FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2001.

PURE MATHEMATICS
PAPER-I

TIME ALLOWED: THREE HOURS — MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including question No.8 which is **COMPULSORY**. At least select **TWO** questions from each section. All questions carry **EQUAL** marks.

SECTION-I

1. (a) Prove that any group G can be embedded in a group of bijective mappings of a certain set. **(10)**

   (b) Prove that the number of elements in a conjugacy class Ca of an element "a" in a group G is equal to the index of its normalizer. **(10)**

2. (a) Let G be a group, prove that: **(12)**
   - (i) The derived subgroup G' is normal subgroup of G.
   - (ii) G/G' is abelian.
   - (iii) If K is a normal subgroup of G such that G/K is abelian then $k \supseteq G'$.

   (b) Prove that a finite dimensional integral domain is a field. **(08)**

3. (a) Prove that in a commutative ring with identity an ideal M of R is maximal ideal if and only if R/M is a field. **(07)**

   (b) Find rank and nullity of $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$ **(07)**

   (c) Let V be a vector space of polynomials of degree ≤ 3, determin whether the vectors $x^3 - 3x^2 + 5x + 1$, $x^3 - x^2 + 8x + 2$ and $2x^3 - 4x^2 + 9x + 5$ of V are linearly independent. **(06)**

4. (a) Find value of λ for which the following homogeneous system of linear equations has non-trivial solution. Find the solution **(07)**

$$(1-\lambda)x_1 + x_2 + x_3 = 0$$
$$x_1 - \lambda x_2 + x_3 = 0$$
$$x_1 - x_2 + (1-\lambda)x_3 = 0$$

   (b) Find eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$. **(06)**

   (c) Solve the following system of equations by reducing to reduced echlon form: **(07)**

$$2x_1 - x_2 + 3x_3 = 3$$
$$3x_1 + x_2 - 5x_3 = 0$$
$$4x_1 - x_2 + x_3 = 3$$

SECTION-II

5. (a) Find equation of a sphere passing through the points (0,-2,-4), (2,-1,-1) and having the centre on the straight line 2x-3y=0=5y+2z **(08)**

   (b) (i) Discuss the following surface and sketch it $9x^2 - 4y = 9z^2$ **(06)**

   (ii) Find cylindrical and spherical polar coordinates of the point P with rectangular coordinates $(2\sqrt{3}, 2, -2)$. **(06)**

6. (a) Show that the lines: (10)
L: x=3+2t, y=2+t, z=-2-3t
M: x=-3+4s, y=5-4s, z=6-5s
Intersect. Find an equation of the plane containing these lines.

(b) Show that the perpendicular distance D of a point $P(x_1, y_1, z_1)$ from the plane ax+by+cz+d=0 is given by $D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ and hence find distance between the parallel planes 2x+2y-4z+3=0 and 3x+3y-6z+1=0. **(10)**

7. (a) Find length of one arch of the cycloid $x = a(\theta - \text{Sin}\theta), y = a(1 - \text{Cos}\theta)$. **(10)**

(b) Show that for the parabola $y = ax^2 + bx + c$, the curvature is minimum at its vertex. **(10)**

COMPULSORY QUESTION

8. Write only the correct answer in the answer book. Do not reproduce the questions.

(1) The set {i, -i, 1, -1} is:
(a) Semi group under addition (b) Group under addition
(c) Group under multiplication (d) None of these.

(2) Number of subgroups of order one of an infinite group G is:
(a) Zero (b) 1 (c) 2 (d) infinite (e) None of these.

(3) A cycli group of order n is generated by:
(a) n elements (b) (n-1)elements
(c) two elements (d) one element
(e) None of these.

(4) Let H be a subgroup of order m of a group of order n, the number of right cosets of H in G is:
(a) n (b) n - m (c) m – n
(d) $\frac{n}{m}$ (e) None of these.

(5) The dimension of a vector space V is the number of:
(a) Linearly independent vectors in V.
(b) Linearly dependent vectors in V.
(c) Linearly independent vectors spanning V.
(d) None of these.

(6) The characteristic of an integral domain is:
(a) zero (b) a prime (c) zero or a prime (d) None of these.

(7) The eigenvalue is related to the corresponding eigenvector (for a matrix A) as:
(a) $|A - \lambda I \underline{x}| = 0$ (b) $|A - \lambda I|\ \underline{x} = b$
(c) $A\underline{x} = \lambda\ \underline{x}$ (d) None of these.

(8) For two vectors $\vec{A}$ and $\vec{B}$, $\vec{A}.\vec{B}$ gives:
(a) Cos of angle between $\vec{A}$ and $\vec{B}$
(b) Area of parallelogram with $\vec{A}$ and $\vec{B}$ as its adjacent sides.
(c) Vector perpendicular to $\vec{A}$ and $\vec{B}$
(d) Vector parallel to the plane of $\vec{A}$ and $\vec{B}$
(e) None of these.

(9) If θ is angle between two vectors $\vec{A}$ and $\vec{B}$, then $\frac{\vec{A} \times \vec{B}}{|\vec{A}||\vec{B}|}$ gives:
(a) tan θ (b) cos θ
(c) sin θ (d) sec θ
(e) None of these.

FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2001.

PURE MATHEMATICS
PAPER-II

TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including question **No.8** which is **COMPULSORY**. At least select **TWO** questions from each section. All questions carry **EQUAL** marks.

SECTION-I

1. (a) (i) Find $\lim_{x\to a}\frac{x^p - a^p}{x-a}$. **(5 + 5)**

(ii) Find a and b such that $f(x)=\begin{cases} x^3, x<-1 \\ ax+b, -1\le x<1 \\ x^2+2, x>1 \end{cases}$ is continuous for all x.

(b) (i) Find $\frac{dy}{dx}$ when $\text{Sin}(\ln xy) = x + y^2$ **(5 + 5)**

(ii) Use Taylor's theorem to prove that $\ln \sin(x+h) = \ln \sin x + h \cot x - \frac{1}{2}h^2 \operatorname{cosec}^2 x + \frac{1}{3}h^3 \cot x \operatorname{cosec}^2 x + ---$

2. (a) Evaluate $\int \frac{dx}{(1-2x^2)\sqrt{1-x^2}}$ **(08)**

(b) Evaluate $\int e^{3x} \sin 4x dx$ **(06)**

(c) Test the convergence or divergence of the series:
$\frac{2}{5} + \frac{2.4}{5.8} + \frac{2.4.6}{5.8.11} + \frac{2.4.6.8}{5.8.11.14} + ...$ **(06)**

3. (a) Find the asymptotes of the curve $x^2y^2(x^2-y^2)^2 = (x^2+y^2)^2$. **(10)**

(b) Find maxima and minima of the radius vector of the curve:
$\frac{c^2}{r^2} = \frac{a^2}{\text{Sin}^2\theta} + \frac{b^2}{\text{Cos}^2\theta}$. **(10)**

4. (a) Trace the folium of Descartes $x^3 + y^3 = 3axy$. **(08)**

(b) Define an open sphere in a metric space(x,d). Let (x, d_0) be the discrete metric space, write open balls centered at $x \in X$ with radius $1/2$ and $3/2$. **(06)**

(c) Let X = C [a,b] be the set of all real valued continuous defined on [a,b]. Define a function $d: X \times X \to R$ as follows: **(06)**
For $f, g \in X$, $d(f,g) = \int_a^b |f(x) - g(x)| dx$. Prove that (x,d) is a metric space.

SECTION-II

5. (a) Separate into real and imaginary parts $\tan^{-1}(x+iy)$. **(07)**

(b) Show that $\log(1+\cos\theta + i\sin\theta) = \ln(2\cos\frac{\theta}{2}) + i\frac{\theta}{2}$ **(06)**

(c) Sum the series: **(07)**

$$1 + c\cos\theta + \frac{c^2}{2!}\cos 2\theta + \frac{c^3}{3!}\cos 3\theta + \ldots$$

6. (a) Define an analytic function. Prove that the necessary and sufficient condition for a function $W=f(z)=U(x,y)+iV(x,y)$ to be analytic is that U_x, V_x, U_y, V_y exist and are continuous such that $U_x = V_y$, $U_y = -V_x$. **(10)**

(b) Using cauchy's integral formula evaluate $\int_c \frac{dz}{1+z^2}$ where C is part of the parabola $y=4-x^2$ from A(2,0) to B(-2,0). **(10)**

7. (a) Expand $f(z) = \frac{1}{z^2}$ about $z = 2$ using Taylor's series expansion. **(10)**

(b) Consider the transformation $W = e^{\frac{\pi}{4}}.Z$ and determine the region in w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane. **(10)**

COMPULSORY QUESTION

8. Write only the correct answer in the answer book. Do not reproduce the questions.

(1) The function $f(x)=\frac{x^2-a^2}{x-a}$ is discontinuous at:

(a) $x = 1$ (b) $x = a$
(c) $x = 0$ (d) $x = \sqrt{a}$ (e) None of these.

(2) $f(x) = \cos x$ has a maximum value at:

(a) $x = 0$ (b) $x = 1$
(c) $x = \frac{\pi}{2}$ (d) $x = \frac{3\pi}{2}$ (e) None of these.

(3) $\lim_{x\to 0}\frac{\sin x}{x}$ is :

(a) zero (b) 1
(c) undefined (d) -1 (e) None of these.

(4) Derivative of the function $f(x) = \tan x$ at $x = \frac{\pi}{4}$ is:

(a) 2 (b) $\frac{1}{2}$
(c) 1 (d) Zero (e) None of these.

(5) For an increasing function f, let $x_1 < x_2$ then:

(a) $f(x_1) > f(x_2)$ (b) $f(x_1) < f(x_2)$
(c) $f(x_1) = f(x_2)$ (d) None of these..

(6) Area under the curve $f(x) = e^x + 2$ bounded by $x=0$, $x = 2$ and x-axis is given by:

(a) 3 (b) e^3+2
(c) e^2+1 (d) e^2+3 (e) None of these..

(7) Normal to the parabola $y^2 = 12x$ at $(3,-6)$ is:

(a) $y = x + 3$ (b) $y = x - 9$
(c) $y + x + 3 = 0$ (d) None of these.

(8) Equation of tangent to the circle $x^2 + y^2 = a^2$ at (x_1, y_1) is given by:

(a) $x_1^2 + y_1^2 + 2gx + 2fy + c = 0$
(b) $x^2 + y^2 + 2gx_1 + 2fy_1 + c = 0$
(c) $xx_1 + yy_1 + 2gx_1 + 2fy_1 + c = 0$
(d) $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
(e) None of these.

(9) In a complete metric space:

(a) Every sequence is bounded
(b) Every sequence converges
(c) Every cauchy sequence converges
(d) There is no convergent sequence.
(e) None of these.

(10) The open ball of radius 1 and center at zero in R is given by:

(a) (0,1) (b) [0,1]
(c) (-1,1) (d) {0} (e) None of these.

(11) For the two positive term series $\sum_{1}^{\infty} a_n$ and $\sum_{1}^{\infty} b_n$ if

$a_n \le b_n \forall_n = 1,2....$if $\sum_{1}^{\infty} b_n$ is convergent, then:

(a) $\sum_{1}^{\infty} a_n$ diverges (b) $\sum_{1}^{\infty} a_n$ converges
(c) $\sum_{1}^{\infty} a_n$ converges absolutely (d) None of these.

(12) Polar form of the complex number $z = 3 - 4i$ is:

(a) $5e^{i\theta}$ (b) $5e^{-i\theta}$
(c) $5e^{2i\theta}$ (d) $e^{i\theta}$. (e) None of these.

(13) Log $(x + iy)$ is given by $(|z| = \sqrt{x^2 + y^2},\ \ \theta = \tan^{-1}\frac{y}{x})$:

(a) $\log|z| + i\theta$ (b) $\log|z| + i\lambda\theta$
(c) $\log(|z| + i\lambda\theta)$ (d) $\log(|z| + i\theta)$
(e) None of these..

(14) A curve $Z = f(t)$ is smooth if for $t \in [a,b]$:

(a) $f'(t) = 1$ (b) $f'(t) = 0$
(c) $f'(t) \neq 0$ (d) $f(a) = f(b)$
(e) None of these..

(15) On a Simply connected domain D and any closed con. C in D, for an analytic function f(z), $\int_c f(z)dz$ is:

(a) Zero (b) non - zero
(c) 1 (d) $\frac{1}{2}$ (e) None of these.

(16) $\lim_{n\to\infty}(1+\frac{1}{n})^n$ is:

(a) 1 (b) zero
(c) e (d) e^n (e) None of these.

(17) The set of integers together with the operation of multiplication forms:

(a) a semi-group (b) group
(c) Integral domain (d) field (e) None of these.

(18) $\int \tan x dx$ is:

(a) sec x tan x (b) $\sec^2 x$
(c) ln sec x (d) sec x (e) None of these.

(19) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}(2+x)dxdy$ is:

(a) $\frac{\pi}{2}$ (b) 1
(c) 2π (d) Zero (e) None of these.

(20) $\left|\int \frac{dz}{z^2}\right|$ is:

(a) ≤ 2 (b) ≤ 1
(c) 2 (d) 1 (e) None of these.

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2002

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY**. Select at least TWO questions from each of the **SECTIONS I and II**. All questions carry **EQUAL** marks.

Q.No.		Question	Marks
		SECTION – I	
1	(a)	Let G be a finite group and H be its Subgroup. Then prove that the order of H divides the order of G.	10
	(b	State and prove Fundamental theorem of Homomorphism in groups.	10
2	(a)	Define (i) Commutator Sub group G' of a group G. (ii) Subrings. (iii) Integral Domain.	9
	(b	Show that the correspondence $a + ib \rightarrow \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $a, b \in R^*$ is an isomorphism of the field C of complex numbers into the ring of 2×2 matrices over R^*.	11
3	(a)	Let V be a vector space over F and W a non-empty subset of V. Prove that W is a subspace of V iff it is closed under the operation of addition and scalar multiplication.	08
	(b	Show that the yz plane in R^3 is spanned by (0,1,2), (0,2,3) and (0,3,1).	06
	(c)	Let V=R[x] be the vector space of all polynomials in x over R. Show that the mapping: $I : V \rightarrow R$,defined by $(v)I = \int_0^1 v dx$, is linear.	06
4	(a)	If A is an idempotent matrix then prove that (i) $B = I - A$ is an Idempotent matrix, (ii) $AB = BA = 0$.	06
	(b	Find the eigen values and eigen vectors of $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$	06
	(c)	Investigate for what values of a, b the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$, have: (i) No solution (ii) A unique solution (iii) an infinite number of solutions.	08
		SECTION – II	
5	(a)	Find the length of one arc of the cycloid $x = b(\theta - \sin\theta)$, $y = b(1 - \cos\theta)$.	07
	(b)	Find the pedal equation of $r^m = a^m \cos m\theta$.	07

	(c)	Show that the equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$.	06
6	(a)	Find the equation of the plane through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).	06
	(b	Find the Cartesian and spherical polar coordinates of the point P with cylindrical coordinates (4, arc Cos $\frac{4}{5}$, 3).	06
	(c)	Find the equation of the sphere having the straight line joining the points (2,3,4) and (-2,-3,-4) as a diameter.	08
7	(a)	Define the curvature, the unit principal normal vector and the unit binormal vector of a curve C.	06
	(b	Find the torsion of the curve C: r(t) = [a cos t, a sin t, ct].	07
	(c)	Prove the Serre-Frenet's formula $b' = -\tau\rho$.	07

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Don't reproduce the statement.

1	0,0,1 are the direction Cosines of:				
	(a)	x-axis	(b)	y-axis	
	(c)	z-axis	(d)	None of these.	
2	$AB=AC \Rightarrow B=C$ when				
	(a)	A is Non Singular	(b)	$\|A\|=0$	
	(c)	A^{-1} exists	(d)	None of these.	
3	The angle between the planes $x-y-2z+3=0$ and $2x+y-z=5$ is				
	(a)	0	(b)	$\frac{\pi}{2}$ radians	
	(c)	$\frac{\pi}{3}$ radians	(d)	None of these.	
4	If AB = BA, when A and B are square matrices, the multiplication is said to be:				
	(a)	Associative	(b)	Reflexive	
	(c)	Commutative	(d)	None of these.	
5	The perpendicular distance of the point (3,-1,2) from the plane $2x+y-z=4$ is:				
	(a)	2	(b)	4	
	(c)	$\frac{1}{\sqrt{6}}$	(d)	None of these.	

PURE MATHEMATICS, PAPER-I

6	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ represents;			
	(a) Sphere		(b) Ellipsoid	
	(c) Hyperboloid of one sheet		(d) Hyperboloid of two sheets.	
	(e) None of these.			
7	The radius of the sphere $x^2+y^2+z^2-4x+2y-6z=1$ is:			
	(a) 1		(b) 5	
	(c) 10		(d) None of these.	
8	The equation of surface of revolution obtained by revolving the curve $x=z^2$, y = 0 about the x-axis is:			
	(a) $x^2+y^2=z^4$		(b) $x=y^2+z^2$	
	(c) $x^2=z^4$		(d) None of these.	
9	$ax^2+by^2+2hxy+2gx+2fy+c=0$ represents a parabola when:			
	(a) $h^2<ab$		(b) $h^2>ab$	
	(c) $h^2=ab$		(d) None of these.	
10	$\underline{u}$, $\underline{p}$, $\underline{b}$ constitute a triple of orthogonal unit vectors which is:			
	(a) Right Handed		(b) Left Handed	
	(c) Orthonormal		(d) None of these.	
11	An equivalence relation satisfies the following three properties:			
	(a) Reflexive, symmetric, transitive		(b) Reflexive, Anti symmetric, transitive	
	(c) Not Reflexive, symmetric, transitive		(d) Reflexive, symmetric, Not transitive	
	(e) None of these.			
12	If M and N are any two $n\times n$ square matrices, then det (MN) equals:			
	(a) det M + let N		(b) det M det N	
	(c) Matrix MN		(d) None of these.	
13	If A is a square matrix, then:			
	(a) det 3A = det A		(b) det 3A = 3 det A	
	(c) det $A^t\neq$ det A		(d) det A = A	
	(e) None of these.			

14	Which of the following mapping is a Linear Transformation?				
	(a)	$T(a_1, a_2, a_3) = (a_1, a_2)$	(b)	$T(a,b,c) = (a,1)$	
	(c)	$T(x,y,z) = (x+1,y,z)$	(d)	$T(x,y) = (x+1,y+1)$	
	(e)	None of these.			
15	$(R,+,.)$: where R is the set of all real numbers, is a				
	(a)	Field	(b)	Commutative Ring	
	(c)	Ring with Identity	(d)	Division Ring	
	(e)	None of these.			
16	Which of the following are subspaces of R^2?				
	(a)	$\{(a,a): a \in R\}$	(b)	$\{(a,a^2): a \in R\}$	
	(c)	$\{(a,a+1): a \in R\}$	(d)	$\{(a^2,a): a \in R\}$	
	(e)	None of these.			
17	Matrix A is called Involuntory if:				
	(a)	$A^2 = A$	(b)	$A^2 = I$	
	(c)	$A^{K+1} = A$	(d)	$A' = A$	
	(e)	None of these.			
18	Which of the following statements for groups is wrong?				
	(a)	$(g^{-1})^{-1} = g$, for every g in G.	(b)	The inverse of the identity element e is e itself in G.	
	(c)	A group contains at least the identity element.	(d)	There is a concept of an empty group.	
	(e)	None of these.			
19	Given $\psi: G \to G'$, from G into G', is a group homomorphism. Then ψ is called epimorphism if:				
	(a)	$G' = G$	(b)	ψ is 1-1	
	(c)	ψ is onto G'	(d)	ψ is 1 – 1 and onto G', both	
	(e)	None of these.			
20	A cyclic group of order n is generated by:				
	(a)	n elements	(b)	two elements	
	(c)	One element	(d)	n – 1 elements.	
	(e)	None of these.			

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(19)

FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2002

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY**. Select at least **TWO** questions from each of the **SECTIONS I and II**. All questions carry **EQUAL** marks.

Q.No.	Question		Marks
	SECTION – I		
1	(a)	Prove that an open sphere in a metric space X is an open set.	07
	(b)	The intersection of any two open sets and hence of any number of open sets in X is open. (Prove for topological space (X, τ)).	07
	(c)	Define: (i) Interior point of A (ii) Exterior point of A. (iii) Boundary point of A. (iv) Closure of A; where A is a subset of a subset of a topological space X.	06
2	(a)	Let $X = \{x, y, z\}$, $\tau = \{\Phi, X, \{x\}, \{y, z\}\}$. Define $g: X \to X$ by $g(x) = y, g(y) = z, g(z) = x$. Verify whether g is continuous or not.	06
	(b)	Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$	08
	(c)	Show that $\beta(m,n) = \beta(n,m)$; also evaluate $\Gamma(\frac{5}{2})$.	06
3	(a)	Evaluate $\lim_{x \to \frac{\pi}{2}} \dfrac{\tan 3x}{\tan x}$.	06
	(b)	Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a, b, c are positive constants.	08
	(c)	Calculate $\int_1^4 \dfrac{dx}{(x-2)^{\frac{2}{3}}}$.	06
4	(a)	Test the convergence of the series $\sum_{n=1}^{\infty} \dfrac{n}{(n+1)(n+2)}$.	06
	(b)	Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$.	06
	(c)	State and prove Fundamental theorem of Integral calculus.	08
	SECTION – II		
5	(a)	Expand $\cos^5\theta \sin^3\theta$ in series of Sines of multiples of θ.	08
	(b)	Find the 6 Sixth roots of -1.	06
	(c)	Prove that $\cosh^{-1} z = \mathrm{Log}(z + \sqrt{z^2 - 1})$.	06
6	(a)	Expand $f(x) = \sin x$ in a Fourier cosine series in the interval $0 \le x \le \pi$.	07

	(b)	Verify that $u = x^2 - y^2 - y$ is harmonic in C and find a conjugate harmonic function v of u.	07
	(c)	Evaluate $\oint_c \frac{dz}{z-i}$, c is the circle $\|z\| = 2$ (counter clockwise).	06
7	(a)	Find the center and radius of convergence of the power series $\sum_{0}^{\infty} \frac{(z-2i)^n}{5^n}$.	06
	(b)	Define the following terms: (i) Pole (ii) Isolated essential singularity (iii) Zero of an analytic function (iv) Residue.	06
	(c)	Evaluate $\oint_c \frac{z}{z^2 - \frac{1}{4}} dz$, where c is the unit circle (counter clock wise).	08

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Don't reproduce the statement.

1	The function $f(x) = \frac{x^2-9}{x-3}$ is discontinuous at:			
	(a)	$x = 0$	(b)	$x = 3$
	(c)	$x = 1$	(d)	None of these.
2	$f(x) = \sin x$ has a minimum value at:			
	(a)	$x = 0$	(b)	$x = \frac{\pi}{2}$
	(c)	$x = \frac{3\pi}{2}$	(d)	None of these.
3	$\lim_{x \to \infty}\left(1 + \frac{1}{x}\right)^x$ is:			
	(a)	0	(b)	1
	(c)	e	(d)	-e
4	Derivative of the function $f(x) = \ln x$ at $x = 0$ is:			
	(a)	1	(b)	0
	(c)	∞	(d)	None of these.
5	For a decreasing function g, let $x_1 < x_2$; then:			
	(a)	$g(x_1) > g(x_2)$	(b)	$g(x_1) < g(x_2)$
	(c)	$g(x_1) = g(x_2)$	(d)	None of these.

6	Tangent to the parabola $y^2 = 5x$ at (5.5) is:			
	(a)	$y = x + 5$	(b)	$y = x - 5$
	(c)	$y = x$	(d)	None of these.
7	$(2i)(-3 - i)$ is equal to:			
	(a)	(2, -6)	(b)	(-2, 6)
	(c)	(2, 6)	(d)	(-2, -6)
8	Which of the following statements is not correct?			
	(a)	e^z is never zero	(b)	$5z > z$
	(c)	$e^z = 1$ iff z is an integral multiple of $2\pi i$	(d)	$\arg(z_1 z_2) = \arg z_1 + \arg z_2$.
9	$\int_0^{\frac{\pi}{4}} sr^2 x \, dx$ is equal to:			
	(a)	1	(b)	Zero
	(c)	∞	(d)	None of these.
10	$\Gamma(\frac{1}{2})$ is equal to:			
	(a)	π	(b)	$\sqrt{\pi}$
	(c)	$\frac{1}{2}$	(d)	Zero.
11	The Jocobian of the rotation $x = u\cos\alpha - v\sin\alpha$, $y = u\sin\alpha + v\cos\alpha$ is:			
	(a)	Uv	(b)	α
	(c)	1	(d)	None of these.
12	$\int_0^1 \int_1^2 \int_2^3 dx\, dy\, dz$ is equal to:			
	(a)	1	(b)	2
	(c)	3	(d)	None of these.
13	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$ is the expansion of:			
	(a)	cos x	(b)	e^x
	(c)	$\frac{1}{1-x}$	(d)	None of these.

14	Which of the following statements is not correct?			
	(a)	An absolutely convergent series is convergent.	(b)	$\sum_{1}^{\infty} \frac{1}{n}$ is convergent
	(c)	$\sum_{1}^{\infty} n$ is divergent	(d)	$\{1+(-1)^{n+1} n\}$ oscillates infinitely.
15	The period of cos x sin x is:			
	(a)	$\frac{\pi}{2}$	(b)	2π
	(c)	π	(d)	Arbitrary.
16	Let the metric space be R and let $x_0 = 1$ and $r = \frac{1}{2}$. Then $S_{\frac{1}{2}}(1)$ is given by:			
	(a)	$]\frac{1}{2}, 1[$	(b)	$]0, \frac{3}{2}[$
	(c)	$]\frac{1}{2}, \frac{3}{2}[$	(d)	None of these.
17	Which of the following statements is not correct?			
	(a)	If gof is injective, then f is injective.	(b)	If gof is surjective, then g is surjective.
	(c)	If gof is surjective, and g is injective, then f is surjective.	(d)	If gof is injective, and f is surjective, then g is surjective.
	Note: $f: A \to B$ and $g: B \to C$ are functions.			
18	Select the correct statement:			
	(a)	$\text{Int}(\text{Int } A) \neq \text{Int}(A)$	(b)	$\text{Int}(A \cup B) = \text{Int}(A) \cup \text{Int}(B)$
	(c)	$\text{Int}(A \cap B) = \text{Int}(A) \cap \text{Int}(B)$	(d)	$\text{Ext}(A \cup B) \neq \text{Ext}(A) \cap \text{Ext}(B)$
		where A and B are any two subsets of a topological space.		
19	$\oint_c \cot z \, dz$ is equal to:			
	(a)	$2\pi i$	(b)	πi
	(c)	Zero	(d)	None of these.
	where c is the unit circle (Counter clockwise).			
20	The image of the region $1.5 \leq \|z\| < 2.1$ under the mapping $w = z^2$ is:			
	(a)	$2.25 \leq \|w\| < 4.41$	(b)	$1.5 \leq \|w\| < 4.41$
	(c)	$2.25 \leq \|w\| < 2.1$	(d)	None of these.

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(19)

FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2003

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including question **NUMBER- 8** which is **COMPULSORY**. Select at least **TWO** questions from each of the **SECTIONS I** and **II**. All question carry **EQUAL MARKS**.

SECTION -I

1. (a) Let H, K be subgroup of a group G and $HK=\{hK \mid h\in H,\ k\in K\}$. Show that HK is a subgroup of G if and only if HK=KH. **(08)**

(b) Let H be a normal subgroup and K a subgroup of group G. Show that the factor groups HK/H and $K/H\cap K$ exist and are isomorphic to each other. Also give the famous name of this result. **(12)**

2. (a) Give definition of normalizer of a set in group G. Prove that the index of normalizer of an element g in G is equal to the number of elements in conjugacy class C_g of g in G. **(10)**

(b) State the famous Pigeonhole principle. Use this principle to justify the claim "every integral domain is a field". **(10)**

3. (a) What is meant by a basis of vector space V over field F. If $x_1, \ldots, x_m$ are m linearly independent vectors in n – dimensional vector space V over field F then show that $n \geq m$. **(08)**

(b) Give definition of finite extension of a field. If L is a finite extension of field K and K is a finite extension of field F, Then show that L is a finite extension of F. **(12)**

4. (a) Let S and T be linear transformations of finite – dimensional vector space V into itself. Define the rank r(s) of s. Then show that $r(TS)\leq \min\{r(s), r(T)\}$ and that r(ST)=r(TS)=r(T) whenever S is invertible. **(10)**

(b) Let V be an n-dimensional vector space over field F. Let T be a linear transformation from V into itself having all its characteristic roots in F. Show that T satisfies a polynomial of degree n over F. **(10)**

SECTION –II

5. (a) How would you differentiate between hyperbola and parabola? Prove that the lines $y = \frac{b}{a}x$ and $y = \frac{-b}{a}x$ are asymptotes of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$ **(10)**

(b) Give significance of the pedal equation of a plane curve. Show that $p^2 = ar$ is the Pedal equation of the parabola $y^2=2a(x+a)$. **(10)**

6. (a) Express the equation $P = 7 \sin\theta \sin\phi$ in cylindrical and rectangular coordinates. **(10)**

(b) What kind of surfaces in IR^3 are called ellipsoids? Identify the standard name of the surface $x^2 + 9y^2 - 4z^2 - 6x + 18y + 16z + 20 = 0$. **(02+08)**

7. (a) What is the osculating plane of a curve at point P: Show that the osculating planes at any three points of the cubic curve $\vec{r} = (u, u^2, u^3)$ meet at a point lying in the plane determined by the three points. **(10)**

PURE MATHEMATICS, PAPER-I

(b) Find the curvature and torsion of the curve of intersection of the following two quadric surfaces: $a_1x^2+b_1y^2+c_1z^2 = 1$, $a_2x^2+b_2y^2+c_2z^2=1$. **(10)**

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Do not reproduce the question.

(1) The number of identity elements in a group is:
- (a) 0
- (b) 1
- (c) 2
- (d) None of these.

(2) A field must contain at least:
- (a) one element
- (b) Two elements
- (c) Three elements
- (d) None of these.

(3) A basis of Vector space contains:
- (a) only the zero vector
- (b) no zero vectors
- (c) zero as well as non-zero vectors
- (d) None of these.

(4) Every vector space is:
- (a) a group
- (b) a ring
- (c) a field
- (d) None of these.

(5) Matrix A is nilpotent iff:
- (a) $A^n \neq 0, \forall n$
- (b) $A^n = 0$ for some n
- (c) $A^n = 0, \forall n$
- (d) None of these.

(6) A unit matrix of order n has the rank:
- (a) 0
- (b) 1
- (c) n
- (d) None of these.

(7) The matrix equation AX = B has unique solution if:
- (a) 0
- (b) A is singular
- (c) A is not invertible
- (d) None of these.

(8) The determinant of a triangular matrix is the product of its entries on:
- (a) first row
- (b) second row
- (c) main diagonal
- (d) None of these.

(9) In any conic, the harmonic mean between the segments of focal chord is:
- (a) the geometric mean
- (b) zero
- (c) semi-latus-rectum
- (d) None of these.

(10) $a = r\cos\theta$ is an asymptot of the curve:
- (a) $r = a\cos\theta$
- (b) $r = a\sin\theta$
- (c) $r = a\tan\theta$
- (d) None of these.

(11) The radius of curvature of $y = \sqrt{r^2 - x^2}$ for $x \in [-r, r]$ is:
- (a) $\frac{1}{r}$
- (b) r
- (c) 2r
- (d) None of these.

(12) The distance from the origin to the plane $x + 2y - z - 4 = 0$ is:
- (a) $\frac{1}{\sqrt{6}}$
- (b) $\frac{\sqrt{6}}{4}$

(c) $\frac{4}{\sqrt{6}}$ (d) None of these.

(13) The rectangular coordinates of the point with spherical coordinates $(5, .5\pi, .5\pi)$ are:
(a) (5,0,0) (b) (0,5,0)
(c) (0,0,5) (d) None of these.

(14) $a^{-2}x^2 + b^{-2}y^2 - c^{-2}z^2 = -1$ is hyperboloid of:
(a) 1 sheet (b) 2 sheets
(c) 3 sheets (d) None of these.

(15) The principal normal at point P on a curve is the intersection of normal plane at P and:
(a) the curve (b) tangent plane
(c) osculating plane (d) None of these.

(16) A curve is not a straight line iff its curvature is:
(a) zero (b) non-zero
(c) one (d) None of these.

(17) The relations $t' = kn$, $n' = \tau b$, $b' = -\tau n$ are known as
(a) Gauss-Bonnet equations (b) serret – Frenet formulae
(c) Tissot equations (d) None of these.

(18) A set of n+1 vectors in n-dimensional vector space:
(a) must be linearly independent (b) must be linearly dependent
(c) must be a basis (d) None of these.

(19) Which of the following terms is not used in algebra?
(a) homomorphism (b) homeomorphism
(c) epimorphism (d) None of these.

(20) No group of order 28 can have subgroup of order:
(a) 7 (b) 11
(c) 14 (d) None of these.

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2003

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt FIVE questions in all, including question NUMBER- 8 which is COMPULSORY. Select at least TWO questions from each of the SECTIONS I and II. All questions carry EQUAL MARKS.

SECTION -I

1. (a) For every positive integer n, show that $\lim_{x\to 0}\frac{\sin nx}{nx}=1$. (05)

   (b) Discuss the continuity of function f given by

   $$f(x)=\begin{cases} x \text{ if } x \text{ is irrational number} \\ 1-x \text{ if } x \text{ is rational number, at } x=\frac{2}{3} \end{cases}$$ (05)

   (c) Show that any real function $f(x)$ which is differentiable at point x_0 must be continuous at x_0. Further show that the converse generally is not true. (10)

2. (a) Find $\frac{dy}{dx}$ of $(\tan x)^y + y^{\cot x} = b$. (06)

   (b) Find the volume of the solid region bounded above by the sphere $x^2+y^2+z^2=4$ and below by the upper nappe of the cone $z^2=x^2+y^2$. (06)

   (c) Show that radius of the base of an open cylinder of given surface s and greatest volume V is equal to its height. (08)

3. (a) Let A be any set in a metric space X and $x \in X$. Show that x is a closure point of A iff every open sphere about x intersects A. (10)

   (b) Let f be a function from metric space X into a metric space Y and $x_0 \in X$. Prove that f is continuous at x_0 iff $\lim_{n\to\infty} f(x_n) = f(x_0)$ wherever (x_n) is a sequence in X converging to x_0. (10)

4. (a) Examine the series $\sum_{m=1}^{\infty}\frac{\arctan m}{1+m^2}$ for convergence. (09)

   (b) Let $f(x)$ be Riemann integrable function on [a,b] and let there be a differentiable function F on [a,b] such that $F'=f$. Show that $\int_a^b f(x)dx=F(b)-F(a)$. Also give the famous name of this result. (11)

SECTION -II

5. (a) Prove that every complex number has n nth roots, for all positive integer n. (08)

   (b) Deduce the famous Cauchy – Riemann conditions as a necessity for analytic functions. Show also that these conditions are not sufficient to guarantee the analyticity. (12)

6. (a) Give the standard construction of arc tan z and then discuss its analyticity in detail. (08)

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(9) Every Riemann integrable function is:
(a) differentiable (b) analytic
(c) Riemann-steltye's integrable (d) None of these.

(10) Every subset of a finite metric space is closed because:
(a) there exists no closed set
(b) you can not find any limit point of such sets.
(c) such set have no limit points (d) None of these.

(11) Interior of a set A is:
(a) smallest closed superset of A (b) proper open subset of A
(c) largest open subset of A (d) None of these.

(12) Every set is a metric space w.r.t the metric known as:
(a) indiscrete metric (b) discrete metric
(c) normable metric (d) None of these.

(13) A metric space:
(a) is always complete (b) can never be complete
(c) may be complete (d) None of these.

(14) The function $f: X \rightarrow IR$ is continuous if the metric space X is:
(a) complete (b) discrete
(c) incomplete (d) None of these.

(15) $e^{i\theta} = \cos\theta + i\sin\theta$ is called:
(a) Cauchy formula (b) Gauss formula
(c) Euler formula (d) None of these.

(16) $ln(z+\sqrt{z^2+1})$ is equal to:
(a) $\sin^{-1}z$ (b) $\cos h^{-1}z$
(c) $\sin hz$ (d) None of these.

(17) The converse of the cauchy's integral theorem is also known as:
(a) Jordan Theorm (b) Goursat Theorem
(c) Morera's Theorem (d) None of these.

(18) $1-z+\frac{z^2}{2!}-\frac{z^3}{3!}+\frac{z^4}{4!}-\ldots$ converges to:
(a) e^z (b) e^{-z}
(c) $-ze^z$ (d) None of these.

(19) $\Gamma(z+1)$ equals:
(a) $\Gamma(z)$ (b) $z^{-1}\Gamma(z)$
(c) $z\Gamma(z)$ (d) None of these.

(20) For Beta function B(m,n) is equal to:
(a) $\frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}$ (b) $\frac{\Gamma(n)\Gamma(m-n)}{\Gamma(m)}$
(c) $\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (d) None of these.

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17, UNDER THE FEDERAL GOVERNMENT, 2004

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS — MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY**. Select **TWO** questions from each **SECTION**. All questions carry **EQUAL** marks.

SECTION – I

1 (a) If G is a finite group and H is a subgroup of G, prove that the order of H is a divisor of the order of G. (10)

(b) Let G be a group, H a normal subgroup of G, T an automorphism of G. Let $T(H) = \{ T(h) : h \in H \}$. Prove that T(H) is a normal subgroup of G. (10)

2.(a) Let R be a commutative ring with unit element shoes only ideals are (0) and R itself. Prove that R is a field. (10)

(b) Let F be a finite field with q elements and suppose that FCK, where K is also a finite field with [K:F] = n. Prove that K has q^n elements. (10)

3.(a) If $S = \{x_1, x_2, \ldots\ldots\ldots x_n\}$ is a set of non zero vectors spanning a vector space V, prove that S contains a basis of V. (10)

(b) Let $T : U \to V$ be a linear transformation from an n – dimensional vector space U to a vector space V over the same field F.
If $N(T) = \{u \in U : T(u) = 0\}$ and $R(T) = \{ v \in V : T(u) = v$ for same $u \in U\}$,
Prove that dim N(T) + dim R(T) = n. (10)

4.(a) Let A be a n x n matrix. Prove that A.adj A = det A. I_n (8)

(b) Let V be a finite dimensional vector space over F, A(V) the algebra of all linear transformations V to V. For $T \in A(V)$, r (T) denotes rank of T. (12)
If $S, T \in A(V)$,
Prove:
(i) $r(ST) \leq r(T)$
(ii) $r(TS) \leq r(T)$
(iii) $r(ST) = r(TS) = r(T)$, if S maps V onto V.

SECTION-II

5. (a) Prove that the intrinsic equation of the cardiode $r = (1 - \cos\theta)$ is $8\sin^2(\psi/6)$. (10)

(b) Prove that the normal to a given curve is tangent to its evolute (10)

6. (a) Find the equations of tangent plane and the normal to the hyperboloid $x^2 - 3y^2 - z^2 + 3 = 0$ at (2, 1, -2). (10)

(b) Find the envelope of the family of planes $3a^2x - 3ay + z = a^3$, and show that its edge of regression is the curve of intersection of the surfaces $xz = y^2$, $xy = z$. (10)

7.(a) Prove that a space curve whose curvature and torsion are in a constant ratio is a helix. (10)

(b) Find the curvature and torsion of the curve (10)
$x = 3u - u^3$, $y = 3u^2$, $z = 3u + u^3$.

COMPULSORY QUESTION

(8) Write only the correct choice in the Answer Book. Do not reproduce the questions.

(1) Let G be a cyclic group of order 12. Then G has:
(a) 3 distinct subgroup (b) 4 distinct subgroup
(c) 6 distinct subgroup (d) None of these

(2) Let Q and Z be the additive groups of rationals and integers respectively. Then:
(a) The Group Q/Z is cyclic
(b) Every element of Q/Z is of infinite order
(c) Every element of Q/Z is of finite order.
(d) None of these.

(3) Suppose A,B are matrices such that the product AB exits and is zero matrix, then:
(a) A must be zero matrix (b) B must be zero matrix
(c) Neither A nor B need be zero matrix (d) None of these

(4) Let A be an n x n matrix, with rank A < n. Then:
(a) determinant A may be positive (b) determinant A must be positive
(c) determinant A may be negative (d) None of these

(5) A square matrix A such that $A^2 = A$ is called:
(a) involuntary (b) idempotent
(c) nilpotent (d) None of these

(6) Let V be the real vector space of all functions on R to R, and let A = {x, Cos x}. Then:
(a) A is linearly independent (b) A spans V
(c) A is linearly dependent (d) None of these.

(7) The additive group of integers has:
(a) 6 quotient groups of order 6 each (b) 2 quotient groups of order 3 each
(c) 1 quotient group of order 6 (d) None of these

(8) The determinant of a triangular matrix is the product of its entries on:
(a) last row (b) main diagonal
(c) first row (d) None of these.

(9) Every elementary matrix is :
(a) non singular (b) singular
(c) involuntary (d) None of these

(10) The equation $x^2 + y^2 - z^2 = 0$ represents:
(a) quadric cone (b) a hyperbolic cylinder
(c) a hyperbolic paraboloid (d) None of these

(11) Let A be matrix. Then its :
(a) row rank my be greater than its column rank.
(b) Row rank may be less than its column rank.
(c) Row rank = column rank
(d) None of these

(12) A system of m homogeneous linear equations AX = 0 in n variables has a non – trivial solution if and only if:
(a) rank A = n (b) ran A < n
(c) rank A > n (d) None of these.

(13) M_2, R denote all 2 x 2 real matrices and real numbers. Let F: $M_2 \to R$, $f(A) =$ det A, for $A \in M_2$. Then:
(a) f is onto R (b) f is one-to-one
(c) f is neither onto nor one-to-one (d) None of these

(14) If J_n denotes the ring of integers mod n, then:
(a) J_7 is a field (b) J_6 is a field
(c) J_8 is an integral domain (d) None of these

(15) The rectangular coordinates of appoint with spherical coordinates $(3, \frac{\pi}{6}, \frac{\pi}{4})$ are:
(a) (3, 1, -2) (b) $\left(\frac{3\sqrt{6}}{4}, \frac{3\sqrt{2}}{4}, \frac{3\sqrt{2}}{2}\right)$
(c) $\left(\sqrt{3}, \frac{1}{2}, 2\right)$ (d) None of these

(16) The distance of the point (3, 2, 3) from the plane 2x + 3y – z = 5 is :
(a) $\frac{5}{\sqrt{14}}$ (b) $\frac{3}{\sqrt{14}}$ (c) $\frac{4}{\sqrt{14}}$ (d) None of these

(17) Monge's form of the equation of a surface is:
(a) $f(x, y, z) = 0$ (b) $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$
(c) $z = f(x, y)$ (d) None of these

(18) The only space curve whose curvature and torsion are both constant is:
(a) a parabola (b) a circular helix
(c) a circle (d) None of these

(19) If torsion is zero at all points of a curve, the curve is:
(a) a helix (b) a straight line
(c) all on one plane (d) None of these

(20) Let G be a group of order 13. Then:
(a) G is non cyclic (b) G is non abelian
(c) G is commutative (d) None of these.

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2004.

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY.** Select TWO questions from each of the **SECTIONS I AND II**. All questions carry **EQUAL** marks.

SECTION – I

1. (a) Evaluate: $\lim\limits_{x \to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$ (8)

(b) If f is continuous on [a,b], f ′exists on [a,b] and f (a) =f(b), prove that there is a point C in (a,b) such that $f'(c) = 0$. (8)

(c) Find the inclined asymptotes of the curve $x^3 - y^3 + 6xy = 0$. (4)

2. (a) Evaluate $\iint_D xy^2\, dxdy$, where D is the region bounded by the x-axis, the ordinate at x = 4 and arc of the parabola $x^2 = 8y$. (6)

(b) If f (x,y) is continuously differentiable and homogeneous of degree n in a region R, prove that $x f_x(x,y) + y f_y(x,y) = n f(x,y)$. (6)

(c) Find all the maxima and minima of $f(x,y) = x^3 + y^3 - 63(x + y) + 12xy$ (8)

3. (a) Show that the function f in [0,1], where
$f(x) = 1$, x is irrational
$= 0$, x is rational, is not Riemann –integral (6)

(b) Prove that:

$$\int_0^{\pi/2} \ln \cos x\, dx = -\frac{\pi}{2} \ln 2 \qquad (6)$$

(c) Prove that $\int_{\pi}^{0} \frac{\sin x}{x} dx$ converges. (8)

4. (a) Prove that every compact subset of a metric space is closed. (8)

(b) Set Q be the space of all rational numbers with metric $d(x,y) = |x - y|$ for x, y in Q. Show that Q is not complete. (6)

(c) Prove that $\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ is a number e, such that $2 < e < 3$. (6)

SECTION II

5. (a) Let $x_n + iy_n = (1 + i)^n$, n is a positive integer Using DeMoivre's theorem, Prove:

(i) $x_{2n}^2 + y_{2n}^2 = 4^n$ (5)

(ii) $X_{n-1} y_n - X_n y_{n-1} = 2^{n-1}$ (5)

(b) Let $f(z) = u(x,y) + iv(x,y)$ be analytic in a domain D. Using Cauchy – Riemann Conditions, Prove:

$$\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial x}|f(z)|\right]^2 = |f'(z)|^2 \text{ for all z in D.} \qquad (10)$$

6. (a) Let C be a circle with center Z_0 and radius r and let f be analytic in an open set D containing C and its interior. Prove: (8)

$$\left|f^{(n)}(z_0)\right| \le \frac{M\,n!}{r^n} \quad (n = 0,1,2, \ldots\ldots\ldots)$$

where M is the least upper bound of $|f(z)|$ on C.

(b) Show that $\int_c \frac{e^{3z} + 3\operatorname{Cosh} z}{\left(z - i\frac{\pi}{2}\right)^4}\,dz = 8\pi$ (6)

where C is a simple closed contour containing $i\frac{\pi}{2}$ in its interior, and the integral is in the positive direction.

(c) Find the Laurent series expansion, in powers of z, for $\frac{1}{(z-1)(z-3)}$ in the annulus $1 < |z| < 3$. (6)

7. (a) Find the residues of $\frac{\operatorname{Cos} hz}{z^2(z+i\pi)^3}$ at its poles. (10)

(b) Use the method of residues to evaluate $\int_c \frac{e^z\,d^z}{\operatorname{Sin} hz}$, where C is the circle $|z| = 4$ in the positive direction. (10)

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Do not reproduce the question.

(1) The set of all ....number forms a sequence:
(a) Real (b) Rational (c) Irrational (d) None of these

(2) $f(X) = x$, x rational $= 0$, x irrational in $[0,1]$:
(a) f is discontinuous at $x = \frac{1}{2}$ (b) f is discontinuous at $x = 0$
(c) f is continuous at $x = \frac{1}{3}$ (d) None of these

(3) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is converges for:
(a) $p = 1$ (b) $p = \frac{1}{2}$ (c) $p > 1$ (d) None of these

(4) $\Gamma\left(\frac{1}{2}\right)$ equals to:
(a) π (b) $\sqrt{\pi}$ (c) $\frac{1}{2}$ (d) None of these

(5) If f is homogeneous of degree n, $x f_x(x,y) + y f_y(x,y) = n\, f(x,y)$ is called:
(a) Lagrange's formula (b) Euler's formula
(c) Goursat's formula (d) None of these

(6) Every function $X \to Y$ between metric spaces is continuous if:
(a) X is discrete (b) Y is complete
(c) X is complete (d) None of these

(7) If each f_n is continuous and $f_n \to f$ Uniformly on E, then:
(a) f is differentiable on E (b) f is continuous on E
(c) f is discontinuous on E (d) None of these

(8) Every real –valued continuous function on open interval (0,1) is:
(a) bounded (b) Unbounded (c) monotonic (d) None of these

(9) When n is large, $n! = \sqrt{2\pi n}\ n^n\ e^{-n}$ is called:
(a) Hermite's formula (b) Stirling's formula
(c) Euler's formula (d) None of these

(10) $\Gamma(x)\ \Gamma(1-x) = \dfrac{\pi}{\text{Sin } n\pi}$, for:
(a) $0 < x < 1$ (b) $x = 1,2,3,4$..............
(c) $x = \frac{1}{2}$ only (d) None of these

(11) If $\sum_{n=1}^{\infty} A_n$ converges absolutely to A, then any rearrangement of the series:
(a) diverges (b) Converges but not necessary to A
(c) Converges absolutely to A (d) None of these

(12) Every Riemann integrable function is:
(a) Continuous (b) differentiable
(c) monotonic (d) None of these

(13) Every compact metric space is:
(a) discrete (b) complete
(c) Infinite (d) None of these

(14) The set of all points z satisfying $|(z-1)| + |z+1| = 4$ lies on:
(a) a circle (b) a parabola
(c) an ellipse (d) None of these

(15) Let $\sum_{n=1}^{\infty} Z_n$ be a series of Complex numbers:
(a) if $\lim_{n\to\infty} Z_n = 0$ then series converges to zero
(b) if the series converges, then $\lim_{n\to\infty} Z_n = 0$
(c) if the series converges, it converges absolutely
(d) None of these

(16) $(-i)^i$ equal to:
(a) $e^{\pi/2}$ (b) i (c) $\pi/2$ (d) None of these

(17) If C is the circle $|z| = 1$, $\int_c \dfrac{\text{Sin } z\,dz}{z^2+4}$ equals to:
(a) 1 (b) 0 (c) $2\pi i$ (d) None of these

(18) Log (-1-i) equal to:
(a) $1/2 \log z - i\,\frac{3\pi}{4}$ (b) $1/2 \log z + i\,\frac{3\pi}{4}$
(c) $-1/2\ \log z - i\,\frac{3\pi}{4}$ (d) None of these

(19) $f(z) = y + i\,x$ is:
(a) Analytic inside the circle $|z| = 1$ (b) Not analytic in any domain
(c) Is analytic everywhere. (d) None of these

(20) Every meromorphic function of Z is:
(a) monogeric (b) holomorphic
(c) has only poles as singularities (d) None of these

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS–17, UNDER THE FEDERAL GOVERNMENT, 2005

PURE MATHEMATICS, PAPER–I

TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO.8** which is **COMPULSORY**. Select **TWO** question from each **SECTION**, all questions carry **EQUAL** marks.

SECTION - I

1- (a) If f is a homomorphism of a group G into a group G with kernel K,prove that k is a normal subgroup of G. 10

(b) If G is a group, then A(G), the Set of all automorphisms of G,is also a group. 10

2- (a) If D is a commutative integral domain with unity and has finite characteristic n, prove that n is prime number. 08

(b) If R in a commutative ring with unity and M is an ideal of R, prove that M is a maximal ideal of R if and only if R/M is a field. 12

3- (a) If W is a subspace of finite-dimensional vector space V, prove that $\dim W \leq \dim V$ *and* $\dim V/W = \dim V - \dim W$. 08

(b) Let A be an $n \times n$ matrix prove that det A=0 if and only if rank A is less than n. 12

4- (a) For what values of K the equations 08

$$(5-K)x_1 + 4x_2 + 2x_3 = 0$$
$$4x_1 + (5-k)x_2 + 2x_3 = 0$$
$$2x_1 + 2x_2 + (2-k)x_3 = 0$$

have non-trivial solutions. Find the solutions.

(b) Let V be finite dimensional vector space over a field F and A(V) the algebra of linear Transformations on V. prove that $\lambda \in F$ is an eigen value of $T \in A(V)$ if and only if $vT = \lambda v$ for some $v \neq 0$ in V. 12

Contd......

SECTION – II

5- (a) Find the pedal equation of the cardiode $r = a(1+\cos\theta)$. 10

(b) Find the center of curvature of the parabola $x^2 = 4y$ at the point (4,1). 10

6- (a) Find the volume of a tetrahedron whose vertices are (1,-1,2), (2,0,1) (0,-2,1) and (-2,2,1) 10

(b) The normal at a point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the coordinates planes in G_1, G_2, G_3 respectively. Prove that the ratios $PG_1 : PG_2 : PG_3$ are constant. 10

7- (a) Define the involute and evolute of a space curve. Prove that the tangent to the involute is parallel to the principal normal to the given cutve. 10

(b) If the curve of intersection of two surfaces is a line of curvature on both, prove that the surfaces cut at a constant angle. 10

8- (a) Write only the correct choice in the answer book. Do not reproduce the less than n.

(i) The additive group of all rational number is:

(a) Torsion free
(b) Finitely generated
(c) Cyclic
(d) None of these.

(ii) Every group of order 25 must be

(a) Cyclic
(b) Nonabelian
(c) Abelian
(d) None of these.

(iii) The order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 5 & 1 \end{pmatrix}$ is

(a) 5
(b) 6
(c) 7
(d) None of these.

Contd......

(x) The equation of the surface of revolution obtained by rotating the curve $x^2+2y^2=8$, $z=0$ about y axis is.

(a) $x^2+2y^2+2z^2=8$
(b) $x^2+2y^2+z^2=8$
(c) $x^2+2z^2=8$
(d) None of these.

(xi) If the tangent at a point P on a parabola meets the directrix in K, then angle KSP (S focus) is

(a) Right angle
(b) Straight angle
(c) Obtuse angle
(d) None of these

(xii) The sum of the focal distances of a point P on an ellipse is

(a) Variable with P
(b) Greatest when P is at an end of major axis
(c) Constant
(d) None of these.

(xiii) If field F has finite order q, then for every $a \in F$,

(a) $a^{q-1}=0$
(b) $a^q=a$
(c) $a^q=0$
(d) None of these.

(xiv) Let A be an n x n matrix. Then det A=0 if and only if

(a) Rank $A < n$
(b) Rank A > n
(c) Rank A=n
(d) None of these.

(xv) A square matrix A such that $A^n=0$ for some positive integer n is called

(a) Idempotent
(b) Involutory
(c) Nilpotent
(d) None of these.

Contd......

(iv) If a group G has finite order divisible by n then

(a) G contains a subgroup of order n
(b) G contains an element of order n
(c) G need not contain an element of order n.
(d) None of these.

(v) The multiplicative group of non zero elements of a finite field is

(a) Of prime order
(b) Of prime power order
(c) Cyclic
(d) None of these.

(vi) The envelope of the normal plane of a twisted curve is called ________developable

(a) Osculating.
(b) Polar.
(c) Rectifying.
(d) None of these.

(vii) The Gauss curvature of a surface at any point is the ________ of the principal curvatures.

(a) Difference
(b) Sum
(c) Product
(d) None of these.

(viii) The theorem Kn = Kcos θ connecting normal curvature in any direction with the curvature of any other section through the same tangent is called.

(a) Meunier's theorem.
(b) Euler's theorem.
(c) Dupin's theorem.
(d) None of these.

(ix) The envelope of the family $x^2 + y^2 - 4az + 4a^2 = 0$ is

(a) $x^2 + y^2 = yx$
(b) xyz = 1
(c) $x^2 + y^2 = z^2$
(d) None of these.

Contd......

(xvi) Let $f: V \to W$ be a linear map where v is finite-dimensional, then

(a) dim W=dim V + dim (Kerf)
(b) dim V=dim(Kerf) + dim (imf)
(c) dim V = dim W+dim (imf)
(d) None of these.

(xvii) The perpendicular distance of the point (2,2,1) from the line

$\frac{x-1}{2} = \frac{y+1}{3} = Z$ *is*

(a) 2
(b) 3
(c) $\sqrt{\frac{5}{7}}$
(d) None of these.

(xviii) The cylindrical coordinates of a point with spherical polar coordinates $\left(3, \frac{\pi}{6}, \frac{\pi}{4}\right)$ are

(a) $\left(\frac{3}{\sqrt{2}}, \frac{\pi}{6}, \frac{3}{\sqrt{2}}\right)$
(b) $\left(\frac{3}{4}, \frac{\sqrt{2}}{3}, 6\right)$
(c) $\left(2, \frac{\pi}{2}, 1\right)$
(d) None of these.

(xix) A set of 4 vectors in a 3-dimensional vector space must be

(a) Linearly independent
(b) A basis
(c) Linearly dependent
(d) None of these

(xx) If A,B are matrices such that AB exists and is the zero matrix, then

(a) A must be zero matrix.
(b) B must be zero matrix.
(c) Neither A nor B need be zero matrix
(d) None of these.

FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS–17, UNDER THE FEDERAL GOVERNMENT, 2005

PURE MATHEMATICS, PAPER–II

TIME ALLOWED: THREE HOURS — MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO.8** which is **COMPULSORY**. Select **TWO** question from each **SECTION**, all questions carry **EQUAL** marks.

SECTION - I

1 (a) Evaluate : $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{-2x}}{\ln(1+x)}$ 06

(b)If f is continuous on [a,b] and differentiable on (a,b), then there exist a number C in (a,b) such that f(b) - f(a) = b-a f' (c). 06

(c) If $\sum$ an converges absolutely, prove that $\sum$ an converges. Give an example to show that the converse is not true. 08

2 (a) By evaluating both repeated integrals show that: 08

$$\int_0^1\left[\int_0^1\frac{x-y}{(x+y)^3}dy\right]dx \neq \int_0^1\left[\int_0^1\frac{x-y}{(x+y)^3}dx\right]dy$$

(b) Find the whole length of the cardioid r = a (1+sinθ). 06

(c) Let $\sum\limits_{n=1}^{\infty} Mn$ be a convergent series of positive term, and let | f_n(x) |≤ M_n for all x in [a,b] and all n. prove that $\sum\limits_{x=1}^{\infty} fn(x)$ converges uniformly in [a,b]. 06

3 (a) Let f be Riemann integrable on [a,b] and let, F(x) = 10
Prove that F is Continuous on[a,b].if f is continuous at a point c in (a,b), prove that $f'(c)=f(c)$.

(b) let $f(x,y)=\dfrac{xy}{\sqrt{x^2+y^2}}$ $when (x,y)\neq(0,0) and f(0,0)=0$ 10

prove that f is continuouse possesses partial dervatives but is not differentiable at (0,0).

Contd......

PURE MATHEMATICS, PAPER-II:

4 (a) Let x,y be metric spaces, $f: x \to y$ a function and $C \in X$. 10
Prove that f is continous at C if and if $\lim_{n\to\infty} f(xn) = f(C)$
whenever (x_n is a sequence in x coverging to C.

(b) let (x,d) be a metric space. Define the term:Cauchy sequence and completeness. 10
Prove that if (x,d) is complete and A is a closed subset then (A,d) is also complete.
If A is a compact subset of X, is (A,d) complete ? justify your answer.

Section –II

5 (a) Use De Moivre's Theorm to prove that 08

$$\sum_{k=0}^{8} \cos(\frac{2k\pi}{9}) = 0$$

(b) Let $f(Z) = \begin{cases} 0, & z = 0 \\ u(x,y) + iv(x,y), & z \neq 0, \end{cases}$ 12

$where\ u(x,y) = (x^3 - y^3)/(x^3 + y^2)$
$v(x,y) = (x^3 + y^3)/(x^2 + y^2)$
Show that the cauchy-Riemann equations are satisfied at the origin
but $f'(0)$ does not exit.

6 (a) State and prove Liouville's theorem. 06

(b) Use cauchy integral formula to evaluate 08
$\int_c \frac{9z^2 - iz + 4}{z(z^2+1)} dz$, where c is the circle |z|=2 in the positive direction.

(c) Use Taylor's series, prove that: 06

$$\frac{1}{z^2} = \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n (n+1)(\frac{z-2}{2})^n \ for\ (|z-2|<2).$$

7 (a) Find the residues of tan z at its poles. 10

(b) Use the method of residues to evaluate $\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta - 3\theta)d\theta$ 10

Contd......

(vii) $f(x)=\frac{\sin x}{x}, x\in(0,\frac{\pi}{2})$, is

(a) strictly increasing
(b) strictly decreasing
(c) unbounded.
(d) None of these

(iii) $\lim_{n\to\infty}(1+\frac{x}{n})^n$ *equals*

(a) 1
(b) Dose not exit
(c) e^x
(d) None of these.

(ix) Suppose $f(x)=\sum_{n=0}^{\infty}Cnx^n$, where the series is convergent for all |x|<R. then f is

(a) continous but not differentiable
(b) differentiable
(c) monotonic
(d) None of these.

(x) The interval (0,1) is

(a) A countable set.
(b) A copmpact set
(c) An uncountable set
(d) None of the above.

(xi) Let e = $\lim_{n\to\infty}(1+\frac{1}{n})^n$ then e is

(a) Rational
(b) Irrational
(c) Algebraic
(d) None of these

(xii) The series $\sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}$ *is*

(a) convergent
(b) oscillating
(c) divergant
(d) None of these

(xiii) the function $f(z)=z^2e^{-z}$ is

(a) Entire .
(b) meromorphic
(c) bounded
(d) None of these.

Contd……

PURE MATHEMATICS, PAPER-II:

COMPULSORY QUESTION

8. write only the correct choice in the answer book. Do not reproduce the question.

(i) for all real number a, Limit a/n equals
- (a) 0
- (b) ∝
- (c) 1
- (d) none of these.

(ii) the series $\sum_{x=1}^{\infty}\frac{(-1)^x}{x}$ *is*
- (a) divergent.
- (b) Convergent.
- (c) Absolutely convergent.
- (d) None of these.

(iii) If f is Riemanns integrable on [a,b], the f must be
- (a) Continous on [a,b].
- (b) Differentiable on [a,b].
- (c) Monotonic on [a,b].
- (d) None of these.

(iv) Every closed subset of R, the real line, is
- (a) Complete.
- (b) Compact.
- (c) Bounded.
- (d) None of these.

(v) The series $\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$ *is*

Convergent but not absolutely.
- (a) Absolutely convergent.
- (b) Divergent
- (c) None of these.

(vi) $\lim_{x\to\infty} x^n e^{-x}$ $(x = 1,2,3,......)$ equals
- (a) 0
- (b) 1
- (c) ∝
- (d) None of these.

Contd......

(xiv) The converse of cauchy integral theorem is known as

(a) Goursat theorm
(b) Morera theorm
(c) Cauchy's inequality.
(d) None of these.

(xv) A simple closed curve divides the complex palne into__________ disjoint domains

(a) Two.
(b) Three
(c) Four
(d) None of these

(xvi) If a series of complex numbers $\sum_{n=1}^{\infty} z_n$ converges, then $\lim_{n\to\alpha}(-1)^n zn$ is

(a) -1
(b) Zero
(c) 1
(d) None of these.

(xvii) $\lim_{n\to\infty}\frac{(n-i)^3}{2n^3+n+2}$ equals

(a) $\propto$
(b) $\frac{1}{2}$
(c) zero
(d) None of the

(xviii) Log (-1+i) = 1/2log2 + iQ, where Q equals
(a) ¾
(b) -3/4
(c) -1/4
(d) none of these

(xix) Every compact subset of the complex plane is

(a) Open.
(b) Closed and bounded.
(c) Open and unbounded.
(d) None of these.

(xx) If z is not an integer, then $\pi(z)\pi(1-z)$ equals
(a) π
(b) $z\pi(z)$
(c) $\frac{\pi}{\sin \pi z}$
(d) None of these.