2.705 or > 3.5) Contradictory statements score M0 e.g. “significant, do not reject H₀”

4th A1 for a correct comment in context - must mention “cholesterol” and “fats” condone “relationship” or “connection” here but not “correlation”. e.g. “There is evidence of a relationship between cholesterol level and fat intake” No follow through. If e.g hypotheses are the wrong way round A0 here.
### Question Number 2(a)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uni A B C D E F G</td>
<td>M1A1A1</td>
</tr>
<tr>
<td>Staff-Stu</td>
<td></td>
</tr>
<tr>
<td>2 4 3 5 7 1 6</td>
<td></td>
</tr>
<tr>
<td>Satisfaction</td>
<td></td>
</tr>
<tr>
<td>3 2 6 4 5 1 7</td>
<td></td>
</tr>
<tr>
<td>[ d ]</td>
<td>-1 2 -3 1 2 0 -1</td>
</tr>
<tr>
<td>[ d^2 ]</td>
<td>1 4 9 1 4 0 1 20</td>
</tr>
</tbody>
</table>

\[ r_s = 1 - \frac{6 \times 20}{7(49 - 1)} = 0.642857... \quad \text{(accept } \frac{9}{14} \text{)} \quad \text{(awrt } 0.643) \]

\[ dM1A1 \]

\((b)\)

H\(_0\): \( \rho = 0 \)

H\(_1\): \( \rho \neq 0 \) \((\rho > 0)\)

Critical value is \( \pm 0.7857 (\pm 0.7143 \text{ for a one tailed test}) \)

0.643<cv so insufficient evidence to reject H\(_0\)

There is insufficient evidence to suggest a (positive) correlation between staff-student ratio and satisfaction.

### Notes

(a) **1\(^{st}\) M1** for an attempt to rank the staff-students ratio or satisfaction (at least 4 correct)

**1\(^{st}\) A1** for correct rankings for both (one or both may be reversed)

**2\(^{nd}\) A1** for \( \sum d^2 = 20 \) or correct \( d^2 \) row (NB \( \sum d^2 = 92 \) for one set of reversed ranks)

**2\(^{nd}\) dM1** for use of the correct formula, follow through their \( \sum d^2 \) (Dependent on 1\(^{st}\) M1)

If answer is not correct, a correct expression is required.

**3\(^{rd}\) A1** If \( \sum d^2 = 20 \) for awrt 0.643 or if \( \sum d^2 = 92 \) for awrt 0.643 (accept \( \pm \frac{\alpha}{14} \))

(b) **1\(^{st}\) B1** for both hypotheses in terms of \( \rho \), one tail H\(_1\) must be compatible with their ranking

Hypotheses just in words e.g. “no correlation” score B0

**2\(^{nd}\) B1** for cv of 0.7857 or 0.7143 for one-tailed test (accept \( \pm \))

Their cv must be compatible with their H\(_1\) which may be in words

If hypotheses are the wrong way around this must be B0 but 3\(^{rd}\) B1 is possible.

**3\(^{rd}\) B1ft** for a correct contextualised comment. Must mention “ratio” or “no. of students per member of staff” and “satisfaction”

Follow through their \( r_s \) and their cv (provided it is \( |\text{cv}| < 1 \))

Don’t insist on the word “positive” for a one-tailed test

Use of “association” is B0

Independent of 1\(^{st}\) B1 so if \( |r_s| > |\text{cv}| \) must say there is sufficient evidence of …….(o.e.)

and if \( |r_s| < |\text{cv}| \) must say insufficient evidence of … (o.e.) regardless of their hypotheses

Contradictory statements score B0

(This mark is just testing interpretation of comparison of their \( r_s \) and their cv)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 3(a)i | **Quota Sampling:**  
Advantages: Fieldwork can be done quickly, or administering the test is easy, or costs are kept to a minimum (cheap), or gives estimates for each course.  
e.g. OK for large populations or sampling frame not required (o.e.)  
Disadvantages: **Non-random** process or not possible to estimate the sampling errors, or non response not recorded, or interviewer can introduce bias in sample choice. (o.e.) | B1 |
| 3(a)ii | **Stratified Sampling:**  
Advantages: Can give accurate estimates as it is a random process, or gives estimates for each course or representative of [BUT not “proportional” to] the whole population. (o.e.)  
e.g.  
Disadvantages: Sampling frame required, or strata may not be clear as some students overlap courses or not suitable for large populations. (o.e.) | B1 |
| 3(b) | Total enrolments=1000  
Leisure and Sport=$\frac{420}{1000} \times 100 = 42$  
Information Technology=$\frac{337}{1000} \times 100 = 33.7=34$  
Health and Social Care=$\frac{200}{1000} \times 100 = 20$  
Media Studies=$\frac{43}{1000} \times 100 = 4.3=4$ | (2) |
| 3(c) | The college’s information system would be used to identify each student and which course they are enrolled on.  
i.e. idea of sampling frame or list for each course.  
Use of random numbers to select required number of students from each course | (3) |

**Notes**

- Do not penalise for lack of context in part (a)

- **(a)**  
  1st B1 for an advantage and a disadvantage for quota sampling (must be 1st or labelled (i))  
  2nd B1 for an advantage and a disadvantage for stratified sampling (2nd or labelled (ii))  
  Do not allow opposite pairs e.g. “quicker/easier” for quota sampling and “takes a long time/more difficult” for stratified or quota “easy to use” but strat. “hard for large populations”  
  Do not allow same reason for both e.g. “gives estimates for each course”

- **(b)**  
  M1 for one correct calculation, ft their “1000”  
  A1 for 42, 34, 20 and 4 only

- **(c)**  
  1st B1 for some mention of a suitable sampling frame. Need not give the specific term but a suitable source of list is required for all students in each course.  
  2nd B1 for mentioning use of random numbers or some random selection process for each course. If they are describing systematic sampling score B0 here
### Question 4

#### (a)

\[ \bar{x} = \frac{8 \times 1.5 + 12 \times 4 + 13 \times 5.5 + 9 \times 7 + 8 \times 10}{50} = \frac{274.5}{50} = 5.49 \]

\[ s^2 = \frac{8 \times 1.5^2 + 12 \times 4^2 + 13 \times 5.5^2 + 9 \times 7^2 + 8 \times 10^2}{49} - \frac{50}{49} \times 5.49^2 = 6.88 \]

(*B1cso*)

#### (b)

\[ a = 50 \times P(6 < X < 8) = 50 \times P(0.194.. < Z < 0.956..) \]

\[ a = 12.81 \text{ (tables)} \text{ or } 12.68 \text{ (calc)} \]

\[ b = 50 - (28.85 + a) \]

\[ b = 8.34 \text{ (tables)} \text{ or } 8.47 \text{ (calc)} \]

(*M1,A1cso*)

#### (c)

H₀: Normal distribution is a good fit  
H₁: Normal distribution is not a good fit

**Class** |  \( O \) |  \( E \) | \( \frac{O^2}{E} \) | \( \frac{(O-E)^2}{E} \)
--- | --- | --- | --- | ---
0-3 | 8 | 8.56 | 7.4766.. | 0.0366..
3-5 | 12 | 12.73 | 11.31186.. | 0.0418..
5-6 | 13 | 7.56 | 22.354497.. | 3.9144..
6-8 | 9 | 12.68 or (12.81) | (6.32) ~ 6.38801.. | 1.0680..< (1.13)
8-12 | 8 | (8.34) or 8.47 | 7.556080..< (7.67) | (0.013) ~ 0.0260..

\[ \sum \frac{O^2}{E} - N = 5.087 .. ~ 5.1400.. \]

\[ \nu = 5 - 3 = 2 \]

\[ \chi^2 (0.05) = 5.991 \]

\[ 5.09 < 5.991 \text{ so insufficient evidence to reject } H_0 \]

Normal distribution is a good fit.

(*M1*)

**Notes**

#### (a)

B1cso  for denominator of 50 and at least 3 products on num or 274.5 on num

M1  for a correct expression with at least 3 correct products on num or \[ \frac{1844.25}{49} = \frac{1507.005}{49} \]

or \[ \frac{337.245}{49} \text{ or } \left( \frac{7377}{200} - 5.49^2 \right) \times \frac{50}{49} \text{ etc Allow 3sf accuracy} \]

A1cso  for 6.88 with M1 scored and no incorrect working seen

#### (b)

M1  a full method for \( a \) or \( b \) using the normal dist. Correct use of \( (6), 8, 5.49 \) and \( \sqrt{6.88} \) seen

1st A1  for \( a \) in range 12.68 ~ 12.81 or \( b \) in range 8.34 ~ 8.47 or awrt these values

2nd A1ft  for 50 – 28.85 – their \( a \) (or \( b \)) (but requires M1). Allow awrt 3sf. Must add up to 50

#### (c)

1st B1  for both hypotheses. B0 if they include 5.49 or 6.88. Condone \( X \sim N(\mu, \sigma^2) \) etc

1st M1  for attempting \( \frac{(O-E)^2}{E} \) or \( \frac{O^2}{E} \), at least 3 correct expressions or values.

1st A1  for at least 4 correct calcs - 3rd or 4th column. (2 dp or better and allow e.g. 7.47)

Allow any value in the ranges for the last two rows.

2nd A1  for a test statistic that is awrt 5.09 ~ 5.14. Award M1A1A1 if this is obtained.

2nd M1  for a correct statement based on their test statistic (> 1) and their cv (> 3.8)

Contradictory statements score M0 e.g. “significant” do not reject \( H_0 \).

3rd A1  for a correct comment suggesting that normal model is suitable or manager’s belief is correct. **Note**. Condone mention of 5.49 or 6.88 here. Hypotheses wrong way round scores A0

**Total 14**
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 (a)</td>
<td>Let ( L \sim N(50,25) ) and ( S \sim N(15,9) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Let ( X = L - (S_1 + S_2 + S_3) )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( E(X) = 50 - 3 \times 15 = 5 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \text{Var}(X) = 25 + 3 \times 9 = 52 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( P(X &lt; 0) = P \left( Z &lt; \frac{-5}{\sqrt{52}} \right) )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>( = P(Z &lt;-0.693..) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( =0.244 \text{ or } 0.2451 \text{ (tables)} ) (awrt 0.244 ~ 0.245)</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Let ( Y = L - 3S )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( E(Y) = 50 - 3 \times 15 = 5 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \text{Var}(Y) = 25 + 3^2 \times 9 = 106 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( P(Y &gt; 0) = P \left( Z &gt; \frac{-5}{\sqrt{106}} \right) )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>( = P(Z &gt; -0.4856..) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( =0.686 \text{ or } 0.6879 \text{ (tables)} ) (awrt 0.686 ~ 0.688)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Total 12</td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

(a) 1<sup>st</sup> B1 for forming a suitable variable \( X \) explicitly seen. Do not give for \( L - 3S \) but allow \( L - (S + S + S) \)

2<sup>nd</sup> B1 for \( E(X) = 5 \) (or \(-5 \) if their \( X \) is defined the other way around)

1<sup>st</sup> M1 for an attempt at \( \text{Var}(X) = \text{Var}(L) + 3 \text{Var}(S) \). Do not condone 5 for “25” or 3 for “9”

1<sup>st</sup> A1 for 52

2<sup>nd</sup> dM1 for attempting the correct probability and standardising with their mean and sd.

This mark is dependent on 1<sup>st</sup> M1 so if \( X \) is not being used or wrong variance score M0

If their method is not crystal clear then they must be attempting \( P(Z < -\text{ve value}) \)

or \( P(Z > +\text{ve value}) \) i.e. their probability after standardisation should lead to a prob. < 0.5

2<sup>nd</sup> A1 for awrt 0.244 ~ 0.245

Correct ans. only scores 5/6 (or 6/6 if 1<sup>st</sup> B1) but must be clearly labelled as (a) or the first answer.

(b) 1<sup>st</sup> B1 for defining a new variable \( [Y = \pm (L - 3S)] \). May be implied by a correct variance.

2<sup>nd</sup> B1 for \( E(Y) = 5 \) (or \(-5 \) if their \( Y \) is defined as \( Y = 3S - L \) )

1<sup>st</sup> M1 for an attempt at \( \text{Var}(Y) = \text{Var}(L) + 3^2 \text{Var}(S) \). Do not condone 5 for “25” or 3 for “9”

1<sup>st</sup> A1 for 106 only

2<sup>nd</sup> dM1 for attempting the correct probability and standardising with their mean and sd.

This mark is dependent on 1<sup>st</sup> M1 so if \( Y \) is not being used or wrong variance score M0

If their method is not crystal clear then they must be attempting \( P(Z > -\text{ve value}) \)

or \( P(Z < +\text{ve value}) \) i.e. their probability after standardisation should lead to a prob. > 0.5

2<sup>nd</sup> A1 for an awrt 0.686 ~ 0.688

Correct answer only scores 6/6 but must be clearly labelled as (b) or the second answer.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 6 (a) | $H_0 : \mu_{new} - \mu_{old} = 1$  
$H_1 : \mu_{new} - \mu_{old} > 1$  
\[
    z = \frac{7 - 5.5 - 1}{\sqrt{\frac{0.5}{60} + \frac{0.75}{70}}} = 3.62254... \quad \text{(awrt 3.62)}
\]  
Critical value $z = 1.6449$ (allow ±)  
$[3.62 > 1.6449]$ so sufficient evidence to reject $H_0$  
Evidence that the mean yield of new variety is more than 1 kg greater than the old variety. | B1  
B1  
M1 A1 A1  
A1  
B1  
B1  
(9)  
Total 11 |
| (b) | Mean yield is normally distributed  
Sample size is large. Must state or imply that in this case sample size is large | |

<table>
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<tr>
<th>Notes</th>
</tr>
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</table>
| 1st & 2nd B1 for hypotheses. Accept $\mu_1, \mu_2$ or $\mu_A, \mu_B$ etc if there is some indication of which is which e.g. $A \sim N(\mu_A, 0.5)$  
1st M1 for an attempt at se. Condone switching 0.5 and 0.75 $\sqrt{\frac{0.5}{60} + \frac{0.75}{70}}$  
1st A1 for a correct expression for denominator of test statistic or 0.138... or $\sqrt{0.0190...}$  
2nd A1 for a correct numerator of test statistic (must have the $-1$)  
3rd A1 for awrt 3.62  
3rd B1 for $\pm 1.6449$ seen or probability of 0.0002 (tables) or 0.000145...(calc) [allow 0.0001]  
2nd dm1 dep. on 1st M1 for a correct statement based on their normal cv and their test statistic  
2nd A1 for correct comment in context. Must mention “yield” and “varieties” or “old” and “new” and “1”  
If second B mark is B0 award A0 here |

ALT  
Pooled estimate: If they calculate $s_p = \sqrt{0.41845...} = 0.64688...$allow 1st M1, 1st A1 for expression (or awrt 0.114) and 2nd A1 if numerator correct but A0 for test statistic (4.39) |
| (b) | 1st B1 for mention of mean (yield) and normal (distribution)  
2nd B1 for mention of sample (size) being large in this case | |
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (a)</td>
<td>$\bar{x} = \frac{33.29}{8} = 4.16125$ (awrt 4.16)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\sigma^2 = s^2 = \frac{4.12^2 + 5.12^2 + ... - 8 \times \bar{x}^2}{7}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\sigma^2 = s^2 = \frac{141.4035 - 138.528013}{7} = 0.41078...$ (awrt 0.411)</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>$\sum x = 33.29 + 32 \times 4.55 = 178.89$, (awrt 179)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\sum x^2 = &quot;141.4035&quot; + 31 \times 0.25 + 32 \times 4.55^2 (= 811.6335)$ (awrt 812)</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>Combined sample: $s^2 = \frac{811.6335 - 178.89^2}{39} = 0.29724865...$ (awrt 0.297)</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>$\frac{s}{\sqrt{n}} = \frac{\sqrt{0.297}...}{\sqrt{40}} = 0.0862$ (awrt 0.0862)</td>
<td>M1A1</td>
</tr>
<tr>
<td>(c)</td>
<td>$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = \frac{178.89 \pm 1.96 \times 0.67}{40}$</td>
<td>M1B1</td>
</tr>
<tr>
<td></td>
<td>= (4.2646..., 4.67988...) awrt (4.26[or 4.265], 4.68)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(Total 3)</td>
<td></td>
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</tbody>
</table>

**Notes**

(a) M1 for an attempt at $s^2$: correct denom, clear attempt at $\sum x^2$ and ft their $\bar{x}$ Ans only 2/2

(b) B1 for correct sum or mean or fully correct expression (accept mean = awrt 4.47) **May be in (c)**

1st M1 for their 141.4035 + 31 × 0.25 + 32 × 4.55² or “141.4035” + 7.75 + 662.48 (accept 3sf)

**Beware:** 32(0.25 + 4.55²) + “141.4035” = awrt 812 but scores M0A0.

1st A1 for a fully correct expression (all to 3sf or better) or answer only = awrt 812

2nd M1 for a correct expression using their values

3rd M1 dependent on using a changed $s^2$ (not their 0.411 or 0.25) for $\sqrt{\frac{0.297}{40}}$

This $s^2$ must be based on a combination of their 0.411 and 0.25 e.g. 0.661

(c) M1 for $\bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$ for any $z$ (> 1.5) and ft their $\bar{x}$ based on combining their 4.16 and 4.55, do not award for simply using 4.55 or their 4.16. Condone $\sigma = \sqrt{\text{their} 0.297}$ or their (b)

B1 for $z = 1.96$ used in an attempt at a CI, may for example miss $\sqrt{n}$

A1 for both limits awrt 3sf. Allow lower limit of 4.265